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A TEXTBOOK

ON

MECHANICAL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

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MECHANICAL DRAWING

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GEOMETRICAL DRAWING

INSTRUMENTS AND MATERIALS

1. A **drawing** is a representation of objects on a plane surface by means of lines or lines and shades. When done by the use of free hand only, it is called **freehand drawing** or **sketching**; when instruments are used, so that greater exactness may be obtained, it is called **instrumental, or mechanical, drawing.**

2. All the instruments and materials required for the courses in drawing are mentioned in the following descriptions:

The **drawing board** should be made of well-seasoned, straight-grained pine, the grain running lengthwise. For this Course, the student will need a board of the following dimensions: length over all, 22½ inches; width, 16½ inches.

The drawing board illustrated in Fig. 1 is the one furnished in our students' drawing outfits and can be fully recommended as possessing the qualities a good and accurate board should have. It is made of several pieces of pine wood glued together to the required width of the board. A pair of hardwood cleats is screwed to the back of the board, the screws passing through the cleats in oblong slots with iron bushings, which allow the screws to move freely when drawn by the contraction and expansion of the board. Grooves are cut through half the thickness of the board over the entire back side. These grooves take the transverse resistance out of the wood and allow it to be controlled

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by the cleats, at the same time leaving the longitudinal strength nearly unimpaired. In order to provide a perfectly smooth working edge for the head of the **T** square to slide against a strip of hard wood is let into the short edges

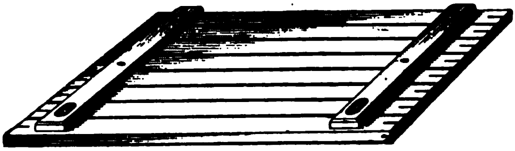

FIG. 1

of the board, and is sawed through in several places, in order to allow for the contraction and expansion of the board. The cleats also raise the board from the table, thus making it

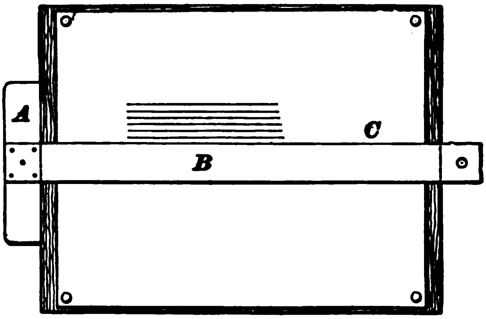

FIG. 2

easier to change the position of the board. When in use, the board is placed so that one of the short edges is at the left of the draftsman, as shown in Fig. 2.

3. The **T square** is used for drawing horizontal straight lines. The head *A* is placed against the left-hand edge of the board, as shown in Fig. 2. The upper edge *C* of the blade *B* is brought very near to the point through which it is desired to pass a line, so that the straight edge *C* of the blade may be used as a guide for the pen or pencil. It is evident that all lines drawn in this manner will be parallel.

Vertical lines are drawn by means of triangles. The triangles most generally used are shown in Figs. 3 and 4, each of which has one right angle. The triangle shown in Fig. 3 has two angles of 45° each, and that in Fig. 4 one of 60° and one of 30°. They are called *45° and 60° triangles*, respectively. To draw a vertical line, place the **T** square in position to draw a horizontal line, and lay the triangle against it, so as to form a right angle. Hold both **T** square and triangle lightly with the left hand, so as to keep them from slipping, and draw the line with the pen or pencil held in the right hand, and against the edge of the triangle. Fig. 5 shows the triangles and **T** square in position.

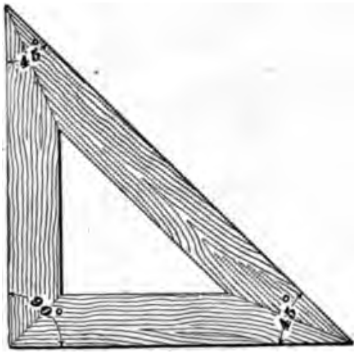

FIG. 3

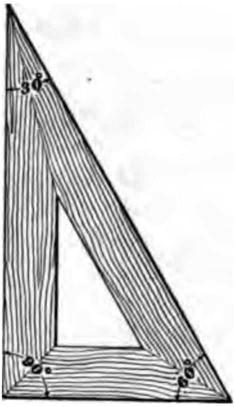

FIG. 4

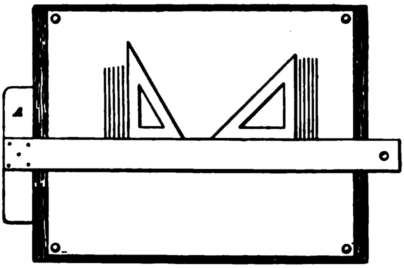

FIG. 5

4. For drawing parallel lines that are neither vertical nor horizontal, the simplest and best way, when the lines are near together, is to place one edge of a triangle, as *a b*, Fig. 6, on the given line *c d*, and lay the other triangle, as *B*, against one of the two edges, holding it fast with the left hand; then move the triangle *A* along the edge of *B*. The edge *a b* will be parallel to the line *c d*; and when the edge *a b* reaches the point *g*, through which it is desired to draw the parallel line, hold both triangles stationary with the left hand and draw the line *e f* by passing the pencil along the edge *a b*. Should the triangle *A* extend too far beyond the edge of the triangle *B* after a number of lines have been drawn, hold *A* stationary with the left hand and shift *B* along the edge of *A* with the right hand and then proceed as before.

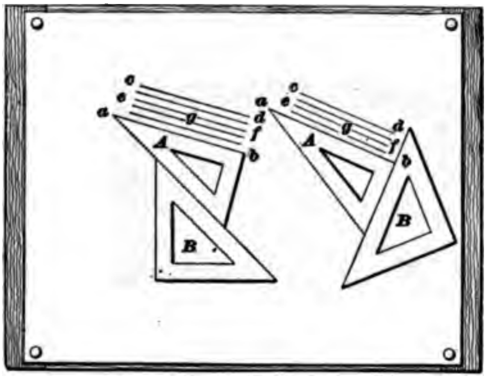

FIG. 6

5. A line may be drawn at right angles to another line which is neither vertical nor horizontal, as illustrated in Fig. 7. Let *c d* be the given line (shown at the left-hand side). Place one of the shorter edges, as *a b*, of the triangle *B* so that it will coincide with the line *c d*; then, keeping the triangle in this position, place the triangle *A* so that its long edge will come against the long edge of *B*. Now, holding *A* securely in place with the left hand, slide *B* along the edge of *A* with the right hand, when the lines *h i*, *m n*, etc. may be drawn perpendicular to *c d* along the edge *b f* of the triangle *B*. The dotted lines show the position of the triangle *B* when moved along the edge of *A*.

6. The right-hand portion of Fig. 7 shows another method of accomplishing the same result, and illustrates how the triangles may be used for drawing a rectangular figure, when the sides of the figure make an angle with the **T** square such that the latter cannot be used.

Let the side *c d* of the figure be given. Place the *long* side of the triangle *B* so as to coincide with the line *c d*, and bring the triangle *A* into position against the lower side of *B*, as shown. Now, holding the triangle *A* in place with the left hand, revolve *B* so that its other short edge will rest against the long edge of *A*, as shown in the dotted position at *B'*. The parallel lines *c e* and *d f* may now be drawn

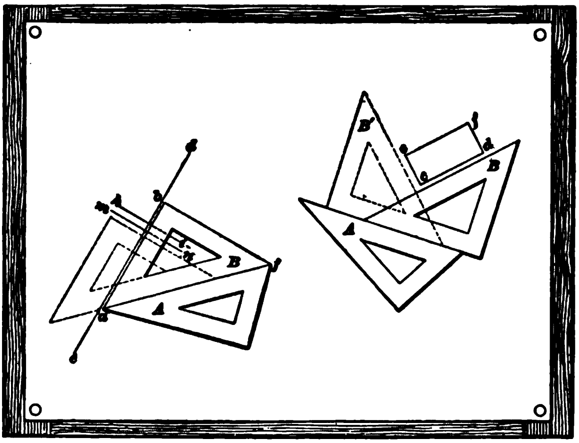

FIG. 7

through the points *c* and *d* by sliding the triangle *B* on the triangle *A*, as described in connection with Fig. 6. Measure off the required width of the figure on the line *c e*, reverse the triangle *B* again to its original position, still holding the triangle *A* in a fixed position with the left hand, and slide *B* upon *A* until the long edge of *B* passes through *e*. Draw the line *e f* through the point *e*, and *e f* will be parallel to *c d*. The student should practice with his triangles before beginning drawing.

7. The **compasses**, next to the **T** square and triangles, are used more than any other instrument. A pencil and pen point are provided, as shown in Fig. 8, either of which may be inserted into a socket in one leg of the instrument, for the drawing of circles in pencil or ink. The other leg is fitted with a needle point, which acts as the center about which the circle is drawn. In all good instruments, the needle point itself is a separate piece of round steel wire, held in place in a socket provided at the end of the leg. The wire should have a square shoulder at its lower end, below which a fine, needle-like point projects. The *lengthening bar*, also shown in the figure, is used to extend the leg carrying the pen and pencil points when circles of large radii are to be drawn.

The joint at the top of the compasses should hold the legs firmly in any position, and at the same time should permit their being opened or closed with one hand. The joint may be tightened or loosened by means of a screwdriver or wrench, which accompanies the compasses.

It will be noticed in Fig. 8 that each leg of the compasses is jointed; this is done so that the compass points may always be kept perpendicular to the paper when drawing circles, as in Fig. 11.

The style of compasses shown in Fig. 8 have what is called a *tongue joint*, in which the head of one leg has a tongue, generally of steel, which moves between two lugs on the other leg. Another common style of joint is the *pivot joint*, in which the head of each leg is shaped like a disk and the two disks are held together in a fork-shaped brace either by means of two pivot screws or by one screw penetrating both disks.

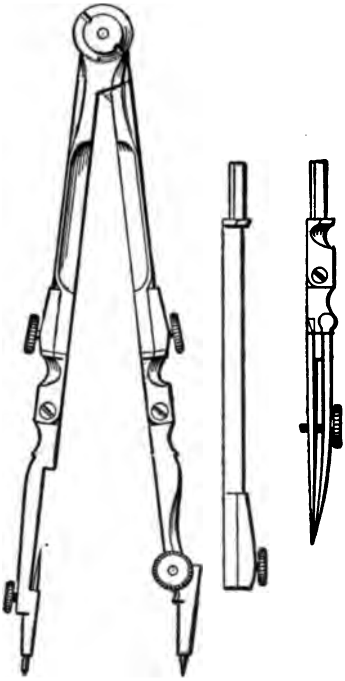

FIG. 8

The brace that forms a part of this joint is generally provided with a handle, as the shape of the joint makes it rather awkward to hold the compasses by the head, as is usual with instruments provided with tongue joints. In Fig. 9 is shown a common style of pivot joint.

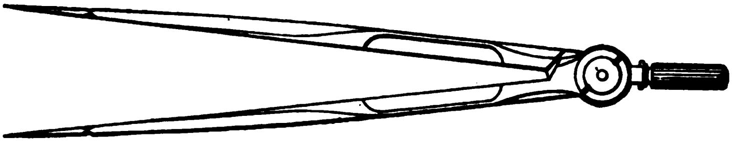

FIG. 9

8. The following suggestions for handling the compasses should be carefully observed by those who are beginning the subject of mechanical drawing. Any draftsman who handles his instruments awkwardly will create a bad impression, no matter how good a workman he may be. The tendency of all beginners is to use both hands for operating the compasses. This is to be avoided. The student should learn at the start to open and close them with one hand, holding them as shown in Fig. 10, with the needle-point leg resting between the thumb and fourth finger, and the other leg between the middle and forefinger. When drawing circles,

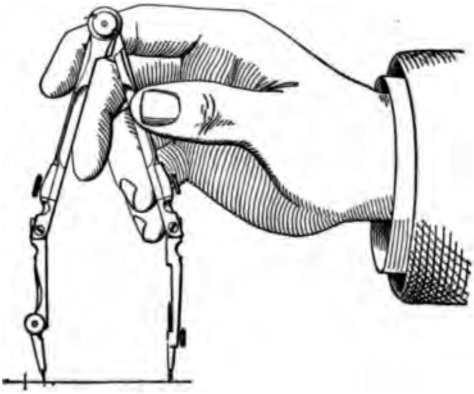

FIG. 10

hold the compasses lightly at the top between the thumb and forefinger, or thumb, forefinger, and middle finger, as in Fig. 11. Another case where both hands should not be used is in locating the needle point at a point on the drawing about which the circle is to be drawn, unless the left hand is used merely to steady the needle point. Hold the compasses as shown in Fig. 10, and incline them until the under side of the hand rests upon the paper. This will steady the hand so that the needle point can be brought to exactly the right place on the drawing. Having placed the needle at the desired point, and with it still resting on the paper, the pen or pencil point may be moved out or in to any desired radius, as indicated in Fig. 10. When the lengthening bar is used, both hands must be employed.

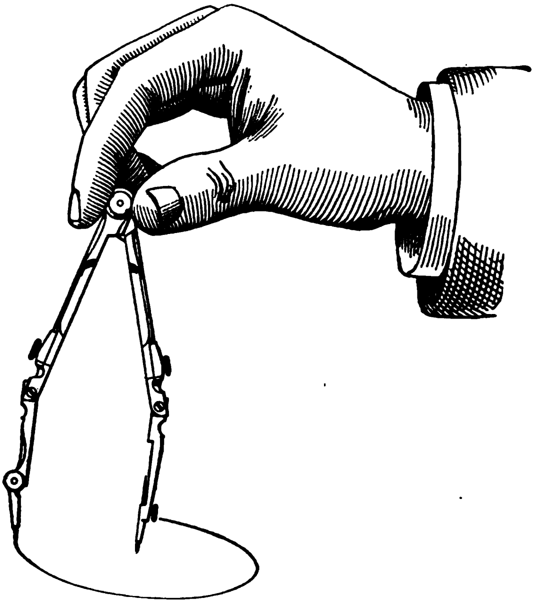

FIG. 11

9. The compasses must be handled in such a manner that the needle point will not dig large holes in the paper. **Keep**

the needle point adjusted so that it will be perpendicular to the paper, when drawing circles, and *do not bear upon it.* A slight pressure will be necessary on the pen or pencil point, *but not on the needle point.*

10. The **dividers**, shown in Figs. 9 and 12, are used for laying off distances upon a drawing, or for dividing straight lines or circles into parts. The points of the dividers should be *very* sharp, so that they will not punch holes in the paper larger than is absolutely necessary to be seen. Compasses are sometimes furnished with two steel divider points, besides the pen and pencil points, so that the instrument may be used either as compasses or dividers. This is the kind illustrated in Fig. 12. When using the dividers to space a line or circle into a number of equal parts, hold them at the top between the thumb and forefinger, as when using the compasses, and step off the spaces, turning the instrument alternately to the right and left. If the line or circle does not space exactly, vary the distance between the divider points and try again; so continue until it is spaced equally. When spacing in this manner, great care must be exercised not to press the divider points into the paper; for, if the points enter the paper, the spacing can never be accurately done. The student should satisfy himself of the truth of this statement by actual trial.

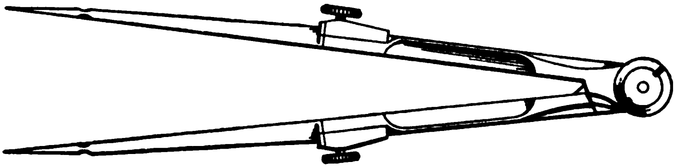

FIG. 12

11. The **bow-pencil** and **bow-pen**, shown in Fig. 13, are convenient for describing small circles. The two points of the instruments must be adjusted to the same length; otherwise, very small circles cannot be drawn. To open or close either of these instruments, support it in a vertical position by resting the needle point on the paper and bearing slightly on the top of it with the forefinger of one hand, and turn the adjusting nut with the thumb and middle finger of the same hand.

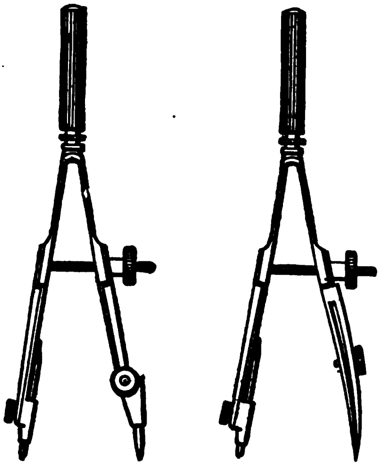

FIG. 13

12. Drawing Paper and Pencils.—The drawing paper recommended for this series of lessons is T. S. Co.'s cold-pressed demy, the size of which is 15" × 20". It takes ink well and withstands considerable erasing. The paper is secured to the drawing board by means of **thumbtacks.** Four are usually sufficient — one at each corner of the sheet (see Fig. 7). Place a piece of paper on the drawing board, and press a thumbtack through one of the corners about ¼ or ⅜ of an inch from each edge. Place the **T** square in position for drawing a horizontal line, as before explained, and straighten the paper so that its upper edge will be parallel to the edge of the **T**-square blade. Pull the corner diagonally opposite that in which the thumbtack was placed, so as to stretch the paper slightly, and push in another thumbtack. Do the same with the remaining two corners. For drawing in pencil, an HHHH pencil of any reputable make should be used The pencil should be sharpened as shown at *A*, Fig. 14. Cut the wood away so as to leave about ¼ or ⅜ of an inch of the lead projecting; then sharpen it flat by rubbing it against a fine file or a piece of

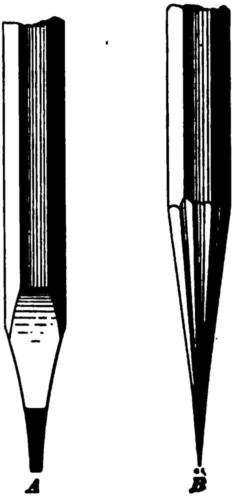

FIG. 14

fine emery cloth or sandpaper that has been fastened to a flat stick. Grind it to a sharp edge like a knife blade, and round the corners very slightly, as shown in the figure. If sharpened to a round point, as shown at *B*, the point will wear away very quickly and make broad lines; when so sharpened it is difficult to draw a line exactly through a point. The lead for the compasses should be sharpened in the same manner as the pencil, but should have its width narrower. ***Be sure that the compass lead is so secured that when circles are struck in either direction, but one line will be drawn with the same radius and center.***

13. Inking.—For drawing ink lines other than arcs of circles, the **ruling pen** (or *right-line pen*, as it is sometimes called) is used. It should be held as nearly perpendicular to the board as possible, with the hand in the position shown in Figs. 15 and 16, bearing lightly against the **T** square or triangle, along the edge of which the line is drawn. After a little practice, this position will become natural, and no difficulty will be experienced.

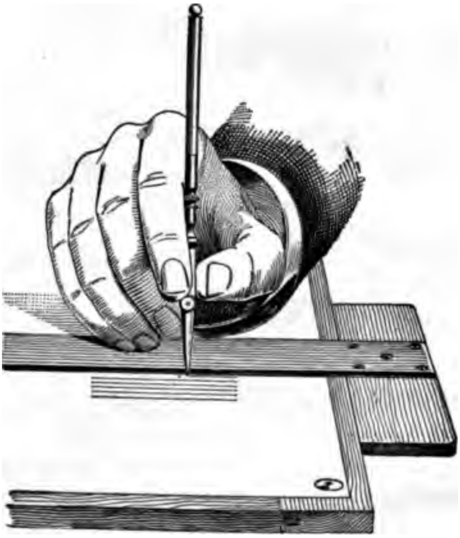

FIG. 15

14. The beginner will find that it is not always easy to make smooth lines. If the pen is held so that only one blade bears on the paper when drawing, the line will almost invariably be ragged on the edge where the blade does not bear. When held at right angles to the paper, as in Fig. 16, however, both blades will rest on the paper, and if the pen is in good condition, smooth lines will result. The pen must not be pressed against the edge of the **T** square or triangle, as the blades will then close together, making the line uneven. The edge should serve as a guide simply.

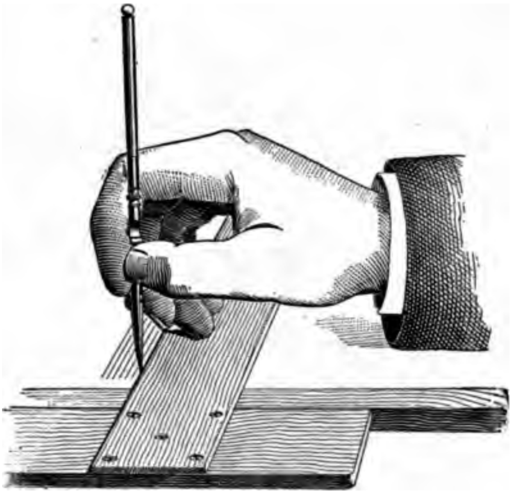

FIG. 16

In drawing circles with the compass pen, the same care should be taken to keep the blades perpendicular to the paper by means of the adjustment at the joint. In both the ruling pen and compass pen, the width of the lines can be altered by means of the screw which holds the blades together. The handles of most ruling pens can be unscrewed, and are provided with a needle point intended for use when copying maps by pricking through the original and the underlying paper, thus locating a series of points through which the outline may be drawn.

15. Drawing Ink.—The ink we recommend for the work in this Course is the T. S. Co.'s superior waterproof liquid India ink. A quill is attached to the cork of every bottle of this ink, by means of which the pen may be filled. Dip the quill into the ink and then pass the end of it between the blades of the drawing pen. Do not put too much ink in the pen, not more than enough to fill it for a quarter of an inch along the blades, otherwise the ink is liable to drop. Many draftsmen prefer to use stick India ink; and for some purposes this is to be preferred to the prepared liquid ink recommended above. In case the stick ink is bought, put enough water in a shallow dish (a common individual butter plate will do) to make enough ink for the drawing, then place one end of the stick in the water, and grind by giving the stick a circular motion. Do not bear hard upon the stick. Test the ink occasionally to see if it is black. Draw a fine line with the pen and hold the paper in a strong light. If it shows brown (or gray), grind a while longer, and test again. Keep grinding until a fine line shows *black*, which will usually take from fifteen minutes to half an hour, depending on the quantity of water used. The ink should always be kept well covered with a flat plate of some kind, to keep out the dust and prevent evaporation. The drawing pen may be filled by dipping an ordinary writing pen into the ink and drawing it through the blades, as previously described when using the quill. If liquid ink is used, all the lines on all the drawings will be of the same color, and no time will be lost in grinding. If stick ink is used, it is poor economy to buy a cheap stick. A small stick of the best quality, costing, say, a dollar, will last as long, perhaps, as five dollars' worth of liquid ink. The only reason for using liquid ink is that all lines are then sure to be of equal blackness and time is saved in grinding.

India ink will dry quickly on the drawing, which is desirable, but it also causes trouble by drying between the blades and refusing to flow, especially when drawing fine lines. *The only remedy is to wipe out the pen frequently* with a cloth. Do not lay the pen down for any great length of time when it contains ink; wipe it out first. The ink may sometimes be started by moistening the end of the finger and touching it to the point, or by drawing a slip of paper between the ends of the blade. *Always keep the bottle corked.*

16. To Sharpen the Drawing Pen.—When the ruling, or compass, pen becomes badly worn, it must be sharpened. For this purpose a fine oilstone should be used. If an oilstone is to be purchased, a small, flat, close-grained stone should be obtained, those having a triangular section being preferable, as the narrow edge can be used on the inside of the blades in case the latter are not made to swing apart so as to permit the use of a thicker edge.

The first step in sharpening is to screw the blades together, and, holding the pen perpendicular to the oilstone, to draw it back and forth over the stone, changing the slope of the pen from downwards and to the right to downwards and to the left for each movement of the pen to the right and left. The object of this is to bring the blades to exactly the same length and shape, and to round them nicely at the point.

This process, of course, makes the edges even duller than before. To sharpen, separate the points by means of the screw, and rub one of the blades to and from the operator in a straight line, giving the pen a slight twisting motion at the same time, and holding it at an angle of about 15° with the face of the stone. Repeat the process for the other blade. To be in good condition, the edges should be fairly sharp and smooth, but not sharp enough to cut the paper. *All the sharpening must be done on the outside of the blades.* The inside of the blades should be rubbed on the stone only enough to remove any burr that may have been formed. Anything more than this will be likely to injure the pen. The whole operation must be done very carefully, bearing on lightly, as it is easy to spoil a pen in the process Examine the points frequently, and keep at work until the pen will draw both *fine* lines and *smooth* heavy lines. Many draftsmen prefer to send the pens to be sharpened to the dealer who sold them, and who is generally willing to do such sharpening at a trifling cost.

17. Irregular Curves.—Curves other than arcs of circles are drawn with the pencil or ruling pen by means of curved or irregular-shaped rulers, called **irregular curves** (see Fig. 17). A series of points is first determined through which the curved line is to pass. The line is then drawn through these points by using such parts of the irregular curve as will pass through several of the points at once, the curve being shifted from time to time as required.

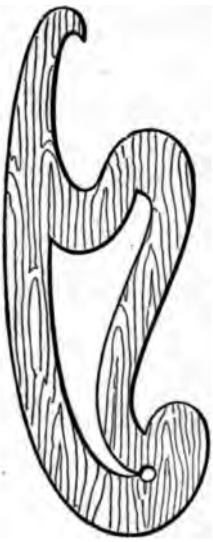

FIG. 17

It is usually difficult to draw a smooth, continuous curve. The tendency is to make it curve out too much between the points, thus giving it a wavy appearance, or else to cause it to change its direction abruptly where the different lines join, making angles at these points. These defects may largely be avoided by always fitting the curve to at least three points, and, when moving it to a new position, by setting it so that it will coincide with part of the line already drawn. It will be found to be a great help if the line be first sketched in freehand, in pencil. It can then be penciled over neatly, or inked, without much difficulty, with the aid of the irregular curve, since the original pencil line will show the general direction in which the curve should be drawn. Whenever the given points are far apart, or fall in such positions that the irregular curve cannot always be made to pass through three of them, the line must invariably be sketched in at first.

As an example, let it be required to draw a curved line through the points *a*, *b*, *c*, *d*, etc., Fig. 18. As just stated, a part of the irregular curve must be used which will pass through at least three points. With the curve set in the first position *A*, its edge is found to coincide with four points *a*, *b*, *c*, and *d*. The line may then be drawn from *a* around to *d*, or, better, to a point between *c* and *d*, since, by not continuing it quite to *d*, there is less liability of there being an angle where the next section joins on. For the next section of the line, the curve should be adjusted so as to coincide with a part of the section already drawn; that is, instead of adjusting it to points *d*, *e*, *f*, etc., it should be placed so as to

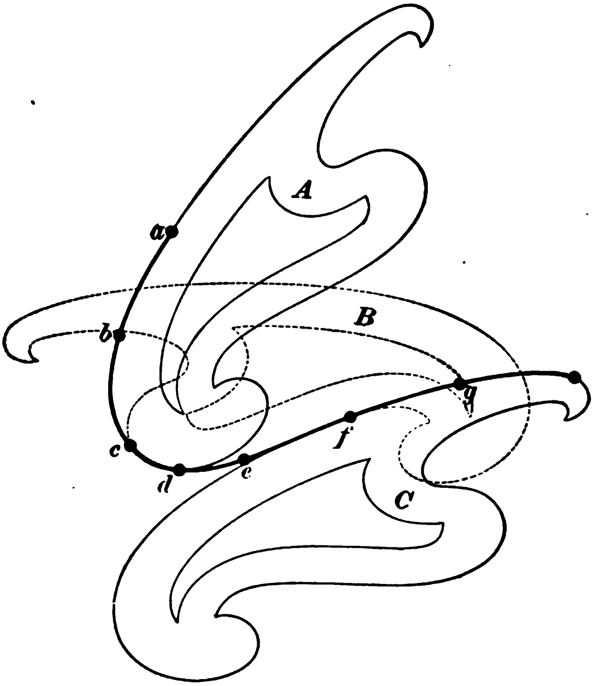

FIG. 18

pass through the point *c*, the part from *c* to *d* being coincident with the corresponding part of the first line drawn. The irregular curve is shown dotted in this position at *B*. Its edge passes through four points *c*, *d*, *e*, and *f*, and the line should be made to stop midway between the last two, as before.

Now, it will be noticed that the points *f* and *g* are so situated that the remainder of the line must curve up, instead of down, as heretofore, the change in curvature occurring at a point between *e* and *f*. In this case, therefore, it is not necessary for the curve to extend back to *e*, through which point the line has already been drawn, but it may be placed in position *C* with its edge just tangent to the line at the point where the curvature changes.

It is to be noticed that in inking with the irregular curve, the blades of the pen must be kept tangent to its edge (i. e., the inside flat surface of the blades must have the same direction as the curve at the point where the pen touches the paper), which requires that the direction of the pen be continually changed.

18. The **scale** is used for obtaining measurements for drawings. The most convenient forms are the usual flat and triangular boxwood scales, having beveled edges, each of which is graduated for a distance of 12 inches. The beveled edges serve to bring the lines of division close to the paper when the scale is lying flat, so that the drawing may be accurately measured, or distances laid off correctly. The use of the graduations on scales will be explained when it is necessary to use the scale.

19. A **protractor** is shown in Fig. 19. The outer edge is a semicircle, with center at *O*, and is divided into 360 parts. Each division is one-half of one degree, and, for convenience, the degrees are numbered from 0° to 180° from

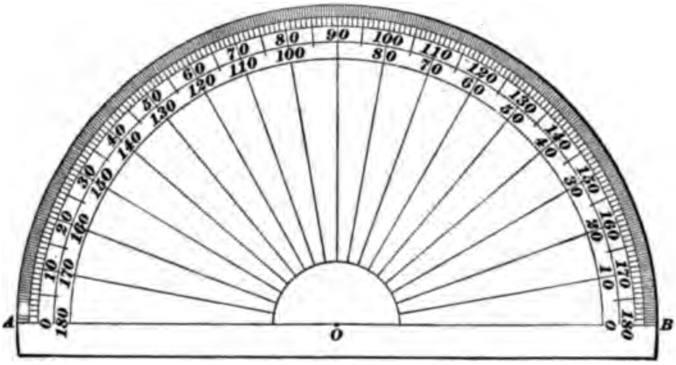

FIG. 19

both *A* and *B*. The protractor is used for laying off or measuring angles. Protractors are often made of metal, in which case the central part is cut away to make the drawing under it visible. When using the protractor, it must be placed so that the line *OB*, Fig. 19, will coincide with the line forming one side of the angle to be laid off or measured, and the center *O* must be at the vertex of the angle.

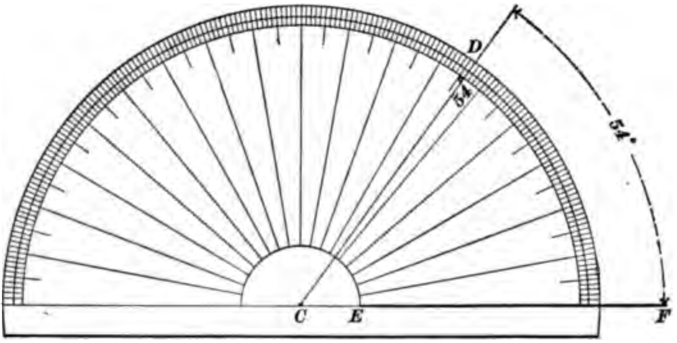

FIG. 20

For example, let it be required to draw a line through the point *C*, making an angle of 54° with the line *EF*, Fig. 20. Place the protractor upon the line *EF*, as just described, with the center *O* upon the point *C*. With a sharp-pointed pencil, make a mark on the paper at the 54° division, as indicated at *D*. A line drawn through *C* and *D* will then make an angle of 54° with *EF*. Greater exactness will be secured if the line *EF* be extended to the left, so that both zero marks (*A* and *B*, Fig. 19) can be placed on the line. This should always be done when possible.

LETTERING

20. In mechanical drawing, all headings, explanatory matter, and dimensions should be neatly printed on the drawing. Ordinary script writing is not permissible.

It is usually difficult for beginners to letter well, and unless the student is skilful at it, he should devote some time to practicing lettering before commencing the drawing. In correcting the plates, the lettering will be considered as well as the drawing. Many students think that it is only necessary to exercise special care when drawing the views on a plate, and that it is not necessary to take particular pains in lettering. This, however, is not the case, for, no matter how well the views may be drawn, if the lettering is poorly done, the finished drawing will not have a neat appearance. In fact, generally speaking, more time is required to make well-executed letters than to make well-executed drawings of objects. We earnestly request the student to practice lettering, and not to think that that part of the work is of no importance. The student should not be too hasty in doing the lettering. It takes an experienced draftsman considerable time to do good lettering, and no draftsman can perform this work as quickly as he can ordinary writing; therefore, no beginner should attempt to do what experienced draftsmen cannot do. In order to letter well, the work must be done slowly. Very frequently more time is spent in lettering a drawing than in inking in the objects represented. Instructions will be given in two styles of freehand lettering, both extensively used in American drafting rooms.

With the exception of the large headings or titles of the plates, the style and size of all lettering used on the original drawing plates of this Course are shown in Fig. 21. This style, although a little more elaborate and difficult in execution, was selected on account of its greater neatness and legibleness. The two styles are very similar in the formation of the letters, and although the student is advised to

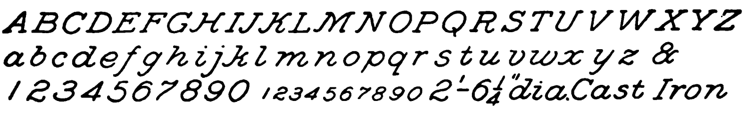
ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz &
1234567890 1234567890 2'-6¼" dia. Cast Iron

FIG. 21

select and use only one of the two on his drawings in this Course, he will find, after having mastered one of the styles, little difficulty in practicing the other.

When lettering, a Gillott's No. 303 pen should be used. The height of the capital letters should be $\frac{3}{32}$", and of the small letters two-thirds of this, or $\frac{1}{16}$". This applies to both styles of freehand lettering. *Do not make them larger than this.*

21. Before beginning to letter, horizontal guide lines should be drawn with the **T** square, to serve as a guide for the tops and bottoms of the letters (see Fig. 22). The outside lines should be $\frac{3}{32}$" apart for the capitals, and the two lower lines $\frac{1}{16}$" apart for the small letters. The letters should be made to extend fully up to the top, and down to the bottom, guide lines. They must not fall short of the guide lines, nor extend beyond them. Failure to observe this point will cause the lettering to look ragged, as in the second word in Fig. 22.

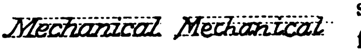

FIG. 22

22. It is very important that all the letters have the same inclination. For example, by referring to Fig. 23 (*a*), it will be seen that the backs of letters like *B*, *E*, *l*, *g*, *d*, *i*, *t*, etc. are parallel and slant the same way. This is also true of both sides of letters like *H*, *M*, *n*, *u*, *h*, *y*, etc. To aid in keeping the slant uniform, draw parallel slanting lines across the guide lines with the 60° triangle, as in Fig. 23 (*b*), and, in lettering, make the backs or sides of the letters parallel with these lines.

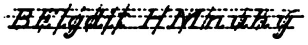

FIG. 23 (*a*)

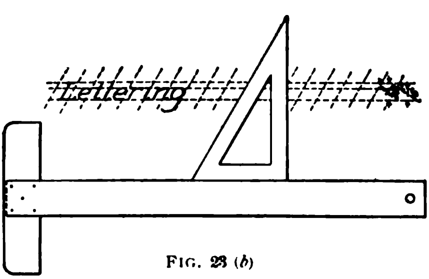

FIG. 23 (*b*)

23. A few points regarding the construction of the letters are illustrated in Fig. 24, in which the letters are shown upon an enlarged scale. The capital letters *A*, *V*, *Y*, *M*, and *W* must be printed so that their general inclination will be the same as for the other letters. To print the *A*, draw the center line *a d*, having the common slant; from *a* draw the sides *a c* and *a b*, so that points *c* and *b* will each be $\frac{3}{64}''$ distant from point *d*. The side *a b* will be nearly perpendicular to the guide lines. The *V* is like an inverted *A*, and is drawn in the same way, the line *b d* being nearly perpendicular.

To make the *Y*, draw the center line *a d*, having the common slant, which gives the slant for the base of the letter. The upper part of the *Y* begins a little below its center, and is similar to the *V*, though somewhat narrower, as the letter should be only $\frac{5}{64}''$ wide at the top. Points *b* and *c* should be at equal distances from point *a*.

The two sides *b c* and *e f* of the *M* are parallel, and have the common slant. The *M* is made as broad as it is high, or $\frac{3}{32}''$. Having drawn the two sides, mark the point *d*, midway between the points *c* and *f*, and connect it with points *b* and *e*. The lines *b d* and *e d* should be slightly curved, as shown.

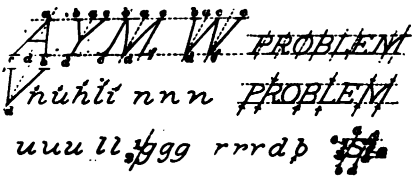

FIG. 24

In the *W* the two outside lines are not parallel, as in the *M*, but are farther apart at the top than at the bottom. Draw the line *a d*, having the common slant. Mark points *b* and *c*, which are exactly $\frac{1}{32}''$ from the point *a*. From *b* and *c*, draw lines *b d* and *c d*. The other half of the *W* is like the first part, *c f* being parallel to *b d* and *e f* parallel to *c d*. It will be seen that the *W* is composed of two narrow *V*'s, each $\frac{1}{16}''$ wide, the width of the whole letter being $\frac{1}{8}''$.

24. Capital letters like *P*, *R*, *B*, *L*, *E*, etc. should be printed so that their top and bottom lines will be *exactly horizontal*. This is illustrated in the two examples of the word *problem* in Fig. 24. In the first example, it will be noticed that the tops of the *P* and *R*, the bottom of the *L*, and the tops and bottoms of the *B* and *E*, all run in the same direction as the guide lines, and coincide with them. In the second example, these lines are not horizontal, which makes the word look very uneven. It is also to be noticed that these lines extend beyond the upright lines in the first word, and that cross-lines are used on the bottom of the *P* and *R*, on the top of the *L*, and on the *M*. In the second word, these lines are omitted at the points indicated by the arrows. These features are found on most of the other capitals.

The small letters *n*, *u*, *h*, *l*, *i*, etc. should have sharp corners at the points indicated by the arrows in Fig. 24. They look much better that way, and are less difficult to make, than when they have round corners. Following these letters are five groups of letters containing *n*, *u*, *l*, *g*, and *r*. The first letter of each group is printed correctly, while the letters following show ways in which they should *not* be printed. In the case of the *g*, point *2* should fall in a slanting direction under point *1*, the slant being the same as *a d* of the preceding letters. The difference between *d* and *b* and the construction of the *s* are also shown in the same figure. The *b* should be made rounding at the point indicated. As a guide in making the *s*, draw the two lines *a b* and *c d*, having the common slant. The *s* should now be drawn so that it will touch these lines at points *1*, *3*, and *4*, but *not* at point *2*. It will be an additional help if the line *e x* is also drawn as a guide for the middle portion of the *s*; but care should be taken not to have it slant more than shown in the copy.

The letters *a*, *o*, *b*, *g*, etc. should be full and round; do not cramp them. It will be necessary to follow the copy closely until familiar with it. Notice that the figures are not made as in writing, particularly the *6*, *4*, *8*, and *9* (see Fig. 21). Try to space the letters evenly. Letter in pencil first, and, if not right, erase and try again.

25. Another style of freehand lettering is shown in Fig. 25. This style is extensively used for the lettering of working drawings. It is more easily and rapidly made than the style previously described, and although not productive

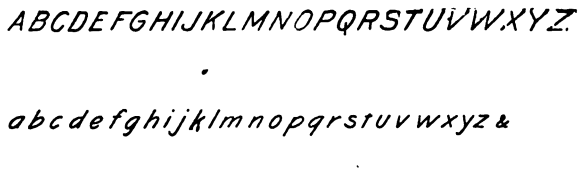
ABCDEFGHIJKLMNOPQRSTUVWXYZ
abcdefghijklmnopqrstuvwxyz &
12345678910 1234567890 2′-6¼″ dia. Cast Iron.

FIG. 25

of as high degree of neatness in appearance will be found very useful and acceptable for general office work.

A comparison between the two systems will disclose a great similarity in the detail formation of the letters.

26. The horizontal and slanting guide lines are drawn exactly in the same manner as for the style previously described, and if not followed, the results will be similar. See the uneven appearance of the second word in Fig. 26.

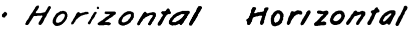

FIG. 26

27. By studying the formation of the letters carefully, it will be found that many of them are formed on the same principle, as shown in Fig. 27.

The ovals of the letters *a*, *b*, *d*, *g*, *p*, and *q* are formed exactly alike and have a slant of 45° with the horizontal. These ovals should be made a little wider at the top than at the bottom. Care should be taken that the straight downward strokes are made parallel to the slanting guide lines. The letters *c* and *e* are commenced in the same way, but the upper loop in *e* should be formed in such a manner that its axis will be at an angle of 45° with the horizontal. The *r* is made by having the down stroke parallel to the slanting guide line

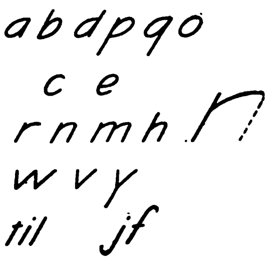
a b d p q o
c e
r n m h
w v y
t i l j f

FIG. 27

and the up stroke slightly curved in the same way as in the letter *n* (see Fig. 27). The strokes in the letters *j* and *f* are the same, with the position of the hook part reversed.

28. The capital letters shown in Fig. 28 are formed very nearly in the same manner as those shown in Art. **23,** but differ slightly by omitting the short spurs that give to the letters a more finished appearance.

In the capital *M*, however, there is a decided variation. The *M* is made with four strokes, putting in the parallel sides first. The two other strokes should join midway between these sides and at a distance from the top of about $\frac{1}{3}$ of the height of the letter. These strokes, as will be seen, are straight and not curved.

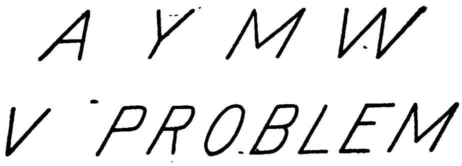

FIG. 28

29. The *numerals* should be $\frac{5}{32}''$ high and of the style shown in Fig. 25; fractions should be $\frac{1}{8}''$ high over all. In Fig. 29 the numerals are illustrated to a larger scale, and a comparison with the style shown in Fig. 21 will disclose several variations.

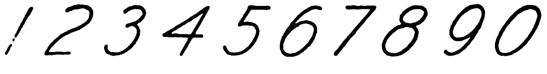

FIG. 29

The loops of the *2*, *3*, *5*, *6*, and *9* should be formed so that their axes will be at an angle of 45° with the horizontal. It will be noted that the *7* differs widely from the style shown in Fig. 21, the down stroke not curving but having a straight slant of 45°. The axis of the *0* and the loops of the *8* should slant at an angle of 60°.

Diligent practice for a short time and careful observation of the forms of letters and numerals, as shown in Figs. 21–29, will soon enable the student to acquire skill and speed in this branch of drawing.

30. The alphabet shown in Fig. 30, called the **block letter,** is to be used for the large headings or titles of plates, as shown on the copy plates. This alphabet is *not* to be used on the first five geometrical drawing plates. The letters and figures are to be made $\frac{5}{16}''$ high and $\frac{1}{4}''$ wide, except *M*, which is $\frac{5}{16}''$ wide, and *W*, which is $\frac{3}{8}''$ wide. The thickness of all the lines forming the letters is $\frac{1}{16}''$, measured horizontally. The distance between any two letters of a word is $\frac{1}{16}''$, except where *A* follows *P* or *F*; where *V*, *W*, or *Y* follows *L*; where *J* follows *F*, *P*, *T*, *V*, *W*, or *Y*; where *T* and *A* are adjacent, or *A* and *V*, *W*, or *Y* are adjacent; in this case, the bottom extremity of *A* and the top extremity of *P*, *T*, *V*, *W* are in the same vertical line, etc.

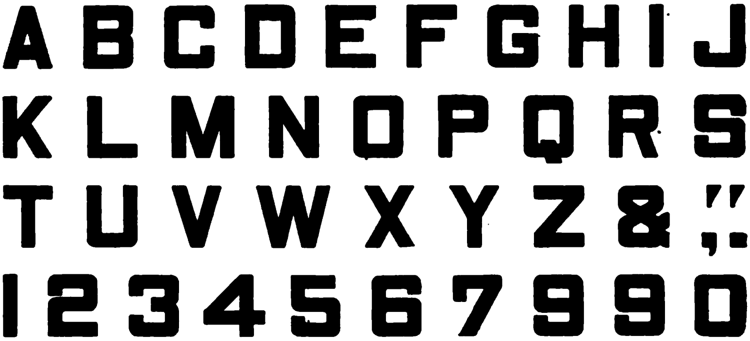

FIG. 30

Since these letters are composed of straight lines, they can be made with the T square and triangle. In lettering the title of the drawing plates, the student should draw six horizontal lines $\frac{1}{16}''$ apart in lead pencil, to represent the thickness of the letters at the top, center, and bottom; then, by use of the triangle, he should draw in the width of the letters and the spaces between them in lead pencil. Having the letters all laid out, he can very easily ink them in. Use the ruling pen for inking in the straight outlines of the letters, and the lettering pen for rounding the corners and filling in between the outlines. It is well to ink in all the perpendicular lines first, next the horizontal lines, and then the oblique lines.

PLATES

31. Preliminary Directions.—The size of each plate over all will be 14" × 18", having a border line ½" from each edge all around, thus making the size of the space on which the drawing is to be made 13" × 17". The sheet itself must be larger than this when first placed upon the board, so that the thumbtack holes may be cut out; the extra margin is also very convenient for testing the pen, in order to see whether the ink is flowing well and whether the lines are of the proper thickness.

32. The first five plates will consist of practical geometrical problems which constantly arise in practice when making drawings. The method of solving every one of these problems should be carefully memorized, so that they can be instantly applied when the occasion requires, without being obliged to refer to the text for help. Particular attention should be paid to the lettering. Whenever any dimensions are specified, they should be laid off as accurately as possible. All drawings should be made as neat as possible, and the penciling entirely finished before inking in any part of it. Great care should be taken in distributing the different views, parts, details, etc. on the drawing, so that when the drawing is completed, one view will not be so near to another as to mar the appearance of the drawing. The hands should be perfectly clean, and should not touch the paper except when necessary. No lines should be erased except when *absolutely* necessary; for, whenever a line has once been erased, the dirt flying around in the air

and constantly falling on the drawing will stick to any spot where an erasure has been made, and it is then very difficult, if not impossible, to entirely remove it. For this reason, all construction lines that are to be removed, or that are liable to be changed, should be drawn lightly, that the finish of the paper may not be destroyed when erasing them. When it is found necessary to erase an ink blot or a line that has been inked in, only an *ink eraser* or *sand rubber* should be used. After the erasure has been made, the roughened part of the surface of the paper can be smoothed by rubbing with some hard, smooth substance, as a piece of ivory or the handle of a knife.

PLATE I

33. Take a sheet of drawing paper 15" wide and 20" long (demy size), and fasten it to the board as previously described. On this draw the outlines of the size of the plate, 14" × 18", and draw the border line all around $\frac{1}{2}$" from the edge of the outline, leaving the space inside for the drawing 13" × 17". When the word *drawing* is used hereafter, it refers only to the space inside the border lines and the objects drawn upon it. To understand clearly what follows, refer to Plate I. Divide the drawing into two equal parts by means of a faint horizontal line. This line is shown dotted in Plate I, above referred to. Divide each of these halves into three equal parts, as shown by the dotted lines; this divides the drawing into six rectangular spaces. *These division lines are not to be inked in, but must be erased when the plate is completed.* On the first five plates, space for the lettering must be taken into account. For each of the six equal spaces, the lettering will take up one or two lines. The height of all capital letters on these plates will be $\frac{3}{32}$", and of the small letters $\frac{2}{3}$ of this, or $\frac{1}{16}$". The distance between any two lines of lettering will also be $\frac{3}{32}$". The distance between the tops of the letters on the first line of lettering and the top line of the equal divisions of the drawing is to be $\frac{1}{2}''$; and the space between the bottoms of the letters and the topmost point of the figure represented on the drawing within one of these six divisions must also be not less than $\frac{1}{2}''$. This makes a very neat arrangement, if the figure is so placed that the outermost points of the bounding lines are equally distant from the sides of one of the equal rectangular spaces. Consequently, if there is one line of lettering, no point of the figure drawn should come nearer than $\frac{1}{2}'' + \frac{3}{32}'' + \frac{1}{2}'' = 1\frac{3}{32}''$ to the top line of the space within which it is represented; or, if there are two lines of lettering, nearer than $\frac{1}{2}'' + \frac{3}{32}'' + \frac{3}{32}'' + \frac{3}{32}'' + \frac{1}{2}'' = 1\frac{9}{32}''$. The letter heading for each figure on the first five plates will be printed in heavy-faced type at the beginning of the directions explaining each problem. The student must judge for himself by the length of the heading whether it will take up one line or two, and make due allowance for the space it takes up. This is a necessary precaution, because the lettering should never be done until the rest of the drawing is entirely finished and inked in.

PROBLEM 1.—**To bisect a straight line.**

See Fig. 31; also 1 of Plate I.

CONSTRUCTION.—Draw a straight line *A B*, $3\frac{1}{2}''$ long. With one extremity *A* as a center, and a radius greater than one-half of the length of the line, describe an arc of a circle on each side of the given line; with the other extremity *B* as a center, and the same radius, describe arcs intersecting the first two in the points *C* and *D*. Join *C* and *D* by the line *C D*, and the point *P*, where it intersects *A B*, will be the required point; that is, *A P* = *P B*, and *P* is the middle point

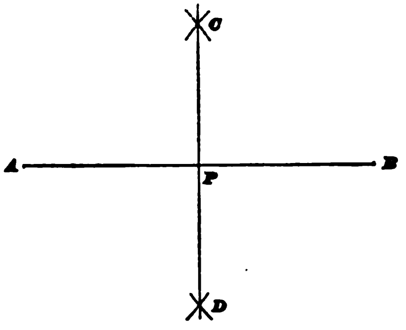

FIG. 31

of *A B*. Since *C D* is perpendicular to *A B*, this construction also gives a *perpendicular to a straight line at its middle point.*

PROBLEM 2.—To draw a perpendicular to a straight line from a given point in that line.

NOTE.—As there are two cases of this problem, requiring two figures on the plate, the line of letters will be run clear across both figures, as shown in Plate I.

Case I.—*When the point is at or near the center of the line.* See Fig. 32; also 2, Case I, of Plate I.

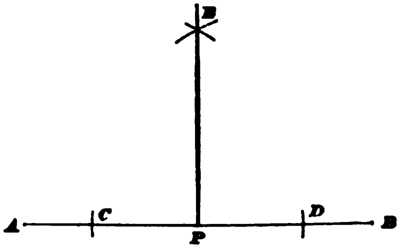

FIG. 32

CONSTRUCTION. — Draw *A B* 3½" long. Let *P* be the given point. With *P* as a center, and any radius, as *P D*, describe two short arcs cutting *A B* in the points *C* and *D*. With *C* and *D* as centers, and any convenient radius greater than *P D*, describe two arcs intersecting in *E*. Draw *P E*, and it will be perpendicular to *A B* at the point *P*.

Case II.—*When the point is near the end of the line.* See Fig. 33; also 2, Case II, of Plate I.

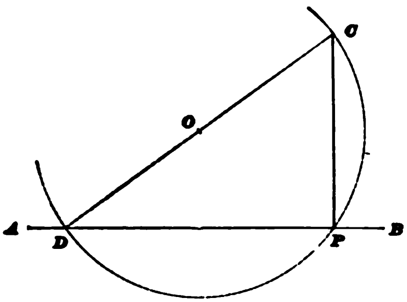

FIG. 33

Draw *A B* 3½" long. Take the given point *P* about ⅜" from the end of the line. With any point *O* as a center, and a radius *O P*, describe an arc cutting *A B* in *P* and *D*. Draw *D O*, and prolong it until it intersects the arc in the point *C*. A line drawn through *C* and *P* will be perpendicular to *A B* at the point *P*.

PROBLEM 3.—**To draw a perpendicular to a straight line from a point without it.**

As in Problem 2, there are two cases.

Case I.—*When the point lies nearly over the center of the line.* See Fig. 34; also 3, Case I, of Plate I.

CONSTRUCTION.—Draw *A B* 3½" long. Let *P* be the given point. With *P* as a center, and any radius *P D* greater than the distance from *P* to *A B*, describe an arc cutting *A B* in *C* and *D*. With *C* and *D* as centers, and any convenient radius, describe short arcs intersecting in *E*. A line drawn through *P* and *E* will be perpendicular to *AB* at *F*.

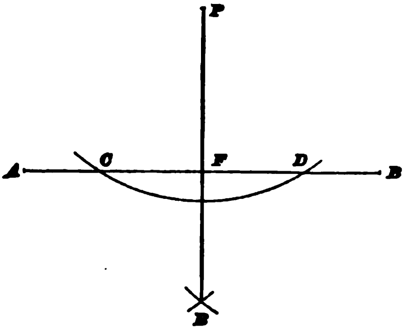

FIG. 34

Case II.—*When the point lies nearly over one end of the line.* See Fig. 35; also 3, Case II, of Plate I.

Draw *A B* 3½" long, and let *P* be the given point. With any point *C* on the line *A B* as a center, and the distance *C P* as a radius, describe an arc *P E D* cutting *A B* in *E*. With *E* as a center, and the distance *E P* as a radius, describe an arc cutting the arc *P E D* in *D*. The line joining the points *P* and *D* will be perpendicular to *A B*.

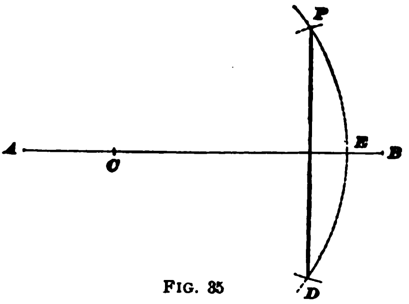

FIG. 35

PROBLEM 4.—**Through a given point, to draw a straight line parallel to a given straight line.**

See Fig. 36; also 4 of Plate I.

CONSTRUCTION.—Let *P* be the given point, and *A B* the given straight line 3½" long. With *P* as a center, and any convenient radius, describe an arc CD intersecting AB in D. With D as a center, and the same radius, describe the arc PE. With D as a center, and a radius equal to the chord of the arc PE, describe an arc intersecting CD in C. A straight line drawn through P and C will be parallel to AB.

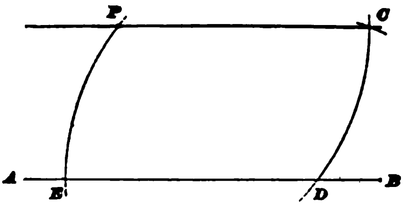

FIG. 36

34. These four problems form Plate I. They should be carefully and accurately drawn in with lead-pencil lines and then inked in. It will be noticed that on Plate I, and Figs. 31 to 36, the given lines are *light*, the required lines *heavy*, and the construction lines, which in a practical working drawing would be left out, are *light dotted*. This system must also be followed in the four plates which are to follow. A single glance enables one to see at once the reason for drawing the figure, and the eye is directed immediately to the required line.

In the first five plates, accuracy and neatness are the main things to be looked out for. The student should be certain that the lines are of *precisely* the length that is specified in the description. When drawing a line through two points, be sure that the line goes through the points; if it does not pass exactly through the points, erase it and draw it over again. If a line is supposed to end at some particular point, make it end there—do not let it extend beyond or fall short. Thus, in Fig. 36, if the line PC does not pass through the points P and C, it is not parallel to AB. By paying careful attention to these points, the student saves himself a great deal of trouble in the future. *Do not hurry your work.*

First ink in all of the light lines and light dotted lines (which have the same thickness); then ink in the heavy required lines after the pen has been readjusted. Now do the lettering (first read carefully the paragraphs under the head "Lettering"), and finally draw the heavy border lines, which should be thicker than any other line on the drawing. The word "Plate" and its number should be printed at the top of the sheet, outside the border lines, and midway of its length, as shown. The student's name, followed by the words "Class" and "No.," and after this his Course letter and *class number* should be printed in the lower right-hand corner below the border line, as shown. Thus, John Smith, Class No. C 4529. The date on which the drawing was completed should be placed in the lower left-hand corner, below the border line. *All of this lettering is to be in capitals $\frac{3}{32}''$ high.* Erase the division lines, and clean the drawing by rubbing very gently with the eraser. Care must be exercised when doing this, or the inked lines will also be erased. It is best to use a so-called "Sponge Rubber" for this purpose, as it will not injure the inked lines. *If any part of a line has been erased or weakened, it must be redrawn.* Then write with the lead pencil your name and address in full on the back of your drawing, after which put your drawing in the empty tube which was sent you, and send it to the Schools.

HINTS FOR PLATE I

35. *Do not forget to make a distinction between the width of the given and required lines, nor forget to make the construction lines dotted.*

When drawing dotted lines, take pains to have the dots and spaces uniform in length. Make the dots about $\frac{1}{16}''$ long and the spaces only about one-third the length of the dots.

Try to get the work accurate. The constructions must be accurate, and all lines or figures should be drawn of the length or size previously stated. To this end, work carefully and keep the pencil leads very sharp, so that the lines will be fine.

The lettering on the first few plates, as well as on the succeeding plates, is fully as important as the drawing, and should be done in the neatest possible manner. Drawings sent in for correction with the lettering omitted will be returned for completion.

The reference letters like A, B, C, etc., as shown in Fig. 31, are not to be put on the plates.

Do not neglect to trim the plates to the required size. Do not punch large holes in the paper with the dividers or compasses. Remember that the division lines are to be erased—not inked in

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PLATE II

36. Draw the division lines in the same manner as described for Plate I. The following five problems (5 to 9, inclusive) are to be drawn in regular order, as was done in Plate I, with problems from 1 to 4. The letter headings are given in heavy-faced type after the problem number.

PROBLEM 5.—**To bisect a given angle.***

Case I.—*When the sides intersect within the limits of the drawing.* See Fig. 37.

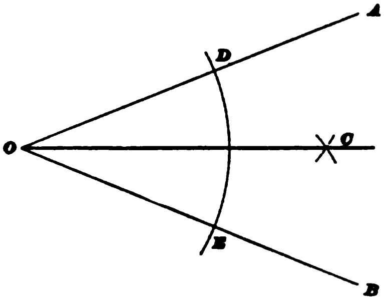

FIG. 37

CONSTRUCTION.—Let $A\,O\,B$ be the angle to be bisected. Draw the sides OA and OB 3½″ long. With the vertex O as a center, and any convenient radius, describe an arc $D\,E$ intersecting $O\,A$ at D and $O\,B$ at E. With D and E as centers, and a radius greater than the chord of half the arc DE, describe two arcs intersecting at C. The line drawn through C and O will bisect the angle; that is, $A\,O\,C = C\,O\,B$.

Case II.—*When the sides do not intersect within the limits of the drawing.* See Fig. 38.

CONSTRUCTION.—Draw two lines, $A\,B$ and $C\,D$, each 3½″ long, and inclined towards each other as shown. With any point *E* on *C D* as a center and any convenient radius, describe arc *F I G H*; with *G* as a center and same radius, describe arc *H L E F*, intersecting *F I G H* in *H* and *F*. With *L* as a center and same radius, describe arc *K G J*; with *I* as a center and same radius, describe arc *J E K*, intersecting *K G J* in *K* and *J*. Draw *H F* and *J K*; they intersect at *O*, a point on the bisecting line. With *O* as a center and the same or any convenient radius, describe an arc intersecting *A B* and *C D* in *M* and *N*. With *M* and *N* as centers and any radius greater than one-half *M N*, describe arcs intersecting at *P*. A line drawn through *O* and *P* is the required bisecting line.

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* Since the letter heading in this problem is very short, it will be better to place it over each of the two cases separately, instead of running it over the division line, as was done with the long headings of the two cases in Plate I. Put Case I and Case II under the heading, as in the previous plate.

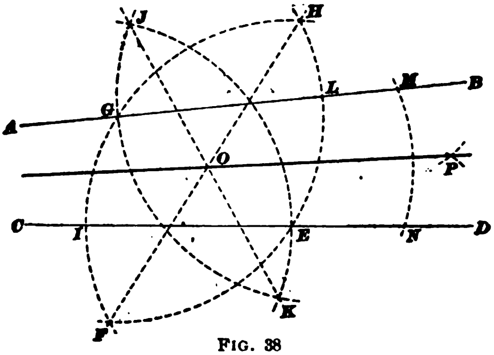


FIG. 38

PROBLEM 6.—**To divide a given straight line into any required number of equal parts.**

See Fig. 39 (*a*).

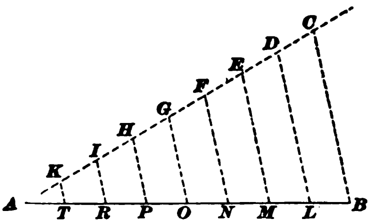


FIG. 39 (*a*)

CONSTRUCTION.—*A B* is the given line $3\frac{7}{16}''$ long. It is required to divide it into eight equal parts. Through one extremity *A* of the line, draw an indefinite straight line *A C*, making any angle with *A B*. Set the dividers to any convenient distance, and space off eight equal divisions on *A C*, as *A K*, *K I*, *I H*, etc. Join *C* and *B* by the straight line $C B$, and through the points D, E, F, G, etc. draw lines $D L$, $E M$, etc. parallel to $C B$, by using the two triangles; these parallels intersect $A B$ in the points L, M, N, etc., which are equally distant apart. The spaces $L M$, $M N$, $N O$, etc. are each equal to $\frac{1}{8} A B$. Proceed in a similar way for any number of equal parts into which $A B$ is to be divided.

Another method is shown in Fig. 39 (*b*). Draw $A B$ as before, and erect the perpendicular $B C$. Now divide the length of $A B$ by the number denoting the number of equal parts into which $A B$ is to be divided, obtaining, in this case, $3\frac{7}{16}'' \div 8 = \frac{55}{128}''$. As $A C$ is longer than $A B$, the equal divisions $A K$, $K I$, etc. are longer than $A T$, $T R$, etc. and may be made any convenient length greater than $A B \div 8$. In this case, $\frac{1}{2}''$ is the most convenient fraction nearest to and greater than $\frac{55}{128}''$; hence, consider $A K$, $K I$, etc. to be each $\frac{1}{2}''$ in length, thus making the length of $A C$ $8 \times \frac{1}{2}'' = 4''$. With A as a center and a radius equal to 4'', describe an arc cutting $B C$ in C, and draw $A C$. Then with a scale lay off $A K = K I =$ etc. $= \frac{1}{2}''$, and project K, I, H, etc. upon $A B$, in T, R, P, etc., the required points. The advantage of this method over the other is that the **T** square and triangle can be used throughout, thus making it very much easier to draw the parallels $D L$, $E M$, etc.

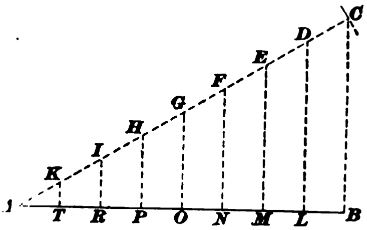


FIG. 39 (*b*)

The student, when drawing this plate, is at liberty to use either of the two methods given in this problem.

PROBLEM 7.—**To draw a straight line through any given point on a given straight line to make any required angle with that line.**

CONSTRUCTION.—In Fig. 40, $A B$ is the given line $3\frac{1}{2}''$ long, P is the given point, and $E O F$ is the given angle. With the vertex O as a center, and any convenient radius, describe an arc EF cutting OE and OF in E and F. With P as a center, and the same radius, describe an arc CD. With D as a center, and a radius equal to the chord of the arc EF, describe an arc cutting CD in C. A line drawn through the points P and C will make an angle with AB equal to the angle O, or $CPD = EOF$.

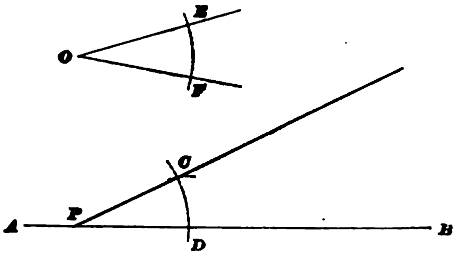


FIG. 40

PROBLEM 8.—**To draw an equilateral triangle, one side being given.**

CONSTRUCTION.—In Fig. 41, AB is the given side 2½″ long. With AB as a radius, and A and B as centers, describe two arcs intersecting in C. Draw CA and CB, and CAB is an equilateral triangle.

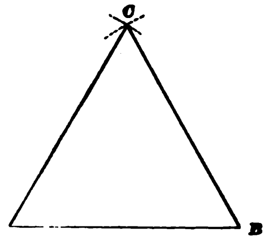


FIG. 41

PROBLEM 9.—**The altitude of an equilateral triangle being given, to draw the triangle.**

CONSTRUCTION.—In Fig. 42, AB is the altitude 2¼″ long. Through the extremities of AB draw the parallel lines CD and EF perpendicular to AB. With B as a center, and any convenient radius, describe the semicircle $CHKD$ intersecting CD in C and D. With C and D as centers, and the same radius, describe arcs cutting the semicircle in H and K. Draw BH and BK, and prolong them to meet EF in E and F. BEF is the required equilateral triangle.

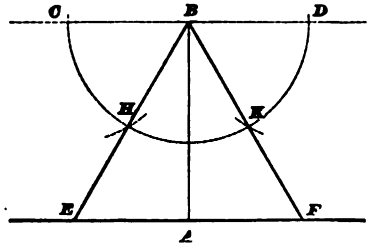


FIG. 42

This problem finishes Plate II. The directions for inking in, lettering, etc. are the same as for Plate I.

PLATE III

37. This plate is to be divided up like Plates I and II, and the six following problems are to be drawn in a similar manner:

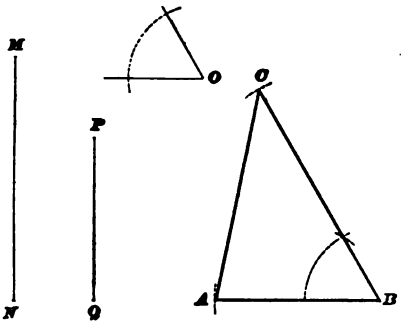


FIG. 43

PROBLEM 10.—**Two sides and the included angle of a triangle being given, to construct the triangle.**

CONSTRUCTION.—In Fig. 43, make the given sides *M N* 2½" long and *P Q* 1⅞" long. Let *O* be the given angle. Draw *A B*, and make it equal in length to *P Q*. Make the angle *C B A* equal to the given angle *O*, and make *C B* equal in length to the line *M N*. Draw *C A*, and *C A B* is the required triangle.

PROBLEM 11.—**To draw a parallelogram when the sides and one of the angles are given.**

CONSTRUCTION.—In Fig. 44, make the given sides *M N* 2½" long and *P Q* 1⅞" long. Let *O* be the given angle. Draw *A B* equal to *M N*, and draw *B C*, making an angle with *A B* equal to the given angle *O*. Make *B C* equal to *P Q*. With *C* as a center, and a radius equal to *M N*, describe an arc at *D*. With *A* as a center, and a radius equal to *P Q*, describe an arc intersecting the other arc in *D*. Draw *A D* and *C D*, and *A B C D* is the required parallelogram.

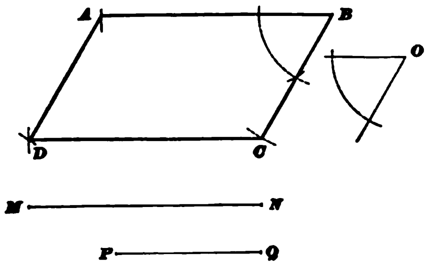


FIG. 44

PROBLEM 12.—**An arc and its radius being given, to find the center.**

CONSTRUCTION.—In Fig. 45, *A C D B* is the arc, and *M N*, 1¾″ long, is the radius. With *M N* as a radius, and any point *C* in the given arc as a center, describe an arc at *O*. With any other point *D* in the given arc as a center, and the same radius, describe an arc intersecting the first in *O*. *O* is the required center.

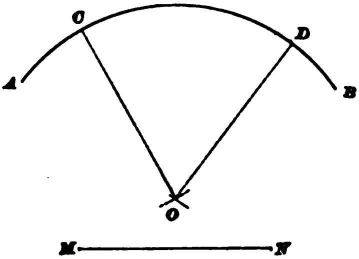


FIG. 45

PROBLEM 13.—**To pass a circumference through any three points not in the same straight line.**

CONSTRUCTION.—In Fig. 46, *A*, *B*, and *C* are the given points. With *A* and *B* as centers, and any convenient radius, describe arcs intersecting each other in *K* and *I*. With *B* and *C* as centers, and any convenient radius, describe arcs intersecting each other in *D* and *E*. Through *I* and *K* and through *D* and *E*, draw lines intersecting at *O*. With *O* as a center, and *O A* as a radius, describe a circle; it will pass through *A*, *B*, and *C*.

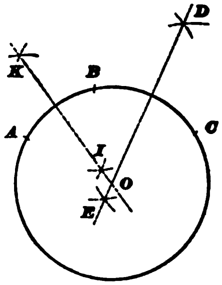


FIG. 46

PROBLEM 14.—**To inscribe a square in a given circle.**

CONSTRUCTION.—In Fig. 47, the circle *A B C D* is 3½″ in diameter. Draw two diameters, *A C* and *D B*, at right angles to each other. Draw the lines *A B*, *B C*, *C D*, and *D A*, joining the points of intersection of these diameters with the circumference of the circle, and they will be the sides of the square.

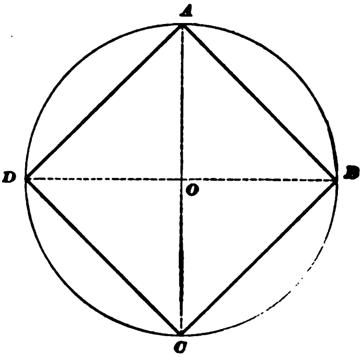


FIG. 47

PROBLEM 15—To inscribe a regular hexagon in a given circle.

CONSTRUCTION.—In Fig. 48, from *O* as a center, with the dividers set to 1¾″, describe the circle *A B C D E F*. Draw the diameter *D O A*, and from the points *D* and *A*, with the dividers set equal to the radius of the circle, describe arcs intersecting the circle at *E*, *C*, *F*, and *B*. Join these points by straight lines, and they will form the sides of the hexagon. This problem completes Plate III.

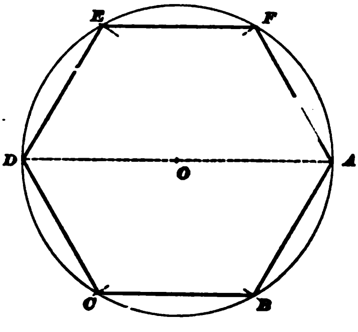


FIG. 48

PLATE IV

38. The first four problems on this plate are more difficult than any on the preceding plates and will require very careful construction. All the sides of each polygon must be of exactly the same length, so that they will space around evenly with the dividers. The figures should not be inked in until the pencil construction is done accurately. The preliminary directions for this plate are the same as for the preceding ones.

PROBLEM 16.—To inscribe a regular pentagon in a given circle.

CONSTRUCTION.—In Fig. 49, from *O* as a center, with the dividers set to 1¾″, describe the circle *A B C D*. Draw the two diameters *A C* and *D B* at right angles to each other. Bisect one of the radii, as *O B*, at *I*. With *I* as a center, and *I A* as a radius, describe the arc *A J* cutting *D O* at *J*.

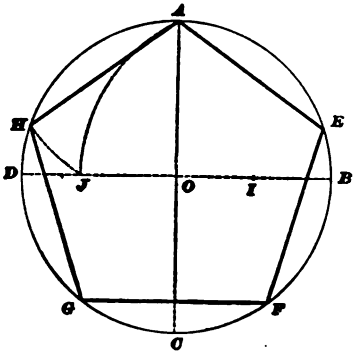


FIG. 49

With A as a center, and $A\ J$ as a radius, describe an arc $J\ H$ cutting the circumference at H. The chord $A\ H$ is one side of the pentagon.

Problem 17.—**To inscribe a regular octagon in a given circle.**

Construction.—In Fig. 50, from O as a center, with the dividers set to $1\frac{3}{4}''$, describe the circle $A\ B\ C\ D\ E\ F\ G\ H$. Draw the two diameters $A\ E$ and $G\ C$ at right angles to each other. Bisect one of the four equal arcs, as $A\ G$ at H, and draw the diameter $H\ O\ D$. Bisect another of the equal arcs, as $A\ C$ at B, and draw the diameter $B\ O\ F$. Straight lines drawn from A to B, from B to C, etc. will form the required octagon.

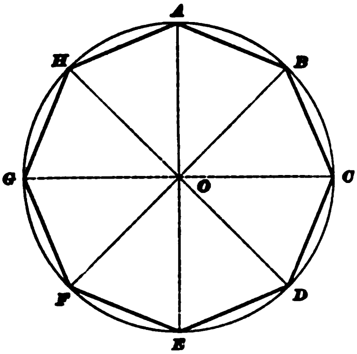


Fig. 50

Problem 18.—**To inscribe a regular polygon of any number of sides in a given circle.**

Construction.—In Fig. 51, from O as a center, with the dividers set to $1\frac{3}{4}''$, describe the circle $A\ 7\ C\ D$. Draw the two diameters $D\ 7$ and $A\ C$ at right angles to each other. Divide the diameter $D\ 7$ into as many equal parts as the polygon has sides (in this case seven). Prolong the diameter $A\ C$ and make $3'A$ equal to three-fourths of the radius $O\ A$. Through $3'$ and 2, the second division from D on the diameter $D\ 7$, draw the line $3'I$, cutting the circumference at I. Draw the chord $D\ I$, and it is one side of the required polygon. The others may be spaced off around the circumference.

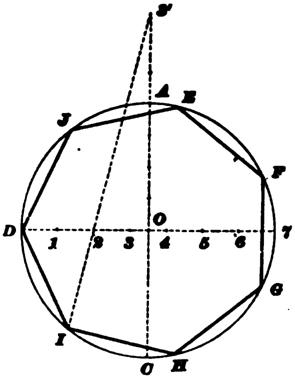


Fig. 51

PROBLEM 19.—**The side of a regular polygon being given, to construct the polygon.**

CONSTRUCTION.—In Fig. 52, let *A C* be the given side. If the polygon is to have eight sides, the line *A C* should be, for this plate, 1¼" long. Produce *A C* to *B*. From *C* as center, with a radius equal to *C A*, describe the semicircle *A 1 2 3 4 5 6 7 B*, and divide it into as many equal parts as there are sides in the required polygon (in this case eight). From the point *C*, and through the second division from *B*, as *6*, draw the straight line *C 6*. Bisect the lines *A C* and *C 6* by perpendiculars intersecting in *O*. From *O* as a center, and with *O C* as a radius, describe the circle *C A H G F E D 6*. From *C*, and through the points *1*, *2*, *3*, *4*, *5* in the semicircle, draw lines *C H*, *C G*, *C F*, etc., meeting the circumference. Joining the points *6* and *D*, *D* and *E*, *E* and *F*, etc., by straight lines, will complete the required polygon.

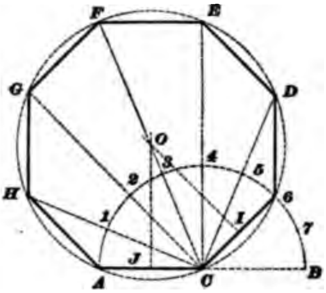


FIG. 52

PROBLEM 20.—**To find an arc of a circle having a known radius, which shall be equal in length to a given straight line.**

NOTE.—There is no exact method, but the following approximate method is close enough for all practical purposes, when the required arc does not exceed ⅛ of the circumference.

CONSTRUCTION.—In Fig. 53, let *A C* be the given line 3¼" long. At *A*, erect the perpendicular *A O*, and make it equal in length to the given radius, say 4" long.

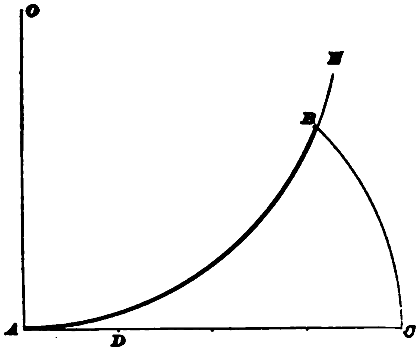


FIG. 53

With OA as a radius, and O as a center, describe the arc ABE. Divide AC into four equal parts, AD being the first of these parts, counting from A. With D as a center, and a radius DC, describe the arc CB intersecting ABE in B. The length of the arc AB very nearly equals the length of the straight line AC.

Problem 21.—**An arc of a circle being given, to find a straight line of the same length.**

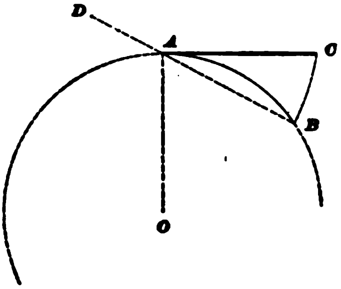


FIG. 54

This is also an approximate method, but close enough for practical purposes, when the arc does not exceed ⅙ of the circumference.

Construction.—In Fig. 54, let AB be the given arc; find the center O of the arc, and draw the radius OA. For this problem, choose the arc so that the radius will not exceed 1¾″. At A, draw AC perpendicular to the radius (and, of course, tangent to the arc). Draw the chord AB, and prolong it to D, so that $AD = \frac{1}{2}$ the chord AB. With D as a center, and a radius DB, describe the arc BC cutting AC in C. AC will be very nearly equal to the arc AB.

PLATE V

39. On this plate there are five problems instead of six. It should be divided into six equal parts or divisions, as the previous ones. The two right-hand end divisions are used to draw in the last figure of Plate V, which is too large to put in one division.

Problem 22.—**To draw an egg-shaped oval.**

Construction.—In Fig. 55, on the diameter AB, which is 2¾″ long, describe a circle $ACBG$. Through the center O, draw OC perpendicular to AB, cutting the circumference $ACBG$ in C. Draw the straight lines BCF and ACE. With B and A as centers, and the diameter AB as a radius, describe arcs terminating in D and H, the points of intersection with BF and AE. With C as a center, and CD as a radius, describe the arc DH. The curve $ADHBG$ is the required oval.

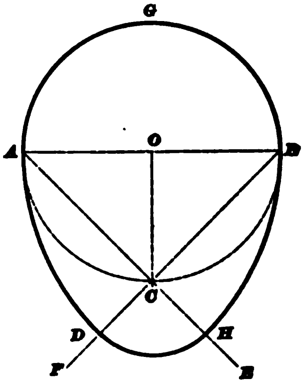


FIG. 55

PROBLEM 23.—**To draw an ellipse, the diameters being given.** The exact method.

CONSTRUCTION.—In Fig. 56, let BD, the long diameter, or major axis, which is $3\frac{1}{2}''$ long, and AC, the short diameter, or minor axis, which is $2\frac{1}{4}''$ long, intersect at right angles to each other in the center O, so that $DO = OB$ and $AO = OC$. With O as a center, and OC as a radius, describe a circle; with the same center, and OD as a radius, describe another circle. Divide both circles into the same number of equal parts, as *1–2*, *2–3*, etc. This is best done by first dividing the larger circle into the required number of parts, beginning at the center line AC, and then drawing radial lines through the points of division on this circle, to the center O of the circles, as shown in the upper right-hand quarter of the figure. The radial lines will divide the smaller circle into the same number of parts that the larger one has been divided into. Through the points of division on the smaller circle, draw horizontal lines, and, through the points of division on the larger circle, draw vertical

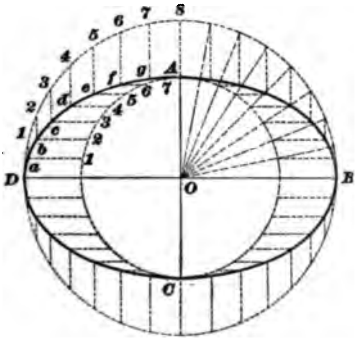


FIG. 56

lines; the points of intersection of these lines are points on the ellipse. Thus, the horizontal line *3c* and the vertical line *3c* intersecting at *c* give the point *c* of the ellipse. Trace a curve through the points thus found by placing an irregular curve on the drawing in such a manner that one of its bounding lines will pass through three or more points, judging with the eye whether the curve so traced bulges out too much or is too flat. Then adjust the curve again, so that its bounding line will pass through several more points, and so on, until the curve is completed. Care should be taken to make all changes in curvature as gradual as possible, and all curves drawn in this manner should be drawn in pencil before being inked in. It requires considerable practice to be able to draw a good curved line in this manner by means of an irregular curve, and the general appearance of a curve thus drawn depends a great deal upon the student's taste and the accuracy of his eye.

Problem 24.—**To draw an ellipse by circular arcs.**

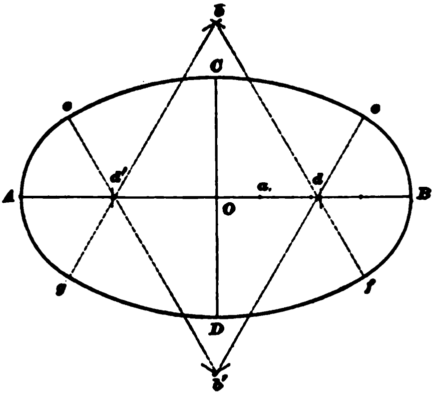


Fig. 57

This is not a true ellipse, but is very convenient for many purposes.

Construction.—In Fig. 57, use the same dimensions as before. On the major axis *A B*, set off *A a* = *C D*, the minor axis, and divide *a B* into three equal parts. With *O* as a center, and a radius equal to the length of two of these parts, describe arcs cutting *A B* in *d* and *d'*. Upon *d d'* as a side, construct two equilateral triangles *d b d'* and *d b' d'*. With *b* as a center, and a radius equal to *b D*, describe the arc *g D f* intersecting *b d f* and *b d' g* in *f* and *g*. With the same radius, and *b'* as a center, describe the arc *c C e* intersecting $b'd'c$ and $b'de$ in c and e. With A and B as centers, and a radius equal to the chord of the arcs $A c$ or $B e$, describe arcs cutting $A B$ very near to d' and d. From the points of intersection of these arcs with $A B$ as centers, and the same radius, describe the arcs $c A g$ and $e B f$.

PROBLEM 25.—**To draw a parabola, the axis and longest double ordinate being given.**

EXPLANATION.—The curve shown in Fig. 58 is called a **parabola.** This curve and the ellipse are the bounding lines of certain sections of a cone. The line $O A$, which bisects the area included between the curve and the line $B C$, is called the **axis.** Any line, $B A$ or $A C$, drawn perpendicular to $O A$, and whose length is included between $O A$ and the curve, is called an **ordinate.** Any line, as $B C$, both of whose extremities rest on the curve, and is perpendicular to the axis, is called a **double ordinate.** The point O is called the **vertex.**

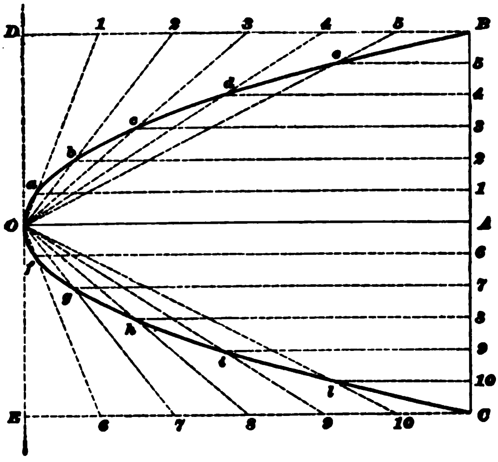


FIG. 58

CONSTRUCTION.—Make the axis $O A$ equal to 3½", and the longest double ordinate $B C$ equal to 3". $B A$, of course, equals $A C$. Draw $D E$ through the other extremity of the axis and perpendicular to it; also draw *B D* and *C E* parallel to *O A* and intersecting *D E* in *D* and *E*. Divide *D B* and *A B* into the same number of equal parts, as shown (in this case six); through the vertex *O*, draw *O 1*, *O 2*, etc. to the points of division on *D B*, and through the corresponding points *1*, *2*, etc., on *A B*, draw lines parallel to the axis. The points of intersection of these lines, *a*, *b*, *c*, etc., are points on the curve, through which it may be traced. In a similar manner, draw the lower half *O f g h i l C* of the curve.

PROBLEM 26.—**To draw a helix, the pitch and the diameter being given.**

EXPLANATION.—The helix is a curve formed by a point moving around the cylinder and at the same time advancing along its length a certain distance; this forms the winding curved line shown in Fig. 59. The center line *A O*, drawn through the cylinder, is called the **axis** of the helix, and any line perpendicular to the axis and terminated by the helix is of the same length, being equal to the radius of the cylinder. The distance *B 12* that the point advances lengthwise during one revolution is called the **pitch.**

CONSTRUCTION.—As mentioned before, this figure occupies two spaces of the plate. The diameter of the cylinder is 3½", the pitch is 2", and a turn and a half of the helix is to be shown. The rectangle *F B E D* is a side view of the cylinder, and the circle *1'*, *2'*, *3'*, *4'*, etc. is a bottom view. It will be noticed that one-half of a turn of the helix is shown dotted; this is because that part of it is on the other side of the cylinder, and cannot be seen. Lines that are hidden are drawn dotted. Draw the axis *O A* in the center of the space. Draw *F D* 3½" long and 4" from the top border line; on it construct a rectangle whose height *F B* = 3". Take the center *O* of the circle 2¾" below the point *H* on the axis *A O*, and describe a circle having a diameter of 3½", equal to the diameter of the cylinder. Lay off the pitch from *B* to *12* equal to 2", and divide it into a convenient number of equal parts (in this case 12), and divide the circle into the same number of equal parts, beginning at one extremity of the diameter *12′ O 6′*, drawn parallel to *B E*. At the point *1′* on the circle divisions, erect *1′–1′* perpendicular to *B E*; through the point *1* of the pitch divisions, draw *1–1′* parallel to *B E*, intersecting the perpendicular in *1′*, which is a point on the helix. Through the point *2′*, erect a perpendicular *2′–2′*, intersecting *2–2′* in *2′*, which is another point on the helix.

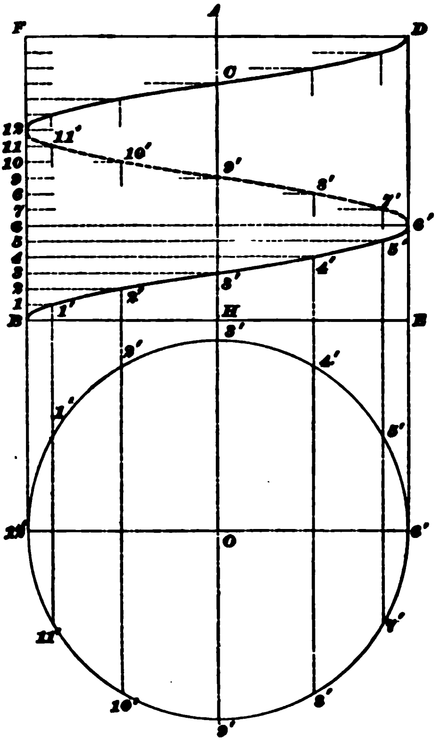


FIG. 59

So proceed until the point *6* is reached; from here on, until the point *12* of the helix is reached, the curve will be dotted. It will be noticed that the points of division *7′*, *8′*, *9′*, *10′*, and *11′* on the circle are directly opposite the points *5′*, *4′*, *3′*, *2′*, and *1′*; hence, it was not necessary to draw the lower half of the circle, since the point *5′* could have been the starting point, and the operation could have been conducted backwards to find the points on the dotted upper half of the helix. The other full-curved line of the helix can be drawn in exactly the same manner as the first half.

This ends the subject of practical geometry. Mechanical drawing, or the representation of objects on plane surfaces, will now be commenced.

THE REPRESENTATION OF OBJECTS

40. There are five kinds of lines used in mechanical drawing, thus:

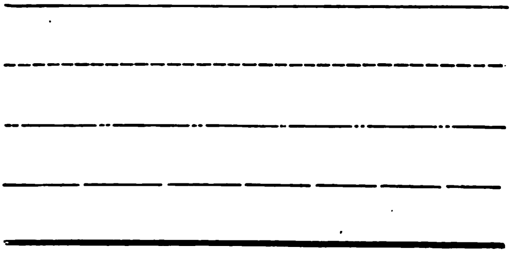
The light full line.

The dotted line.

The broken-and-dotted line.

The broken line.

The heavy full line.



The light full line is used the most; it is used for drawing the outlines of figures, and all other parts that can be seen by the eye.

The dotted line, consisting of a series of very short dashes, is used in showing the position and shape of that part of the object represented by the drawing which is concealed from the eye in the view shown; for example, a hollow prism closed on all sides. The hollow part cannot be seen; hence its size, shape, and position are represented by dotted lines.

The broken-and-dotted line, consisting of a long dash, and two dots or very short dashes repeated regularly, is used to indicate the center lines of the figure or parts of the figure, and also to indicate where a section has been taken when a sectional view is shown. This line is sometimes used for construction lines in geometrical figures.

The broken line, consisting of a series of long dashes, is used in putting in the dimensions, and serves to prevent the dimension lines from being mistaken for lines of the drawing.

The heavy full lines are made not less than twice as thick as the light full lines, and are used for shade lines.

Further explanations in regard to these lines will be given when the necessity for using them arises.

41. The illustrations in this and the following paragraphs should be carefully studied, but the student is not required to send in drawings from same. In Fig. 60 is shown a perspective view of a frustum of a pyramid having a rectangular base and a hole passing through the center of the frustum. This figure represents the frustum as it actually appears when the eye of the observer is in a certain position. The angles at A, B, C, and D are right angles, the hole is round, and the sides $A\ B$ and $D\ C$ are of equal lengths; so also are $A\ D$ and $B\ C$; but, if they were measured on the drawing, it would be found that their lengths are all different. The same difficulty would be met with in trying to measure the angles and edges of the sides $A\ B\ F\ E$, $B\ F\ G\ C$, etc. The real length of any line can be found only by a person perfectly familiar with perspective drawing, and then only with great difficulty. Consequently, this method of representing objects is of no use to a patternmaker, carpenter, machinist, or engineer, except to show what the object looks like. In order to represent the object in such a manner that any line or angle can be measured directly, what is termed **projection drawing**, or *orthographic projection*, is universally employed. In the perspective drawing shown in Fig. 60, three sides of the frustum are shown, and the other three are hidden; in a projection drawing, but one side is usually shown, the other five being hidden.

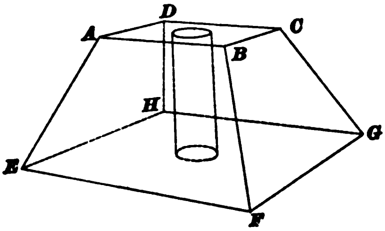


FIG. 60

A line or surface is *projected* upon a plane, by drawing perpendicular lines from points on the line or surface to the plane, and joining them.

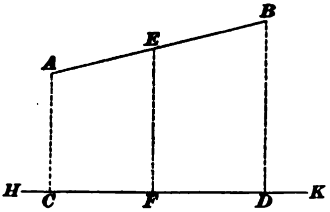


FIG. 61

Thus, if perpendiculars be drawn from the extremities of a line, as *AB*, to another line *HK*, as shown in Fig. 61, that portion of *HK* included between the feet of these perpendiculars is called the **projection** of *AB* upon *HK*. Thus, *CD* is the projection of *AB* upon *HK*, the point *C* is the projection of the point *A* upon *HK*, and the point *D* is the projection of the point *B* upon *HK*.

The projection of any point of *AB*, as *E*, can be found by drawing a perpendicular from *E* to *HK*, and the point where this perpendicular intersects *HK* is its projection. In this case, the point *F* is the projection of the point *E* upon *HK*.

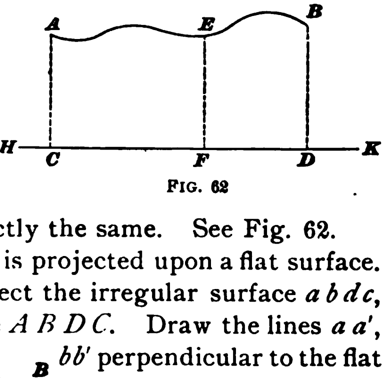


FIG. 62

It makes no difference whether the line is straight or curved—the method of finding the projection is exactly the same. See Fig. 62.

In a similar way, a surface is projected upon a flat surface. Thus, it is desired to project the irregular surface *abdc*, Fig. 63, upon the flat surface *ABDC*. Draw the lines aa', bb' perpendicular to the flat surface; join the points a' and b', where these perpendiculars intersect the flat surface *ABDC*, by a straight line $a'b'$, and $a'b'$ is the projection of the line *ab* upon *ABDC*.

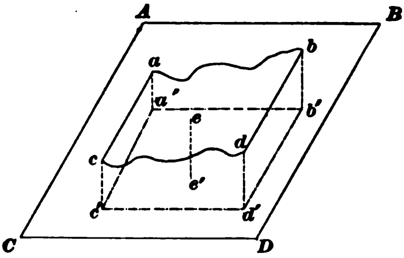


FIG. 63

In the same way, $a'c'$ is found to be the projection of *ac*; $c'd'$, the projection of *cd*; and $d'b'$, the projection of *db*. Hence, the projection of the irregular surface *a b d c* upon the flat surface *A B D C* is the quadrilateral *a′ b′ d′ c′*.

The projection of any point, as *e*, is found as before, by drawing a perpendicular from the point *e* to the surface; thus, *e′* is the projection of the point *e* upon the plane *A B D C*.

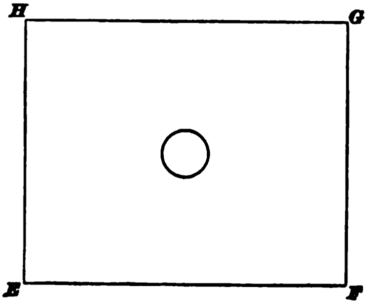


FIG. 64

Suppose that the frustum, Fig. 60, were placed on a plane surface (a surface perfectly flat, like a surface plate), and the outline of the bottom were traced by passing a pencil along its edges, including the round hole, the result would look like Fig. 64, in which the rectangle *E F G H* represents the bottom of the frustum and the circle represents the hole.

The angles and lengths of the sides are exactly the same as they are on the frustum itself; a similar drawing could be made to represent the top, but it is unnecessary, for the reason that the top can be projected on Fig. 64, and both objects accomplished in one drawing. Fig. 65 illustrates the meaning of the last statement. Here *A′ B′* is the projection of the edge *A B*, Fig. 60; *B′ C′*, of *B C*, etc. *A′ E′* is the projection of the edge *A E*; *B′ F′*, of *B F*, etc. This drawing shows the figure as it would look if the eye were directly over it. A drawing which represents the object as if it were resting on a horizontal plane, and the observer looking at it from above, is called a **top view**, or **plan.** The line of vision is thus perpendicular to the faces *A B C D* and *E F G H* of the frustum. The lines *A B*,

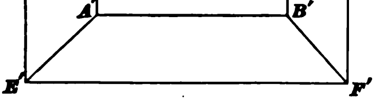


FIG. 65

B C, etc., *E F*, *F G*, etc., and the diameter of the hole, can be measured directly. The drawing is not yet complete, since it does not show whether the ends and sides are rounding, hollowed out, or flat. For this purpose, two more views are necessary—*a vertical projection*, or *front view*, commonly called a **front elevation**, and a *side projection*, or **side view.** A front view (elevation) is drawn by imagining the eye to be so situated that the observer looks directly at the front of the object; in other words, the line of vision is parallel to the faces of the frustum. The side looked at is then drawn as if it were projected on a vertical plane at right angles to the horizontal plane, the vertical plane being also parallel to the edges *E F* and *H G* of the frustum shown in Fig. 60. The drawing would then look like Fig. 66. Here the trapezoid *A B F E* represents the side *A B F E* of the frustum; the altitude of the trapezoid being the same as the altitude of the frustum, it can be measured directly. The hole cannot be seen when the observer looks at the frustum in this position; hence, it is indicated by dotted lines. The projections of the lines *A B* and *D C* (also, of *E F* and *H G*, of *A E* and *D H*, and of *B F* and *C G*) coincide.

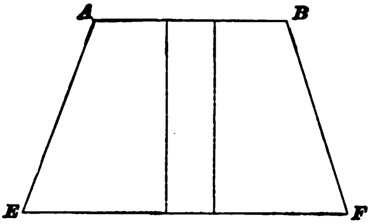


FIG. 66

To draw the side view (sometimes called a **side elevation**), imagine the frustum to be revolved around on its axis 90° to the left, and then draw it in precisely the same manner as the front elevation, by projecting the different lines upon a plane at right angles to the horizontal plane, and perpendicular to the edges *E F* and *H G*, that is, parallel to *B C* and *F G*. The side elevation would then be drawn as shown in Fig. 67. In this view the lines *A D*

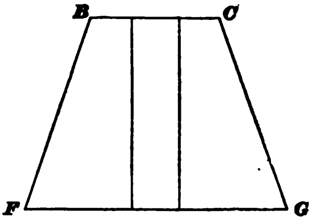


FIG. 67

and *B C* (also, *E H* and *F G*, *D H* and *C G*, and *A E* and *B F*) coincide.

42. In order to show clearly the different views, and to guard against one view being mistaken for another, they are always arranged on the drawing in a certain fixed and invariable manner. Fig. 68 shows this method of arrangement.

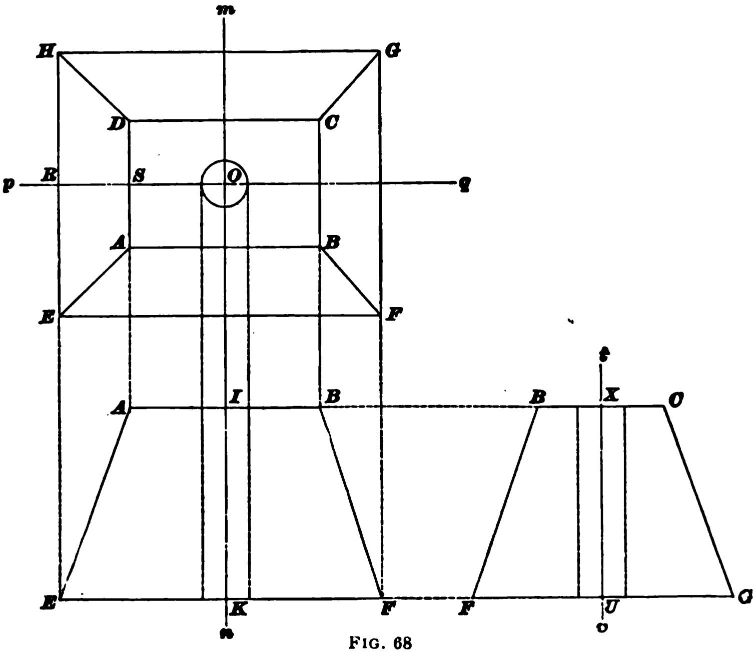


FIG. 68

The plan is drawn first, then the two elevations. It is usually immaterial which of these views is drawn first, but the general arrangement is as shown. Any departure from this method of arrangement should be distinctly specified on the drawing in writing, unless the purpose of the draftsman is so clearly evident that no explanation is needed. The broken and dotted lines are the **center lines**; they serve to show the connection existing between the different views of the object, and to indicate axes of cylindrical surfaces of any kind. It will be noticed that, in the plan view, the two center lines cross each other at right angles, and that their point of intersection *O* is the center of the circle which represents the hole. Whenever a circle is drawn, two center lines should also be drawn through its center at right angles to each other; this enables any one looking at a drawing to instantly locate the center of any circle. This remark also applies to ellipses, semicircles etc.

To draw the frustum as shown in the last figure, either the front elevation or the plan is drawn first—whichever happens to be more convenient. Suppose the front elevation to be drawn first. Draw the vertical center line *m n*; measure the altitude of the frustum, and lay it off on this line, locating the points *I* and *K*; through these points, draw the lines *A B* and *E F* perpendicular to *m n*; make $A I = I B = \frac{1}{2} A B$, measured on the frustum; also $E K = K F = \frac{1}{2} E F$, measured on the frustum, and draw *A E* and *B F*. Lay off the radius of the circular hole on both sides of the center line *m n*, and draw the dotted lines parallel to *m n* through the extremities of these radii to represent the hole. The front elevation is now complete. To draw the plan, decide where the center is to be located on *m n*, and draw the horizontal center line *p q*. With the point of intersection *O* of the two center lines as a center, and with a radius equal to the radius of the hole, describe a circle. Through the points *A*, *B*, *E*, and *F*, draw indefinite straight lines parallel to *m n*. On both sides of the center line *p q*, lay off on these lines *D S* and *S A*, equal to $\frac{1}{2}$ *D A*, and *H R* and *R E*, equal to $\frac{1}{2}$ *H E*, both *D A* and *H E* being measured on the frustum. Through the points *H*, *E*, *D*, and *A*, draw the lines *H G*, *E F*, *D C*, and *A B*, and join the points *H* and *D*, *E* and *A*, *F* and *B*, and *G* and *C* by straight lines, as shown. The figure thus drawn will be the plan.

To draw the side elevation, prolong the lines *A B* and *E F*, and draw the center line *t v*. Lay off, on each side of *t v*, *F U* and *U G* equal to $\frac{1}{2}$ *F G*, measured on the frustum, and *B X* and *X C* equal to $\frac{1}{2}$ *B C*, measured on the frustum. Join *B* and *F*, and *C* and *G*, by the straight lines

B F and *C G*, and draw the hole dotted as in the front elevation. The drawing is now complete.

The student should have by this time a good idea of how simple objects may be represented by the different views of a drawing, and can now begin on the next plate.

DRAWING PLATE, TITLE: PROJECTIONS—I

43. In making actual drawings of objects when the size of the plate is limited, it is usually impossible to divide it up into a certain number of parts, as in the case of the preceding plates, for the various figures differ widely in their sizes. These drawings should be so made that no part shall come nearer than ¾" to the border line, and the figures should be so arranged as to present a pleasing appearance to the eye, and not be scattered aimlessly all over the drawing.

Fig. 1 represents a **rectangular prism** 2" long, 1½" wide, and ¾" thick. The prism is represented as if it were standing on one of its small ends, with the broad side towards the observer. The elevation *A B D C* is drawn first; in this case, it will be a rectangle 2" × 1½". The top view, or plan, *F E B A* is next drawn; this is a rectangle 1½" × ¾", the side *A B* being the projection of the front of the prism, and the side *F E* of its back. Lastly, the side elevation is drawn; this is another rectangle *B E H D*, 2" × ¾", the side *B D* representing the projection of the front of the prism, and the side *B E* corresponding to the right-hand end *B E* of the plan.

Fig. 2 is a **wedge** standing on one of its triangular ends. It is the rectangle shown in Fig. 1, cut diagonally through the corner from *E* to *A* on the plan. It will be noticed that the two elevations are exactly the same as in Fig. 1, the plan showing the difference between the two figures.

Fig. 3 is another wedge, standing on one of its rectangular sides, formed by cutting through the prism, in Fig. 1, from *A* to *D*. The plan and side elevation are the same as in Fig. 1. Here, the front elevation shows the difference between Figs. 1 and 3. The point *D* of the elevation is projected on the plan in the point *D*, and the point opposite *D*, perpendicular to the plane of the paper, is the point *H*, shown in all of the side elevations.

Fig. 4 is also a wedge; it is formed by cutting through the prism in Fig. 1 from *B* to *H*. The front elevation and plan are the same as shown in Fig. 1, the side elevation being different. The point *H* in the side elevation opposite *D* is here projected in the point *H* of the plan; the point opposite *C* in the front elevation, and opposite *H* in the side elevation, is projected in the point *K* of the plan, the line *K H* being opposite *C D* in the plane of the base.

Fig. 5 shows a **cylinder** 1¼" in diameter and 2" long. The side elevation is not given, since all elevations of a cylinder whose bases are perpendicular to its axis are the same. Either view may be drawn first, according to convenience.

Fig. 6 shows a **hexagonal prism** 2" long; the distance between any two parallel sides is 1¼". In this case, the plan (a regular hexagon) must be drawn first. It is desired also that two of the parallel sides shall be horizontal. To draw the plan in this position, with the dimensions given, choose the center *O* of the hexagon; draw two center lines at right angles to each other, as *m n* and *p q*. With *O* as a center, and a radius equal to one-half of the distance between two parallel sides (1¼" × ½ = ⅝"), describe a circle. Now, use the T square to draw two horizontal lines through the points of intersection of this circle with the center line *m n*. By means of the T square and 60° triangle, draw *A B* and *C D* through *O*, in such a manner that the angles *A O q* and *C O p* each equal 60°; this is done by keeping the longer of the two short sides of the triangle vertical, and passing the pencil along the hypotenuse. Through *E* and *H*, the points of intersection of *A B* with the two parallel lines, draw *E K* and *H G* parallel to *C D*; and through *F* and *I*, the points of intersection of *C D* with the two parallels, draw *F G* and *K I* parallel to *A B*. This completes the hexagon, and also the plan of the prism. To draw the front elevation, measure

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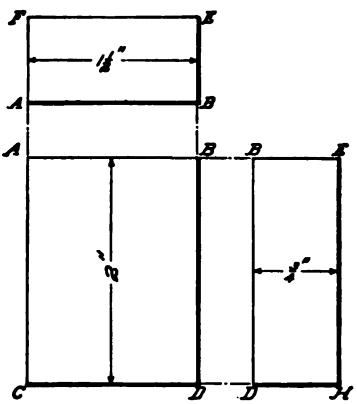
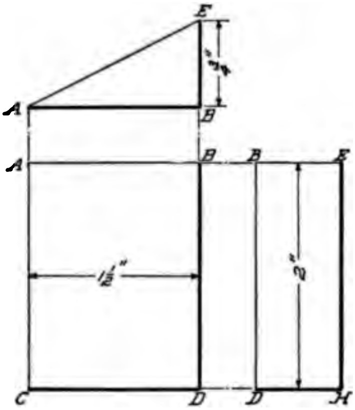


Fig.1.



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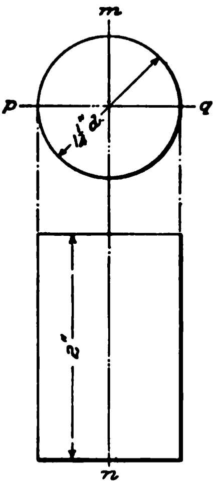


Fig. 5

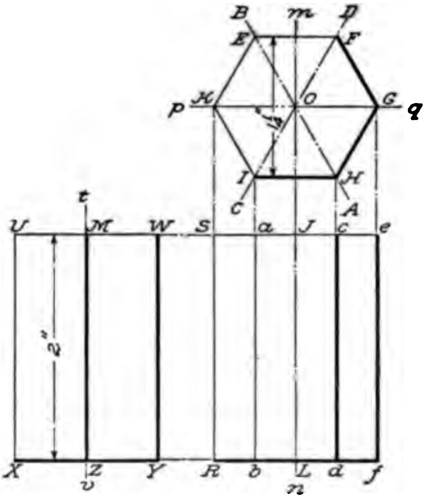
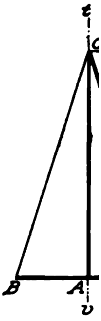


Fig. 6.



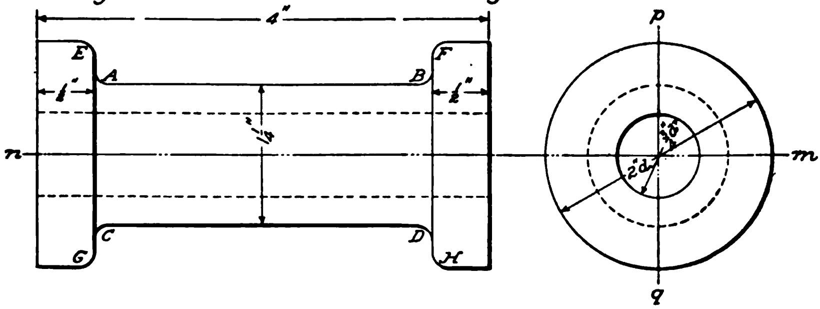


Fig. 10.

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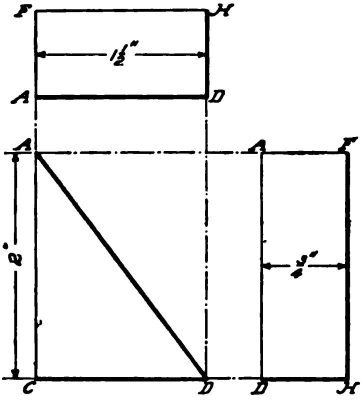

Fig. 3.

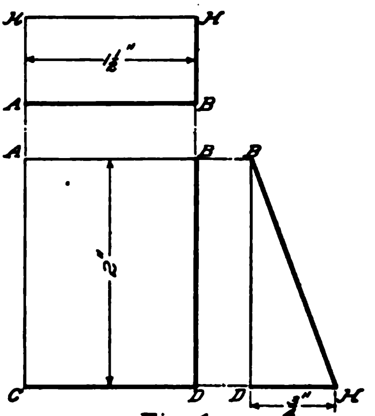

Fig. 4.

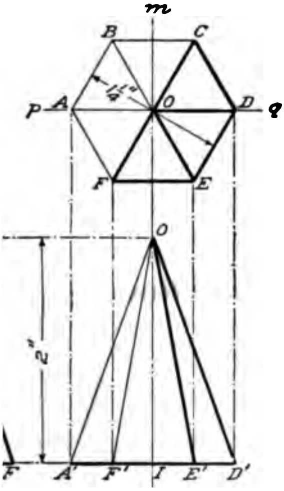

Fig. 7.

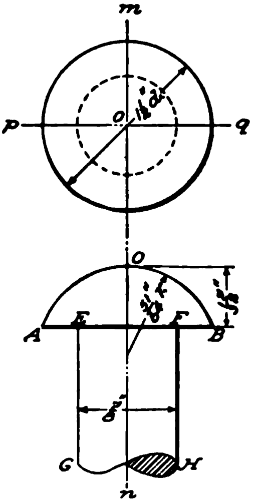

Fig. 8.

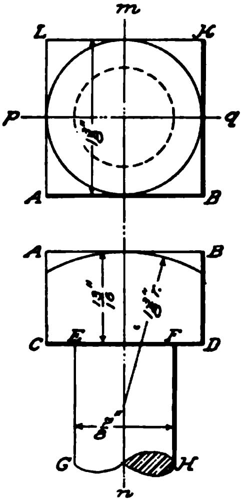

Fig. 9.

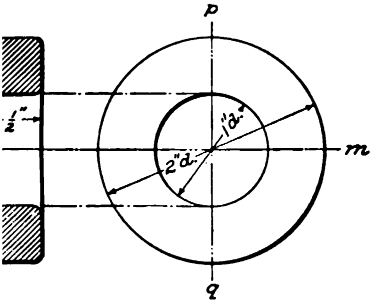

Fig. 11.

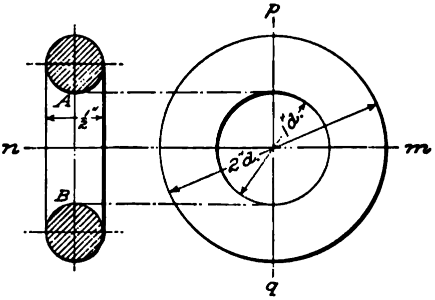

Fig. 12.

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JOHN SMITH, CLASS № 4529.

off, on the center line *m n*, the distance *J L* equal to 2″, and through the points *J* and *L* draw the two horizontal lines *S e* and *R f*. Project the points *K*, *I*, *H*, and *G* upon *S e*, as shown by the dotted lines; and through the points of intersection of these dotted lines with *S e*, draw the vertical lines *S R*, *a b*, *c d*, and *e f*, thus completing the front elevation. To draw the side elevation, extend the lines *S e* and *R f*, and draw the center line *t v*. Make *U W* equal to 1¼″, which is equal to the distance between the parallel sides, and draw *U X* and *W Y*; also, *M Z*, the point *M* corresponding to the point *K* of the plan.

Fig. 7 represents a **hexagonal pyramid**; the distance between two parallel sides of the base is 1¼″, and the altitude is 2″. As in Fig. 6, the plan must be drawn first. Then, to draw the front elevation, lay off *O I* on the center line *m n* equal to the altitude, and through *I* draw the base line *A′D′*. Project the points *D*, *E*, etc. of the plan upon *A′D′*, as shown by the dotted lines, and join them with the point *O* by the straight lines *A′O*, *F′O*, *E′O*, and *D′O*; these lines are the vertical projections of the edges of the pyramid; the horizontal projections of the edges are *F O*, *E O*, *D O*, etc. The side elevation can be easily drawn, and does not require a special description, the length of the base *B F* being equal to the distance between the parallel sides, or 1¼″.

Fig. 8 shows a **rivet** ⅞″ in diameter, having a button head 1¼″ in diameter. The side elevation is not given, since it is exactly the same as the front elevation. Either of the two views may be drawn first, according to convenience. Suppose that the elevation is first drawn. Draw the center line *m n*, and the line *A B* for the base of the head. On the center line lay off from the line *A B*, or the base of the head, a point *O*, at a distance of $\frac{17}{32}$″, the height of the head. With the compasses set to a radius of $\frac{51}{64}$″, and from a point on the center line *m n*, describe an arc *A O B*, taking care to pass this arc through the point *O*. Lay off from, and on both sides of, the center line *m n* a distance of $\frac{7}{16}$″, or ½ of the diameter of the rivet, and draw *E G* and *F H*. Draw the other center line *p q* of the plan, and with *O* as a center, and a radius equal to the radius of the button head, describe a circle. With the same center, and a radius equal to $\frac{7}{16}''$, describe the dotted circle, the horizontal projection of the rivet. The irregular line *G H* indicates that only a part of the rivet is shown. This is done so as not to take up too much space on the drawing.

Fig. 9 shows an ordinary **square-headed bolt** $\frac{7}{8}''$ in diameter, having a head $1\frac{3}{8}''$ square and $\frac{13}{16}''$ thick. Draw the center lines *m n* and *p q*. Construct the rectangle *A B D C*, $1\frac{3}{8}'' \times \frac{13}{16}''$, the elevation of the head. Locate the points *E* and *F* at a distance of $\frac{7}{16}''$ from each side of the center line, and draw *E G* and *F H*. With the compasses set to a radius of $1\frac{3}{8}''$ and from a point on the center line *m n*, describe the arc representing the chamfering of the head. Draw the plan of the head *L K B A* (a square whose edge measures $1\frac{3}{8}''$), and the dotted circle $\frac{7}{8}''$ in diameter, the projection of the body of the bolt, which cannot be seen in this view.

Fig. 10 shows a **distance piece** used to separate two other parts, and to keep them a certain distance apart. The arrangement of the views of this figure is somewhat different from the preceding ones, in order to make room for it on the drawing. Draw the center line *n m*, and construct the figure according to the dimensions marked on the plate. Use a radius of $\frac{1}{8}''$ for the fillets at *A*, *B*, *C*, and *D*, and an equal radius to round the corners at *E*, *F*, *G*, and *H*.

Fig. 11 shows a **square cast-iron washer.** Instead of making an elevation and plan as usual, a section is taken through *p q*; that is, the washer is imagined to be cut on the line *p q*, with all that part of the figure to the left of *p q* removed, and an elevation drawn of the remaining part. In order to distinguish a sectional drawing without any possibility of mistake, the so-called section lines are employed. These are usually drawn by laying a 45° triangle against the edge of the **T** square, and drawing a series of parallel lines as nearly equally distant apart as can be judged by the eye. For cast iron, these lines are full, thin lines, all of the same thickness, and must not be drawn too near together. The method of sectioning for other materials will be given later on. It is not usual to draw the section lines in pencil, but to wait until the outlines of the drawing have been inked in, and then section directly with the drawing pen. The shortest distance apart of the section lines should rarely be *less* than $\frac{1}{16}''$, unless the drawing is of such small dimensions as to cause a sectioning of this width to look coarse. This is the case with Figs. 11 and 12 of this plate. In these two figures make the section lines a full $\frac{1}{32}''$ apart. Only that part of the figure is sectioned which is touched by the cutting plane, the rest of the figure being drawn as if it were projected upon the cutting plane. The corners of this figure should be rounded with a radius of $\frac{1}{16}''$; the other dimensions can be obtained from the plate.

Fig. 12 is a **cast-iron cylindrical ring.** It is shown in plan and section. The dimensions given suffice for the drawing of the figure without further explanations. The inner circle of plan is the projection of the innermost points of the ring which form a circle whose diameter is *A B*.

44. When inking in a drawing, it is generally best to draw the circles and other curved lines first, and the straight lines afterwards. This enables the draftsman to easily blend into one line the straight lines meeting the curves, so that their points of meeting cannot be detected; it enables the tangent lines to be drawn with better success, and also shortens the time of inking in a drawing. It will be noticed that some of the straight lines are heavy and some light, and that parts of the full-line circles are heavy and the rest of the circle light. These are the shade lines; they are described later on. The student may make all of the full lines except the border lines of this plate, and the three following plates, of the same thickness, if he so desires. The dotted lines used to indicate those parts of the figures that are hidden must be of the same thickness as full lines, while the construction lines and center lines should be very thin

45. Dimensions.—The dimension lines and figures on this and succeeding plates are to be inked in by the student.

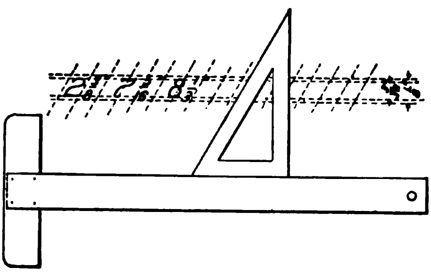

FIG. 69

Make the dimension figures $\frac{3}{32}''$ high, and of the same style as those shown in Art. **20.** Fractions should be $\frac{1}{8}''$ high over all. If there is not room for figures of this size, great care should be taken to make them *clear*.

Until after the student has obtained sufficient practice in lettering, he should draw guide lines in pencil for the dimension figures, as in Fig. 69, unless he can make them look well without. All the figures should have the same slant of 60°, and, when printing fractional dimensions, the *whole* fraction should have the same slant as the figures; that is, the denominator should be under the numerator in a *slanting* direction, and not straight below it. Make the dividing line between the numerator and denominator horizontal, not slanting.

Dimension and extension lines must be light, broken lines of the same thickness as the center and construction lines. Care should be exercised to make the arrowheads as neatly as possible and of a uniform size. They are made with a Gillott's No. 303 pen, and their points must touch the extension lines, as illustrated in Fig. 70. Do not make arrowheads too flaring.

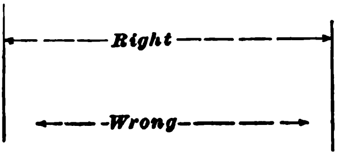

FIG. 70

When putting in the dimensions, care should be taken to give *all* that would be needed to make the piece which the drawing represents, but do not repeat the same dimension on different views. Thus, in Fig. 1 of this plate, the length is given in the front elevation as 2″, and it is obviously unnecessary to give the same dimension in the side elevation.

Again, the dimension lines should be put where they would be most likely to be looked for. In Fig. 10 of this plate, the diameter of the central part of the distance piece is marked 1¼" in the elevation: it could have been marked on the side elevation, as the diameter of the dotted circle, but a person wishing to find the size of this part of the piece would naturally look for it in the front elevation. This is also true of the diameter of the flange. The diameter of the hole could be on the plan or elevation, but it is put on the plan because it is denoted there by a full line, while in the elevation the hole is dotted. Never cross one dimension line by another, if it can well be avoided. Thus, in Figs. 2 and 4 of this plate, the bounding lines of the triangular views are extended by fine broken lines, in order that the dimension lines (¾") may not cross the lines marking the length and width of the wedge.

The student should ink in all the figures used for dimensions shown on this and succeeding plates, on his drawing, but should omit the letters used to describe the different objects. The titles should be made in block letters as shown on sample copies. The date, name, course letter, and class number are to be put on as in the preceding plates.

DRAWING PLATE, TITLE: PROJECTIONS—II

46. The figures on the last plate were drawn under the supposition that the center lines, and at least one flat side, were parallel to the plane of the paper—the center lines were also either vertical or horizontal. This is always possible in detail drawings, where each piece is drawn separately by itself, but in the case of machines, where the parts are placed at different angles, they cannot always be drawn in this manner. The figures on this plate are so drawn that they show objects similar to those in the last plate, but at different angles. The student should exercise particular care to understand this plate and the

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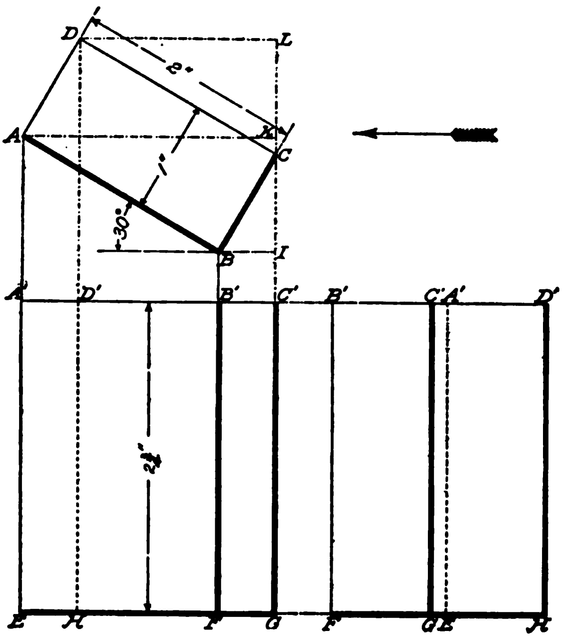
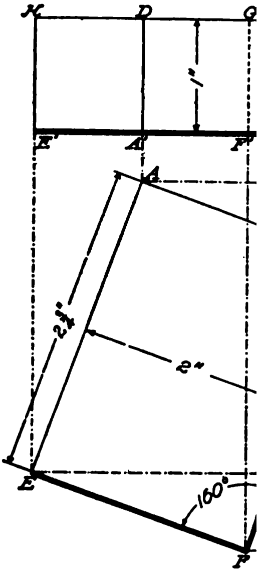


Fig. 1.



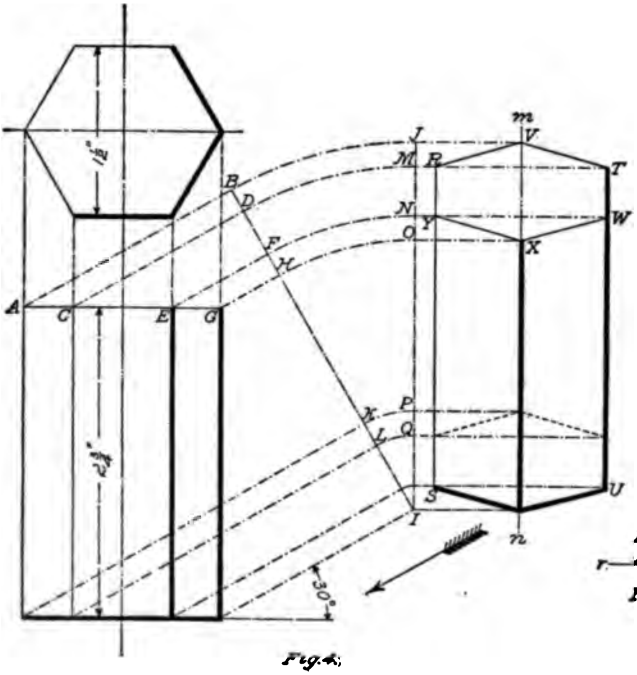


Fig. 4.

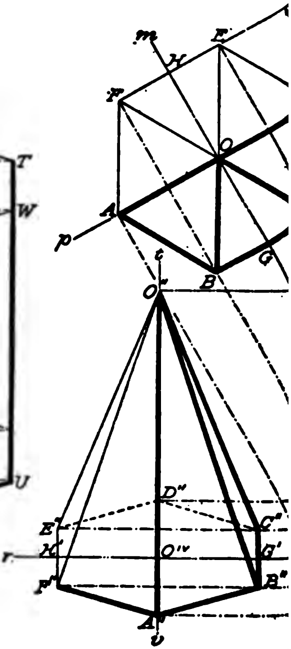


Fig.

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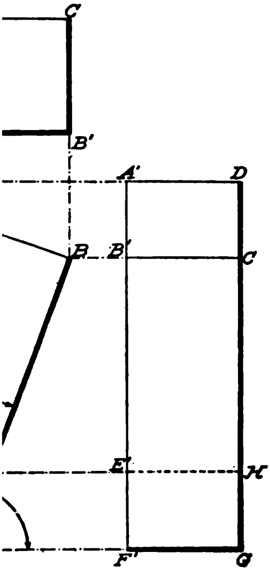
TIONS-II.



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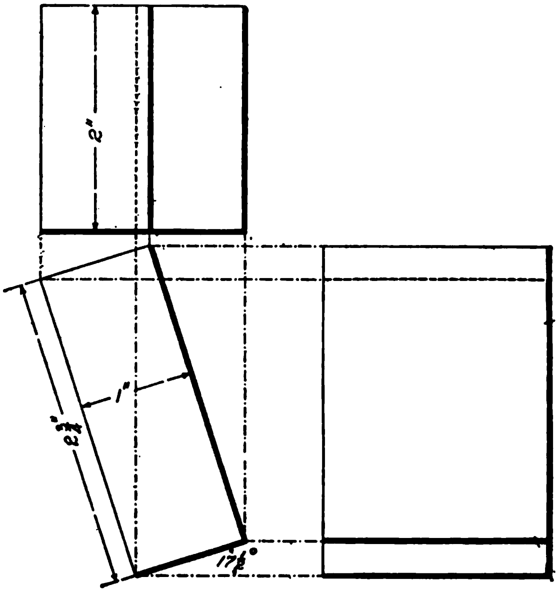
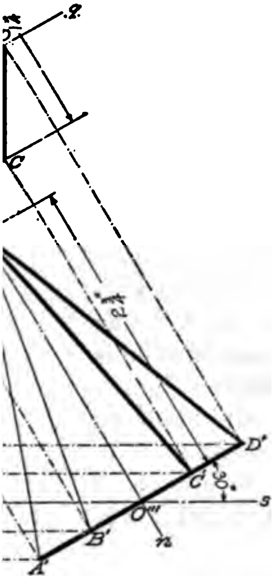


Fig. 3.



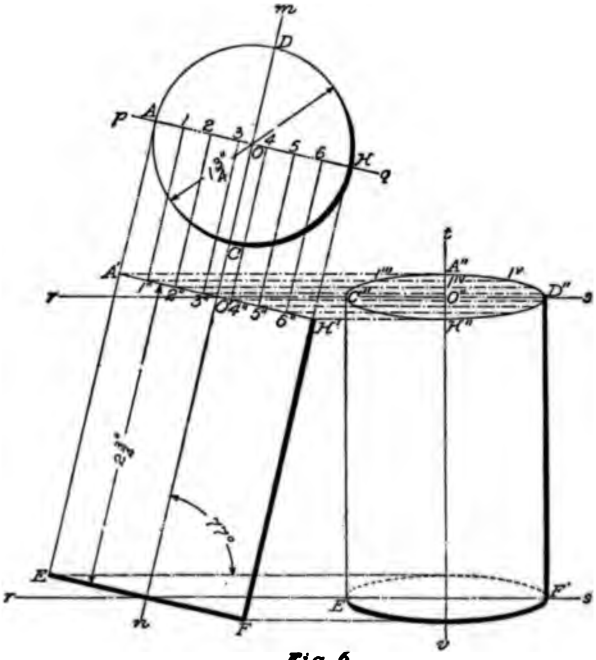


Fig. 6.

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projection or front elevation of the prism. The method of drawing the plan and side elevation is apparent without further explanation.

Fig. 3 is the same **prism** shown in Figs. 1 and 2, but with the narrow sides parallel to the plane of the paper, and tipped until the base makes an angle of 17½° with the horizontal. The sizes are the same as in the two preceding figures, and it should be drawn without further explanation, the front elevation being drawn first.

Fig. 4 shows a **hexagonal prism** having two of its parallel sides parallel to the plane of the paper, and its axis vertical; instead of a side elevation at right angles to the horizontal, a side elevation is desired, as if the vertical prism were looked at in the direction of the arrow, or at an angle of 30° with the horizontal. Draw the plan first and then the front elevation from the dimensions given. To draw the other view, first draw the center line *m n*, and then, by use of the **T** square and 30° triangle, draw the lines *A B*, *C D*, *E F*, and *G H*, from the points *A*, *C*, *E*, and *G*, as shown. Also draw in a similar manner the other four dotted lines at the base of the prism; then draw the line *I B* at a right angle to the lines *A B*, *C D*, etc. At the point *I*, draw the line *I J* parallel to the center line *m n*, and, with *I* as a center, and the points *B*, *D*, *F*, *H*, *K*, *L*, etc. as radii describe arcs, as shown, cutting the vertical line *I J* at the points *J*, *M*, *N*, *O*, *P*, *Q*, etc. Through the points *J*, *M*, *N*, *O*, *P*, *Q*, etc. draw horizontal lines as shown. On each side of the vertical center line *m n*, lay off a distance of ¾", or one-half the distance between the parallel sides of the prism, which is 1½", as shown in the plan, and draw the lines *R S* and *T U*. This view is then completed by drawing the lines *V R*, *V T*, *W X*, and *Y X*, as shown. The lines at the base are drawn in a similar manner.

Fig. 5 represents a **hexagonal pyramid** whose axis is parallel to the plane of the paper, the base making an angle of 30° with the horizontal. It is desired to find the vertical projection of the side elevation. Having drawn the plan *A B C D E F* and the side elevation *O'A'B'C'D'*, as shown from the dimensions marked on the drawing, choose the position of the vertical center line $t\,v$; project O' and O''' upon it in the points O'' and O^{IV}, and, through O^{IV} and O''', draw a fourth center line $r\,s$. On this, lay off $O^{IV}G'$ and $O^{IV}H'$ equal to $O\,G$ and $O\,H$, and construct the projection $A''\,B''\,C''\,D''\,E''\,F''$, as indicated by the broken and dotted lines. Join $O''\,E''$, $O''\,F''$, etc. by straight lines, and it will be the required projection. The figure thus drawn represents the pyramid as it would appear placed so that its base made an angle of 30° with the horizon, the line of vision being horizontal to the observer looking at it from the left side.

Fig. 6 shows a **cylinder** whose axis is parallel to the plane of the paper and makes an angle of 77° with the horizontal. The vertical side projection is required. Draw the plan and front projection as shown from the dimensions given. Draw the center line $t\,v$ vertical, and project the center O' upon it in O''; also, A' in A'', and H' in H''. To find the remaining points on the projected circle, divide the diameter $A\,H$ of the plan into a convenient number of equal parts, in this case 7, as $A\,1$, 1–2, 2–3, etc. Through the points thus laid off, draw the lines 1–$1''$, 2–$2''$, 3–$3''$, etc., parallel to the center line $m\,n$. Through the points A', $1''$, $2''$, $3''$, etc., draw the horizontal lines as shown by the dotted lines. From and on each side of the vertical center line $t\,v$, lay off distances on each side of the horizontal lines just drawn equal to the length of that part of the lines 1–$1''$, 2–$2''$, 3–$3''$, etc. included between the center line $p\,q$ and the semicircle $A\,C\,H$; thus, on the horizontal line drawn through the point O', the distances $O''C''$ and $O''D''$ are each equal to $O\,C$ in the plan. The distances 1^{IV}–$1'''$ and 1^{IV}–1^{V} are each equal to the distance from 1 to the point of intersection of the semicircle on the line 1–$1''$. The remaining distances are laid off in a similar manner. A curve traced through the points thus found will be the required projection of the upper base of the cylinder. The projection of the lower base is found in exactly the same way. Drawing $C''E'$ and $D''F'$ completes the required projection.

DRAWING PLATE, TITLE: CONIC SECTIONS

47. This plate shows the different forms of the curves formed by the intersection of a cone or cylinder by a plane. If the plane of intersection is perpendicular to the axis of the cone or cylinder, the curve of the intersection will be a circle; but if it is inclined to the axis, it will be an ellipse in the case of a cylinder, and an ellipse, hyperbola, or parabola in the case of a cone, according to the angle of inclination.

Fig. 1 is a **cone cut by a plane** which does not intersect the base of the cone. *When the cutting plane does not intersect the base*, or the base of the cone extended, the curve of intersection is an **ellipse.**

Draw the plan and front elevation of a right cone whose altitude is $3\frac{5}{8}$ inches and whose base is 3 inches in diameter. Cut this cone by a plane $a\,b$, making an angle of 52° with the base. See figure.

Divide the circle which represents the base of the cone in the plan into any number of parts, in this case 24, and, through the points of division A, E, H, etc., draw the radii $O\,A$, $O\,E$, $O\,H$, etc. to the center O. Draw also from these points straight lines $A\,A'$, $E\,E'$, $H\,H'$, $B\,B'$, etc., parallel to the axis of the cone $O'n$, and cutting the base $A'B'$ in the points E', H', etc. From these points, draw lines to the apex O' of the cone, and cutting the base $A'\,B'$ in points E', H', etc. From these points, draw lines to the apex O' of the cone, as $E'O'$, $H'O'$, etc., cutting the plane $a\,b$ in the points D', F', etc. From these points D', F', etc., draw straight lines $F'F\,F''$, $D'D\,D''$, etc., parallel to the axis $O'n$ of the cone, and intersecting the radii $O\,A$, $O\,E$, $O\,H$, $O\,B$, etc., in the points C, D, F, K, F'', D'', etc., and through these points of intersection draw the ellipse by aid of an irregular curve.

Fig. 2 is a cone of the same size as in the preceding problem; but the cutting plane $a\,b$ is, in this case, parallel to one of the elements* of the cone, and intersects the base. The

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*Any straight line drawn on the surface of a cone and passing through the apex (as $O'H'$, Fig. 1, or $O'A'$, Fig. 2, etc.) is called an **element.**

curve formed by the intersection of a cone by a plane parallel to one of its elements is called a **parabola.** The plan and front elevations of the cone and curve of intersection are found in a manner similar to the method used in the last problem. To find the side elevation, proceed as follows: Draw the side elevation $O''A''B''$ of the cone with the center line $t\,v$ as its axis. Draw the projection lines $F'F'''F^{\text{iv}}$, $D'D'''D^{\text{iv}}$, etc., and make $K'F'''$ and $K'F^{\text{iv}}$ equal to KF and KF''; make $I'D'''$ and $I'D^{\text{iv}}$ equal to ID and ID'', etc., and trace a curve through the points thus found. The result will be the side elevation of the cone when cut by a plane parallel to one of its elements and having the upper part removed. The side elevation of Fig. 1 may be drawn in a similar manner.

Fig. 3 is a cone having the same dimensions as the two preceding problems, but cut by a plane $a\,b$ parallel to the axis of the cone and perpendicular to the vertical plane of projection. When the cutting plane intersects the base of a cone and is not parallel to any element (that is, if the acute angle included between the cutting plane and the base is greater than the angle $O'A'B'$ included between any one element and the base), the curve of intersection is called a **hyperbola.**

The plan and front elevation are constructed as before, the horizontal projection of the curve for this particular case, where the cutting plane is parallel to the axis of the cone, is also a straight line. The side elevation is found as in the last problem, by drawing the lines of projection $F'F'''F^{\text{iv}}$, $D'D'''D^{\text{iv}}$, etc., and making $I'D'''$ and $I'D^{\text{iv}}$ equal to ID and ID'', $K'F'''$ and $K'F^{\text{iv}}$, equal to IF and IF'', etc. The curve drawn through the points thus found will be the required hyperbola.

Fig. 4 shows the **intersection of a cylinder,** 3⅝" long and 2" in diameter, by a plane $a\,b$, making an angle of 57° with the base. The plan and elevation may be drawn as shown, the horizontal projection of the curve being a circle, having the same diameter as the base. To construct the side elevation of the curve, divide the circle representing the

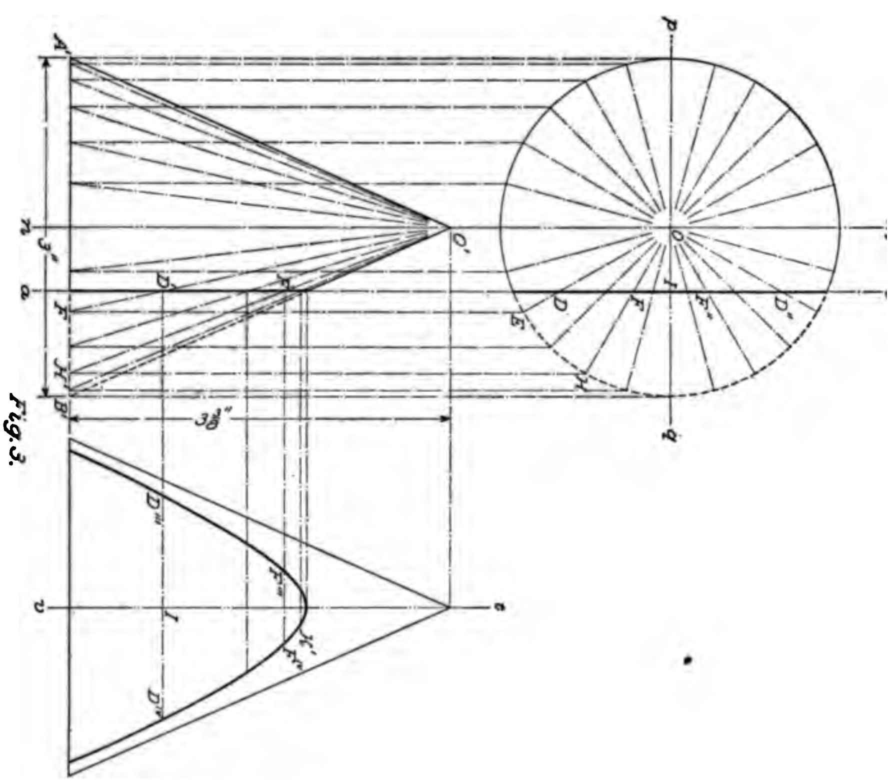

Fig. 3.

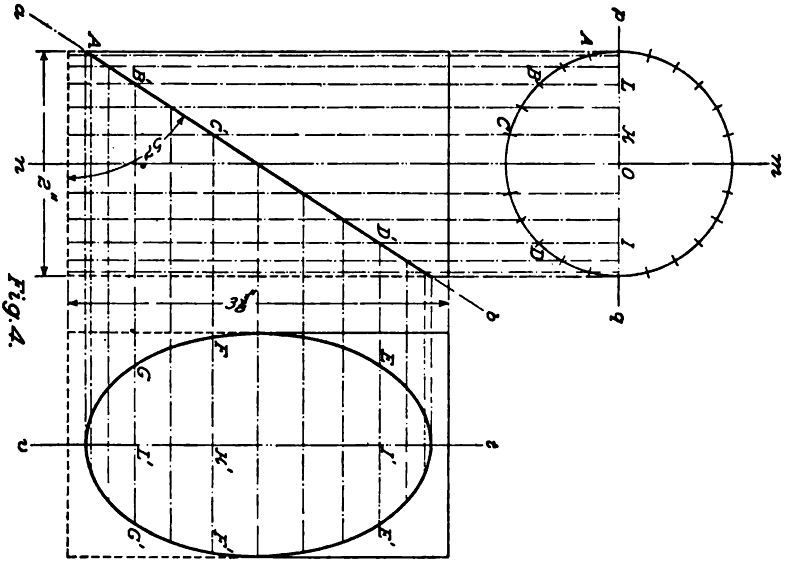

Fig. 4.

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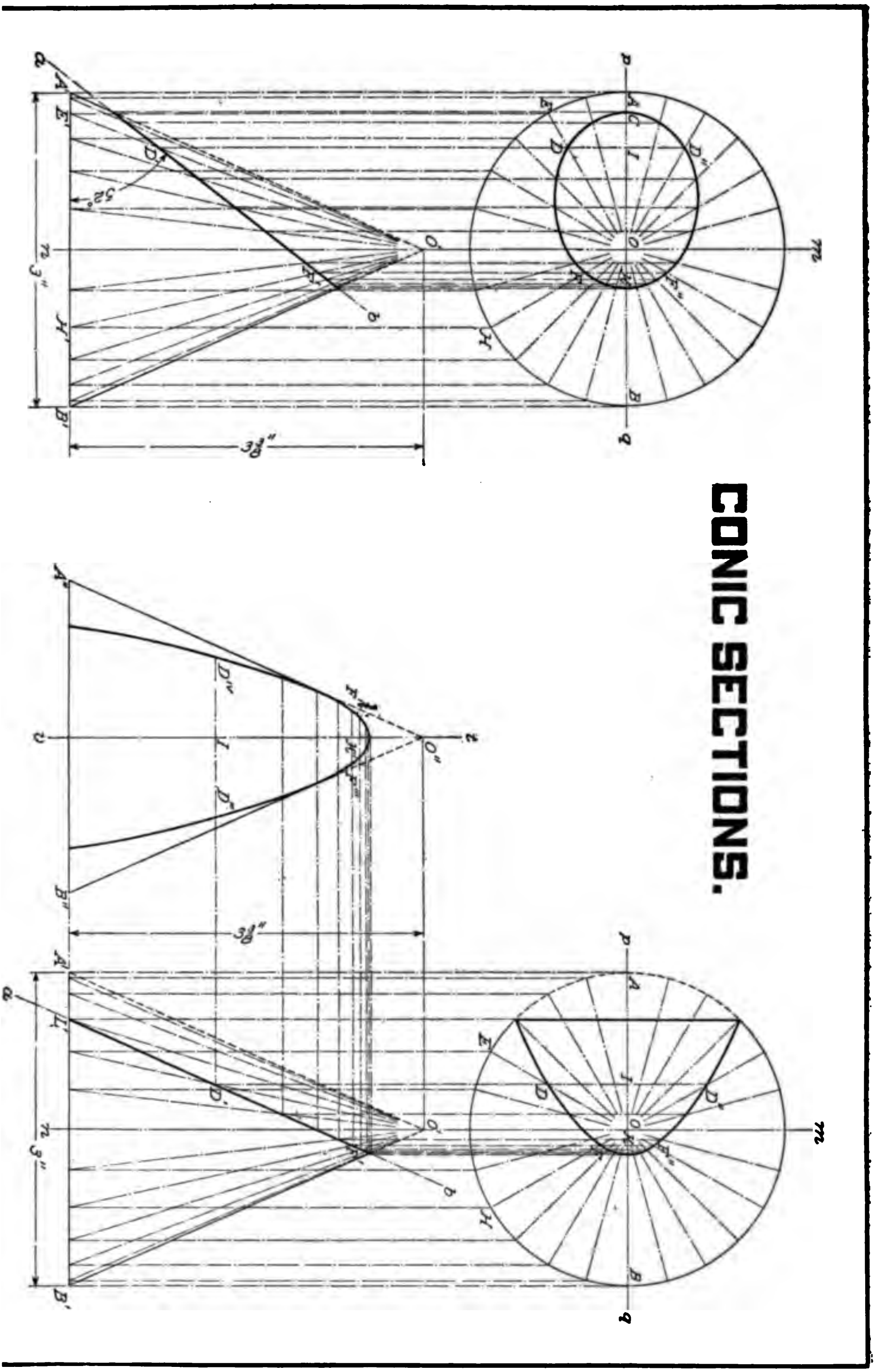

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base of the cylinder in the plan into any number of parts, in this case 24, and through the points of division A, B, C, etc. draw the radii OA, OB, OC, etc. to the center O. Draw also from these points straight lines AA, LBB', KCC', IDD', etc. parallel to the axis mn of the cylinder, and cutting the base in the elevation. From the points A, B', C', D', etc. draw lines $D'E'$, $C'F'$, $B'G'$, etc. at right angles to the axis mn. Make $I'E$ and $I'E'$ each equal to ID; $K'F$ and $K'F'$ each equal to KC; $L'G$ and $L'G'$ each equal to LB, etc. The curve drawn through these points will be the side projection, or side elevation, of the curve of intersection.

DRAWING PLATE, TITLE: INTERSECTIONS AND DEVELOPMENTS

48. On this plate some dimensions are given in decimal fractions instead of common fractions. Such decimal dimensions should be laid off with a decimal scale, if the student has one. A decimal scale is a scale with inches divided into tenths, hundredths, etc. If the student has no decimal scale (and such a scale is not essential), he should take the nearest value of the decimal fraction in thirty-seconds of an inch.

To change a decimal fraction to a common fraction, having a desired denominator, multiply the decimal by the desired denominator of the common fraction, and express the result as a whole number, which whole number will be the numerator of the fraction.

Thus, to express .765" in fourths, we have $.765 \times 4 = 3.06$ fourths = say, $\frac{3}{4}''$. To express .765" in sixteenths, we have $.765 \times 16 = 12.24$ sixteenths = say, $\frac{12}{16}''$. To express .765" in thirty-seconds, we have $.765 \times 32 = 24.48$ thirty-seconds = say, $\frac{24}{32}''$.

The length of the circumference of a circle = the diameter × 3.1416; hence,

The length of circumference of a circle whose diameter is

$$1\tfrac{3}{8}'' = 3.1416 \times 1\tfrac{3}{8}'' = 4.32'' = 4\tfrac{5}{16}''.$$

The length of circumference of a circle whose diameter is

$$1\tfrac{1}{2}'' = 3.1416 \times 1\tfrac{1}{2}'' = 4.71'' = 4\tfrac{23}{32}''.$$

The length of circumference of a circle whose diameter is

$$1\tfrac{1}{4}'' = 3.1416 \times 1\tfrac{1}{4}'' = 3.93'' = 3\tfrac{15}{16}''.$$

The length of circumference of a circle whose diameter is

$$1\tfrac{9}{16}'' = 3.1416 \times 1\tfrac{9}{16}'' = 4.9''.$$

$$4.9'' \div 2 = 2.45'' = 2\tfrac{7}{16}'' \text{ (see Fig. 10).}$$

49. This plate deals with the intersection of surfaces and their development. Fig. 1 shows the intersection of **two unequal cylindrical surfaces** whose axes $p\,q$ and $m\,n$ intersect at right angles. Their dimensions are given in the figure. For the sake of convenience, a bottom view is given, instead of a top view, as usual. First draw the front elevation, omitting, of course, the curve of intersection $E\,Q\,G\,D\,C\,B\,A$, which must be found. Then draw the side elevation and the bottom view, as shown. Divide the circle which represents the side projection of the cylindrical surface $F\,E\,A\,1$ into any convenient number of parts, in this case 12, and draw the projection lines $7\,E$, $6\,Q$, $5\,G$, $4\,D$, $3\,C$, $2\,B$, and $1\,A$ parallel to the axis $p\,q$. Also draw the projection lines 4–$4'$, 3–$3'$, 2–$2'$, 1–$1'$, etc. parallel to the axis $t\,v$. Choose a convenient point O, and through it draw two lines $O\,I$ and $O\,K$ parallel to the axes $p\,q$ and $m\,n$ of the cylinders. Continue the lines 4–$4'$, 3–$3'$, etc. downwards, until they cut $O\,I$ in 8, 7, 6, 5, etc. Now make $O\,8' = O\,8$, $O\,7' = O\,7$, etc.; this may be most conveniently done by taking O as a center, and describing arcs of circles with radii equal to $O\,8$, $O\,7$, $O\,6$, etc., cutting $O\,K$ in $8'$, $7'$, $6'$, etc. Through $8'$, $7'$, $6'$, etc., draw the lines $8'D'$, $7'C'$, $6'B'$, etc. parallel to the center line $r\,s$. Through the points D', C', B', and A', draw the lines $D'D$, $C'G$, and $B'Q$, parallel to the center line $m\,n$, and intersecting the lines $4\,D$, $5\,G$, $6\,Q$, $3\,C$, and $2\,B$ in the points D, G, Q, etc. The curve traced through these points will be the front elevation of the curve of intersection of the two cylindrical surfaces.

Fig. 2 shows the intersection of **two equal cylindrical surfaces** at right angles to each other, as in the case of a pipe elbow. When two cylinders having *equal diameters intersect, and their axes also intersect,* the front elevation of

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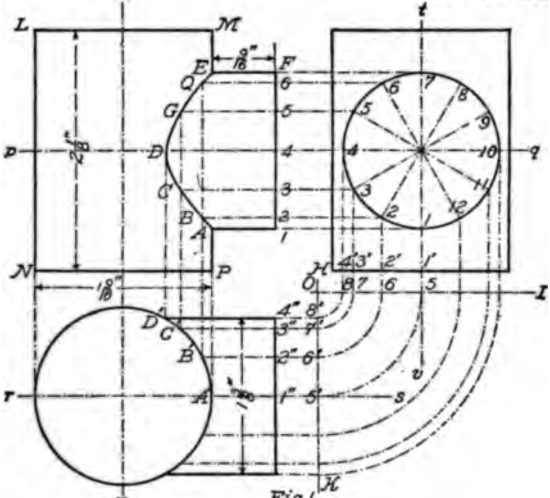

Fig.1.

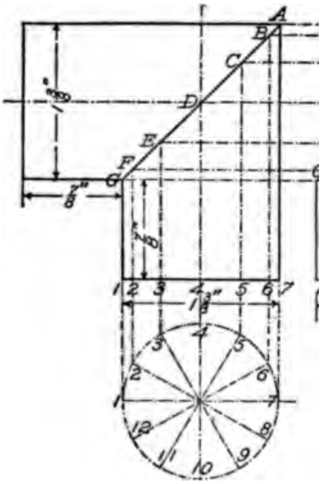

Fig.2.

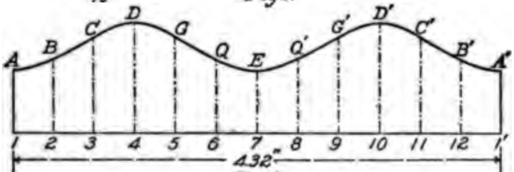

Fig.8.

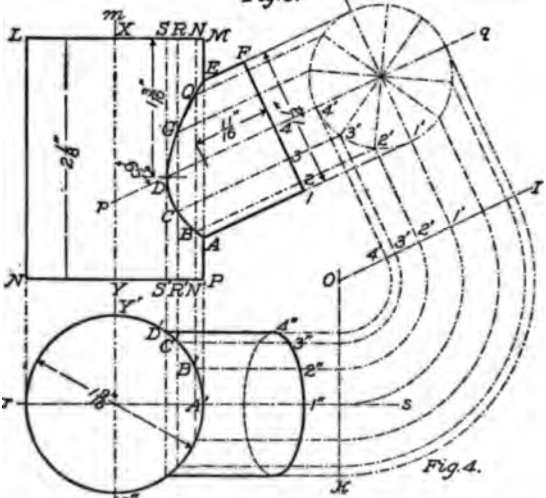

Fig.4.

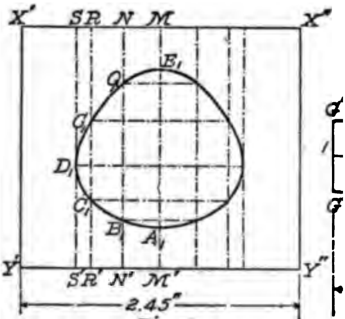

Fig.10.

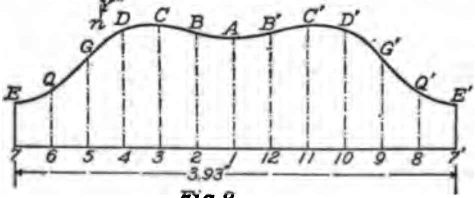

Fig.9.

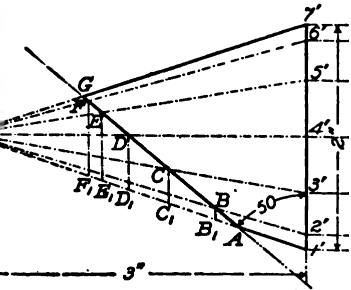

Fig.11.

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DEVELOPMENTS.

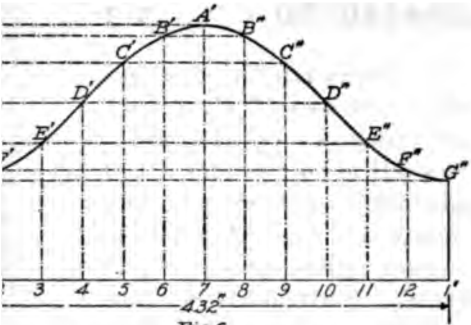


Fig.6.

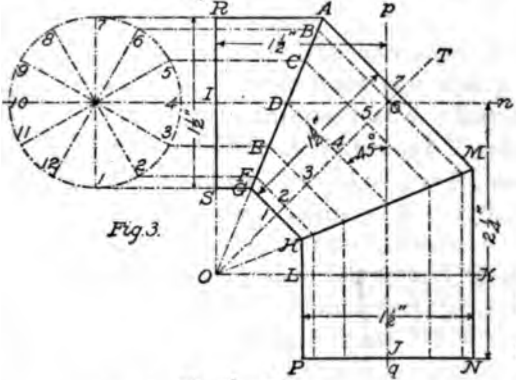


Fig.3.

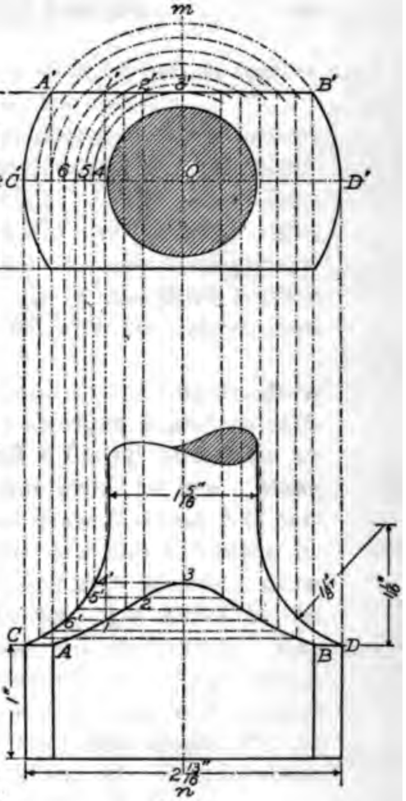


Fig.5.

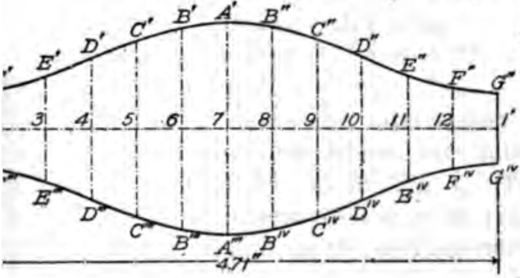
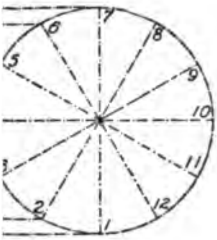


Fig.7.



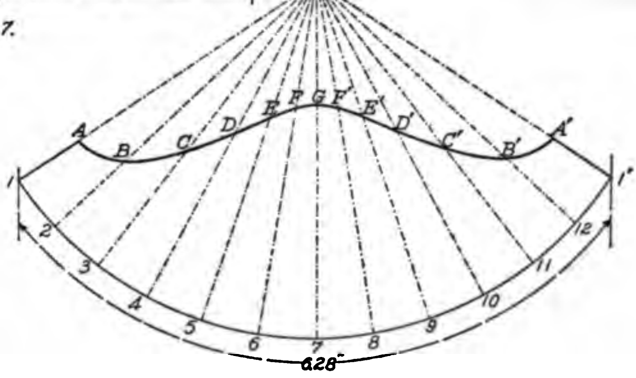


Fig.12.

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the curve of intersection is always a straight line, no matter what angle the two axes make with each other.

Fig. 3 shows a symmetrical **three-jointed elbow** formed by the intersection of three cylindrical surfaces. The diameter of each of the three surfaces is $1\frac{1}{2}''$. The center lines of the surfaces $RAGS$ and $MNPH$ are to be at right angles to each other; then, in order that the arrangement shall be symmetrical, the center line of the third surface $AMHG$ must make an angle of 45° with the center lines of the other two.

To construct the elevation as shown in the figure, draw the two center lines mn and pq at right angles to each other; they intersect at *6*. Lay off $6I = 1\frac{1}{2}''$ and draw an indefinite line RS through I perpendicular to mn. Make IR equal to $IS = 1\frac{1}{2} \times \frac{1}{2} = \frac{3}{4}''$, and draw RA and SG parallel to mn. Draw OK parallel to mn and $1\frac{1}{2}''$ below it. Through the point O, where RS and OK intersect, draw OT passing through *6*, and bisect the angle ROT by the line OA, which intersects RA and SG in A and G. Lay off $6J = 2\frac{1}{4}''$ and draw PJN perpendicular to pq. Make $JP = JN = 1\frac{1}{2} \times \frac{1}{2} = \frac{3}{4}''$, and draw PH and NM parallel to pq. Draw OM so as to bisect the angle TOK; OM intersects PH and NM in H and M. Finally, draw AM and GH.

Fig. 4 shows the intersection of **two unequal cylindrical surfaces** whose axes intersect at an angle of 65° instead of 90°, as in Fig. 1. The method of finding the curve of intersection is in all respects similar to that used in Fig. 1, and, as the corresponding points have been given the same letters or figures, the directions given for Fig. 1 can be applied to Fig. 4 also.

Fig. 5 shows a **cylindrical piece of iron** $2\frac{3}{16}''$ in diameter that has been gradually turned down to $1\frac{5}{16}''$ diameter, and then having the larger part flattened on two sides. The large and small parts of the piece are connected by a graceful curve. The problem is to find the curve of intersection $A\,1\,2\,3\,B$ formed by the flattening. Draw the plan and front elevation from the dimensions given; also draw

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the curve *C 6′ 5′ 4′*, and its equal on the opposite side, so that they look to the eye about as seen in the drawing. In order that all the work sent to us may be alike, the radius of this curve and the position of the center have been given on the drawing. To locate the center, draw an indefinite horizontal straight line $1'' + 1\frac{1}{16}'' = 2\frac{1}{16}''$ above the base of the piece; and with *C* and *D* as centers, and a radius of $1\frac{1}{8}''$, describe short arcs cutting the line just drawn. The points of intersection will be the required centers. With *O* as a center, and radii of convenient lengths, as *O 4*, *O 5*, *O 6*, etc., describe arcs cutting *A′ B′* in *3′*, *2′*, *1′*, etc. Through the points *4*, *5*, *6*, etc. draw the lines *4–4′*, *5–5′*, *6–6′*, etc., parallel to the center line *m n*, and intersecting the curve *C 4′* in *4′*, *5′*, *6′*, *C*, etc. Through the points *A′*, *1′*, *2′*, etc. draw lines *A′A*, *1′–1*, *2′–2*, etc., parallel to *m n*, intersecting horizontal lines drawn through *C*, *6′*, *5′*, *4′*, etc., in *A*, *1*, *2*, *3*, etc. The points *A*, *1*, *2*, *3*, etc. are points on the required curve, and through them the curve may be drawn.

Fig. 6 is the **cylindrical surface** of one section of the elbow *1 7 A G* of Fig. 2 rolled out into a flat plate; hence, if a flat plate were cut into the same shape and size as Fig. 6 and bent into a cylinder so that the ends *1 G′* and *1′G″* touch each other, the vertical projection or front elevation would be the same as shown by *1 7 A G* in Fig. 2. If a second plate were cut out in the same manner and bent into a circle, the two pieces on being brought together, as shown in Fig. 2, would touch at every point. The problem is to find the shape of the curve *G′ A′ G″*. The length of the line *1–1′* is evidently equal to the length of the circumference of a circle whose diameter is $1\frac{3}{8}''$, or 4.32″, very nearly. Produce the line *1–7*, Fig. 2, and make *1–1′* equal in length to 4.32″. Divide the circle *1 2 3 . . . . 12* into a convenient number of equal parts, in this case 12, and erect the perpendiculars *1 G*, *2 F*, *3 E*, etc., cutting the line of intersection *G A* of the cylindrical surfaces in *G*, *F*, *E*, etc. Divide the line *1–1′* into the same number of equal parts that the circle was divided into, thus making the length *1–2* equal length of arc *1–2*; *2–3*, length of arc *2–3*, etc. Through *1*,

2, 3, etc., draw the perpendiculars $1\,G'$, $2\,F'$, $3\,E'$, etc. and project the points G, F, E, etc. upon these perpendiculars, as shown, thus locating the points G', F', E', D', C', B', A' of the left-hand half of the required curve. The points on the right-hand half are found in the same manner, as shown, and the required curve can be drawn through these points.

50. A drawing like Fig. 6 is called the **development** of the cylindrical surface $1\,7\,A\,G$.

Fig. 7 is the **development of the cylindrical surface** $A\,G\,H\,M$ of Fig. 3. Make $1\text{–}1' = 1\tfrac{1}{2} \times 3.1416 = 4.71''$, nearly, and divide it into 12 equal parts to correspond with the 12 equal parts into which the dotted circle is divided. Project the points 6, 5, etc. of the dotted circle upon $O\,A$ as shown, thus locating the points B, C, etc. Through B, C, etc., draw $B\,6$, $C\,5$, etc., perpendicular to $O\,T$. Make $1\,G' = 1\,G''' = 1\,G$, $2\,F' = 2\,F''' = 2\,F$, $3\,E' = 3\,E''' = 3\,E$, etc. Through G', F', E', etc., trace the curve $G'\,F'\,E' \ldots . G''$, and, through G''', F''', E''', etc., trace the curve $G'''\,F'''\,E''' \ldots . G^{\text{iv}}$. Drawing $G'\,G'''$ and $G''\,G^{\text{iv}}$ completes the figure.

Fig. 8 is the development of the cylindrical surface $1\,F\,E\,A$, Fig. 1. The method used here is in all respects similar to the two preceding problems. In this case, the distances $1\,A$, $7\,E$, and $1'\,A'$ are all equal to $1\,A$ or $E\,F$, in Fig. 1; and $2\,B$, $6\,Q$, $8\,Q'$, and $12\,B'$ are all equal to $2\,B$ or $6\,Q$, in Fig. 1. The development of $L\,M\,P\,N$ is not given, for want of room, but the method will be explained in Fig. 10.

Fig. 9 is the development of the cylindrical surface $1\,F\,E\,A$, Fig. 4. The student should have no difficulty in drawing this, after having studied the preceding problems.

Fig. 10 is the development of the cylindrical surface $L\,M\,P\,N$, Fig. 4. Owing to the want of room, only that half of the development is shown which contains the part to be cut out. The length of a circle $1\tfrac{9}{16}''$ in diameter is $4.9''$, nearly; half of this is $2.45''$. Hence, the line $Y'\,Y''$, Fig. 10, which equals the length of the semicircle $Y'\,A'\,Y''$, Fig. 4, is $2.45''$ long. The distance $X'\,Y' = X''\,Y''$ equals the length of the cylinder, $L\,N$ or $M\,P$. Lay off $X'\,S$ equal to the length of the arc $Y'D'$; SR equal to the arc $D'C'$; RN equal to the arc $C'B'$; NM equal to the arc $B'A'$, etc. Find the lengths of these arcs by means of the method given in connection with Fig. 49. Draw through these points the perpendiculars SS', RR', etc. With the spacing dividers, set off SD_1 equal to SD in Fig. 4; RG_1 equal to RG; NQ_1 equal to NQ; and ME_1 equal to ME. Also, $R'C_1$ equal to $R'C$; $N'B_1$ equal to $N'B$; and $M'A_1$ equal to PA. In exactly the same manner, find the points on the right-hand half of the curve. If a plate were cut of the same size and shape as shown in Fig. 10, and rolled into a semicylindrical surface, the diameter of which is $1\frac{9}{16}''$, it would exactly fit the plate cut like Fig. 9 rolled into a cylindrical surface, the diameter of which is $1\frac{1}{4}''$, the two being placed together as shown in Fig. 4.

Fig. 11 shows a **conical surface cut by a plane,** and Fig. 12 shows its **development.** Draw the elevation and horizontal projection of the base as shown in Fig. 11. Divide the projected circle (base of cone) into a convenient number of equal parts, in this case 12, and project the points *1*, *2*, *3*, etc. on the base *1′–7′*, thus locating the points *1′*, *2′*, *3′*, etc. Join these points with the apex O of the cone, by the lines $O1'$, $O2'$, $O3'$, etc., cutting the plane in A, B, C, etc. Now, choose a convenient point O, Fig. 12, and with this as a center, and a radius equal to $O1'$, or $O7'$, Fig. 11, the slant height of the cone, describe an arc *1–1′* of a circle. Make the *length of this arc* equal to the length of the circumference of a circle having the same diameter as the base of the cone. This may be conveniently done as follows: length of arc $= 2 \times 3.1416 = 6.28''$, nearly. Draw a straight line 6.28″ long and divide it into, say, 4 equal parts. Describe an arc having a radius equal to $O1'$, the slant height of the cone, and find the length of a part of this arc equal to $6.28 \div 4 = 1.57''$ by means of the method described in connection with Fig. 48. With the dividers set for the chord of the arc just found, space off the chord four times on the longer arc *1 2 3 . . . . 1′*, Fig. 12. Divide the arc into the same number of equal parts that the circle $1\ 2\ 3 \ldots 12$ has been divided into, that is, 12 parts. Join the points of division 1, 2, 3, etc. with the center O by the lines $O\,1$, $O\,2$, $O\,3$, etc., as shown. Project the points B, C, D, etc., Fig. 11, upon $O\,1'$, in B_1, C_1, D_1, etc., as shown, and lay off $O\,A$ equal to $O\,A'$ equal to $O\,A$, Fig. 11; $O\,B$ equal to $O\,B'$ equal to $O\,B_1$; $O\,C$ equal to $O\,C'$ equal to $O\,C_1$, etc., and through these points draw the curve. A plate cut of the same size and shape as shown by $A\,G\,A'\,1'\,7\,1$ can be bent into the conical surface shown by the elevation $A\,G\,7'\,1'$.

Particular attention must be given to the method explained above for laying out the curve of the development in Fig. 12. It would be entirely wrong to take the measurements from the lines $O\,F$, $O\,E$, $O\,D$, $O\,C$, etc., Fig. 11. The reason for this is that these lines, being on the surface of the cone, are inclined towards the observer, and so do not appear in their true lengths. The line $O\,D$, for example, if measured on the surface of the cone itself, would evidently be of the same length as the line $O\,D_1$; but in the figure it is much shorter. The line $O\,D_1$, however, appears in its true length in the figure, because it is not inclined to the observer in the position shown. The actual distance of point D from the apex O, therefore, is $O\,D_1$, which is the distance to be laid off for point D in the development. The same holds true for the other points.

SHADE LINES

51. The use of the heavy shade line will now be explained In Fig. 71, by means of the shade lines, the draftsman knows, without looking at any other view of the object, that the rectangles *1* and *4* represent square holes, and *2* and *3*, square bosses. When he looks at the other view, it is to find the depth of the holes and the height of the bosses. This explains the use of the shade lines, viz.: to show, from that view of the drawing which is being examined, whether the part looked at is above or below the plane of the surface: that is, for example, whether the rectangles *1*, *2*, *3*, and *4* are the tops of bosses or bottoms of holes, and, consequently, whether they extend above or below the surface of *A B D C*. In order that the shading may be uniform on all drawings, the light is assumed to come in one invariable direction, in such a manner as to be parallel to the plane of the paper, to make an angle of 45° with all horizontal and vertical lines of the drawing, and to come from the upper left-hand corner of the drawing. Each view of the object represented is shaded independently of any of the others; and, when shading, the object is always supposed to stand in such a position that the drawing will represent a top view. Any surface that can be touched by drawing a series of parallel straight lines, making an angle of 45° with the horizontal and vertical lines of the drawing, is called a **light surface**; a surface that cannot be touched by lines having this angle is called a **dark surface**. All of the edges caused by the intersection of a light and dark surface, or two dark surfaces, are usually shaded; that is, the edges thus formed are drawn in heavy lines. Exceptions to this rule are sometimes made by experienced draftsmen, when a rigid adherence to it will produce a bad effect or will render the drawing ambiguous.

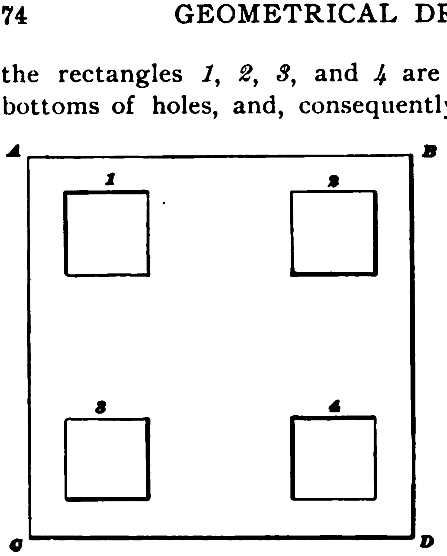

FIG. 71

Fig. 72 shows a plan of a series of triangular wedges radiating from the common center *O*. The top is, of course, a light surface, and, in order to determine whether the perpendicular surfaces are light or not, the 45° triangle may be used. Take the wedge *R O A*. A line drawn at an angle of

45°, the direction of the arrows, would strike the side of which OA is the edge; hence, this side is a light surface, and the top being also a light surface, the line OA must be light. OR, on the contrary, is a heavy line, since the light cannot strike the side of which OR is the edge without passing through the wedge. Hence, this is a dark surface, and its intersection OR with the light surface OAR requires a shaded line. For the same reason, AR is also shaded. The same reasoning as the above applies to the lines OB, OD, OG, OI, OK, and OM; also, to QN, ML, and KJ. CB is not shaded, because the light strikes the surface of which CB is the edge, as shown by the arrow, making CB the intersection of two light surfaces. ON makes an angle of exactly 45° with the horizontal, and is treated as if it were the edge of a light surface; this is done in every case in which the line considered makes an angle of 45° with the horizontal.

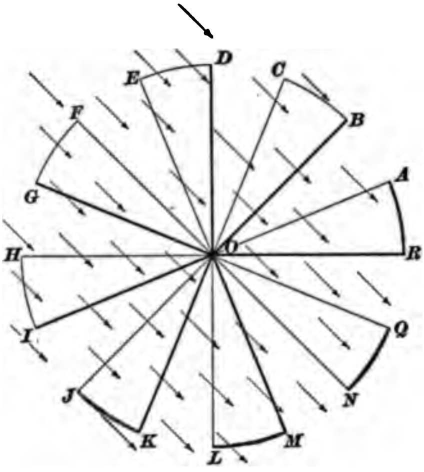

FIG. 72

In shading holes, or any parts of the drawing denoting depressions below the surface under consideration, a slightly different assumption is made. Fig. 73 shows the plan of a square block with a hexagonal hole in the center. If the light passed over the surface $ABCD$, parallel to the plane of the paper as previously assumed, all the inside surfaces would be dark, and the entire outline of the hexagon $EFGHIK$ would be shaded. In order to prevent this and make the work similar to that which has preceded, the rays of light are assumed to make an angle of 45° with the plane of the paper when shading holes and depressions. Hence, the light will strike the surfaces whose edges are *G H*, *H I*, and *I K*, as shown by the arrows, leaving the surfaces whose edges are *K E*, *E F*, and *F G* dark as before. Therefore, these latter edges will be shaded, and the edges *G H*, *H I*, and *I K* will be light. See also Fig. 71.

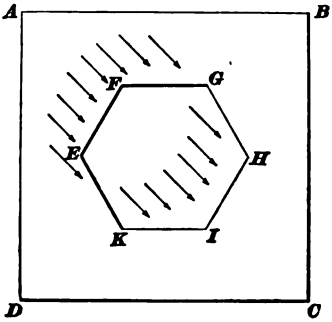

FIG. 73

The conventional method of shading circles which represent the projections of cylinders, or circular holes, is as follows: *A B*, Fig. 74, is the projection or end view of a cylinder having for a base the circular area *A B*. Draw the arrows *E A* and *F B*, making angles of 45° with the horizontal diameter, and tangent to the circle at *A* and *B*. That half of the circle in front of these two points of tangency is to be shaded, and, in order to make the drawing look well, the center point for the compasses is shifted along the line *C H* parallel to *E A* and *F B* in the direction of the arrow an amount equal to the thickness of the desired line. With the same radius that was used to describe the original circle, describe part of another circle, being careful not to run over the first circle, and stopping when the two lines coincide. The directions for shading a hole are precisely the same as for the projection of a cylinder base, except that the half *B C A* of the circle in Fig. 75 is to be shaded, the center being shifted as before, but in the opposite direction, as shown by the arrow.

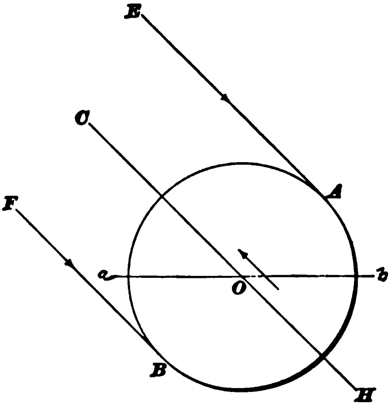

FIG. 74

Vertical projections of cylinders are shaded as shown in the front elevation of Fig. 5, Drawing Plate, title: Projections—I.

After studying the foregoing concerning shade lines, the student should be able to see the reason for the using or omitting of any shade lines on the drawings in the following plates. In the case of an object like the hexagonal prism in Fig. 6, Drawing Plate. title: Projections—I, no part of the upper base or line *Se* is shaded, although, strictly speaking, the part *c e* of the line should be shaded; but, as this would make part of the straight line *Se* heavy and the greater part light, the whole line is drawn light. This is one of the exceptions previously mentioned.

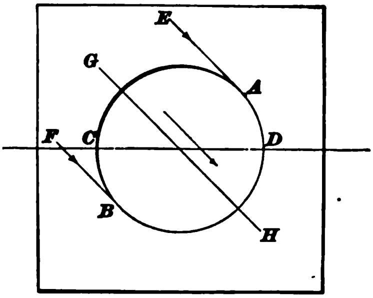

FIG. 75

MECHANICAL DRAWING

CENTER LINES

1. Fig. 1 represents a thick wedge having a cylindrical hole running through its entire length. The lines *m n*, *p q*, and *r s* are called **center lines.** Center lines are usually drawn through the center of anything that is round, such as a cylinder or a cylindrical hole. In the case of a circle, there are usually two center lines, one being at right angles to the other, as shown in view *B*, Fig. 1. By drawing two center lines through a circle in the manner just mentioned, the center of the circle is located by their intersection. The mere presence of these lines shows in most cases that part of the object through which they are drawn is *round*. It is very seldom that center lines appear on drawings unless they are the center lines of cylindrical surfaces. They may sometimes be drawn to indicate that the surface is a regularly curved surface, such as would be formed by circular arcs, or

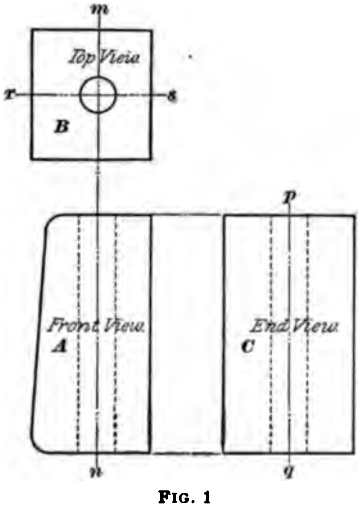

FIG. 1

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one having the shape of an ellipse. In very rare cases, for some special reason, a line that corresponds to a center line may be drawn for some particular purpose, but such a line is not, in the strict sense of the word, a center line.

2. Center lines are an extremely important feature of a drawing, since the workman is guided by them in doing the work called for by the drawing. For instance, suppose that it was required to make a wedge like that shown in Fig. 1, and that the workman was given a piece of cast iron having approximately the shape indicated by the drawing. Suppose, further, that it was necessary to have the hole located exactly as shown in the drawing with reference to the sides of the wedge, and that the sides and ends of the wedge were all to be "finished." The first thing that the workman would probably do would be to drill the hole, and, if the job had to be very accurate, he would drill the hole a little smaller than the drawing calls for and then ream it out to size. He would then face the ends square with the center line of the hole and make the length of the wedge the same as shown on the drawing. The sides of the wedge would then be planed and finally finished with a file, the workman working all the time from the center line *m n*. If the drawing shown in Fig. 1 is intended to be worked to, it would, in most shops, be supplied with proper dimensions.

SECTIONS AND SECTION LINING

3. In order to show the interior of hollow objects, they are often drawn in section, and the kind of material is then usually indicated by certain combinations of lines. Unfortunately, there is no universally adopted standard; thus, a certain combination of lines may indicate that the material is cast iron if drawn in one office; in another office this same combination may have been adopted to represent brass, and so on. As far as working drawings are concerned, there is usually no difficulty experienced on account of this diversity of practice, since as a general rule the material is, and should always be, distinctly specified on the drawing in order to prevent any mistake on the part of the workman.

4. The most commonly used combination of lines for different materials is shown in Fig. 2. Steel of all kinds is indicated as shown in view *A*; view *B* shows the style of sectioning employed for wrought iron. Cast iron is usually sectioned as shown at *C*; brass and other similar copper alloys are sectioned in the manner shown at *D*. For lead, Babbitt, and similar soft metal, the sectioning shown at *E* is extensively used. Wood, when cut across the grain, is usually sectioned as shown in the upper half of view *F*, and when cut along the grain, as shown in the lower half. Wood is also frequently indicated on a drawing by section lines, even when it is not a section. Glass and stone, when in section, are often indicated in the manner shown by the upper half of view *G*; when not in section, they are frequently drawn as shown in the lower half of that view. Concrete may be indicated as in view *H*; view *I* gives a common representation of leather. Rubber and wood fiber are sectioned as in view *J*, firebrick as in *K*, and water as in *L*.

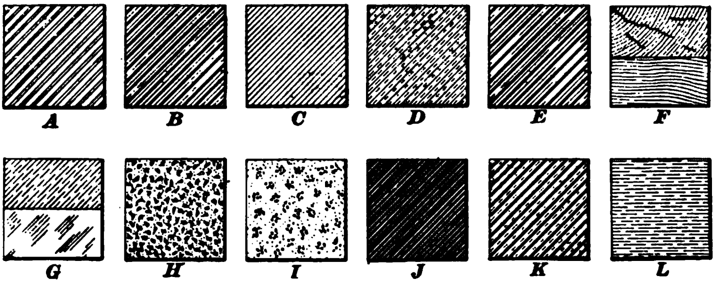


FIG. 2

5. Instead of representing sections by lines, they are occasionally colored, the colors used indicating the different materials. While this practice is very common in Europe, it is very rarely found in the United States.

6. Sections of material that appear too thin on a drawing to be conveniently sectioned, or when it is desired to make the section very prominent, are often blackened in, as shown in Fig. 3. In order to separate different pieces, a white line is then usually left between them. Black sections are most frequently employed for sectional views of structures composed of plates and rolled sections, such as **I** beams, angle irons, bulb angles, rails, **Z** bars.

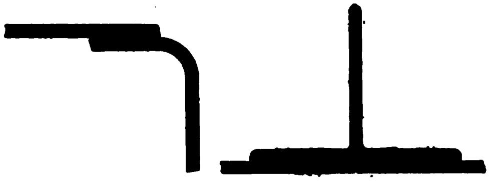


FIG. 3

7. On many sectional views, it will be noticed that the section lines do not run in the same direction. This invariably means that there is more than one piece in the section given. Thus, referring to Fig. 4, it will be seen that the section lining shown at *b*, *b* is at a right angle to the other section lining. It is the general rule among draftsmen that all parts of the same piece shown in section must be section-lined in the same direction, irrespective of the continuity of the section. Thus, referring again to Fig. 4, the fact that all section lining marked *A* is in the same direction immediately establishes the fact that this part of the view is a section of the same piece. Likewise, since the sectioning shown at *b*, *b* runs in the same direction, it follows that *b*, *b* are sectional views of one piece, which is separate from *A*.

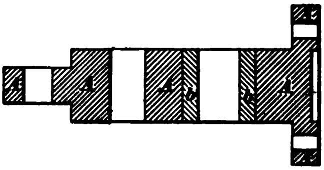
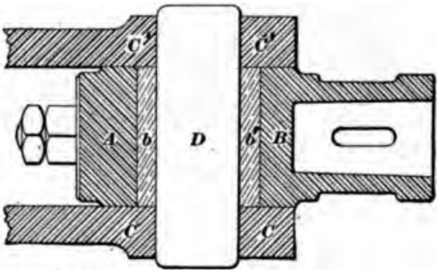
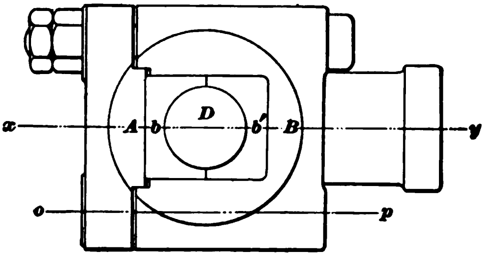


FIG. 4

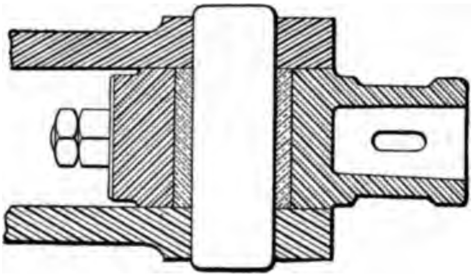
8. The above rule governing the direction of section lines is always adhered to when possible; when any departure is necessary, care is taken to prevent ambiguity. Where only the sectional view is given, it is often very difficult to understand the drawing, and sometimes a violation of the above rule will cause an erroneous conclusion to be drawn. Referring to Fig. 5 (*a*), cover up the front view shown at (*b*). Then, since the sectioning of *A* and *B*, and also that shown at *b* and *b*′, are respectively in the same direction, any one would be perfectly justified in assuming that *A* and *B* was a sectional view of a rod fitted with a solid bushing *b*.



(a)



Rods removed.
(b)



(c)

FIG. 5

Furthermore, since *C*, *C* and *C'*, *C'* are sectioned the same way, the conclusion that they were the jaws of a forked rod would be justifiable. Referring now to view (*b*), it is seen that *b* and *b'* are separate brass boxes; the part *B* is seen to be separate from the cap *A*, and the note "*Rods removed*" indicates that *C* is separate from *C'*. The way the sectional view should have been section-lined to correspond to the front view shown at (*b*) is given in Fig. 5 (*c*).

9. When a cutting plane passes through the axis of a shaft, bolt, rod, or any other solid piece having a curved surface and located in the plane on which the section is taken, it is the general practice not to show such solid pieces in section, but in full. Thus, in Fig. 5 the sectional view is taken on the plane represented by the line *x y*, which passes through the axis of the pin *D*. This pin is shown in full, however. The practice here shown is rarely departed from by experienced draftsmen, since it makes a drawing easier to read and also saves considerable time in making the drawing.

10. Fig. 5 also shows another feature that is frequently met with in shop drawings. Referring to the illustration, it is seen that no bolt is shown in the lower half of the object, as far as the front view (*b*) is concerned. A center line *o p* is drawn in, however; this center line indicates to the workman, who reasons from the symmetry of the object in respect to the center line *x y*, that the lower half of the object is to be supplied with a bolt placed in the plane given by the center line *o p*. In case of symmetrical work, draftsmen will frequently complete only one half of the view and merely indicate the other half by a few lines or not at all, trusting to the judgment of the workman for a correct reading of the drawing. In the best practice, a note is made on the drawing calling attention to the fact that the indicated portion of the view is a duplicate of the complete portion.

BREAKS

11. When a long and comparatively slender object is to be drawn, it often happens that, when drawn to a sufficiently large scale to make it intelligible, it will extend beyond the space available. In such a case, part of the object is broken out and the remaining ends are pushed together. The fact that part of the object is broken away for the sake of convenience is indicated by a so-called **break.** It is always understood that the part broken away and not shown is of the same size and shape as the parts contiguous to the break. In some cases, one end of the object is broken away.

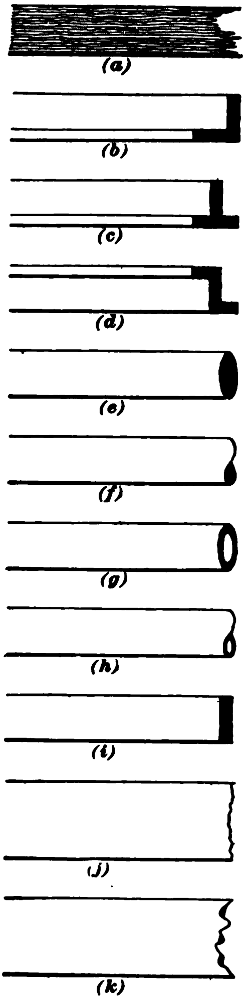


FIG. 6

12. Breaks may be indicated in various ways; most commonly, the break is given an outline that will reveal the shape of the object. Conventional methods of indicating breaks are shown in Fig. 6. Wood is usually shown broken in the manner illustrated at (*a*), angle irons as at (*b*), **T** irons as at (*c*), **Z** bars as at (*d*). Cylindrical objects are occasionally broken as shown at (*e*), but most frequently in the manner shown at (*f*). Pipes and similar hollow cylindrical objects may be broken as shown at (*g*); but, more frequently, the break is made as shown at (*h*). Rectangular objects may be broken in the manner shown at (*i*); plates and objects other than those included between views (*a*) and (*i*) are often shown broken off by drawing a wavy freehand line as in (*j*) and (*k*).

HIDDEN SCREW THREADS

13. When the screw thread is hidden by part of the object and it is deemed necessary to show it in dotted lines, it is usually drawn in one of the four ways illustrated in Fig. 7. Of these the method shown in Fig. 7 (*a*) is probably the clearest; that shown in Fig. 7 (*b*) is fairly good; and the one shown in Fig. 7 (*c*) is cheap. The method illustrated in Fig. 7 (*d*) is practically no representation of a screw thread at all; if used it usually must be supplemented by a note such as "¾″ *stud*," or "1⅛″ *bolt*," and so on.

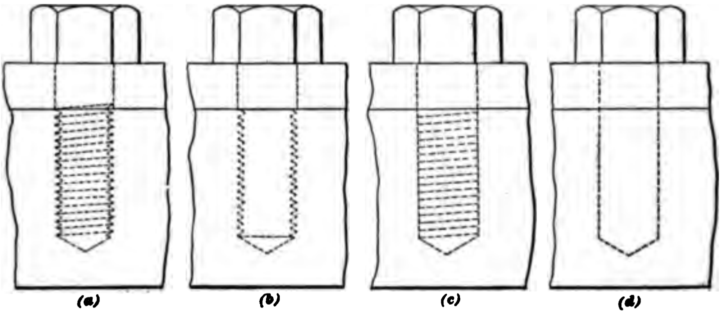


FIG. 7

REPEATED PARTS OF OBJECTS

14. When an object has a relatively large number of similar component parts, they are rarely all shown on a working drawing. Usually, a few of them are shown in full and the rest are merely indicated by showing the position of the center of each part. Sometimes even this is not done, but a note is placed on the drawing calling attention to the fact that some certain part of the object is to be repeated.

15. Fig. 8 is an example of how repeated parts of an object may be treated. Referring to the illustration, which is a top view of a pipe flange, Fig. 8 (*a*) shows three bolts drawn in. The position of the rest of the bolts is indicated by the short radial lines drawn across the bolt circle. In

Fig. 8 (*b*), the bolt circle is drawn in and a note is written along it that is sufficiently definite to convey the idea to the mind of the workman. This latter method is most commonly used on working drawings.

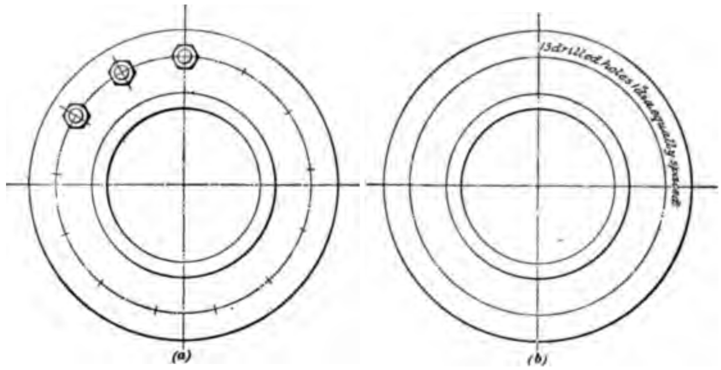


FIG. 8

16. When making a drawing of a gear-wheel, especially when it is a working drawing, it is customary to draw in only two or three teeth and specify how many teeth the wheel is to have. This answers the purpose as well as if all the teeth were drawn and has the advantage that an enormous amount of time is saved in making the drawing. Likewise, in drawings of objects built up of plates or rolled sections, as in boiler and bridge drawings and similar work, only a few rivets, or staybolts, or similar repeated parts are usually shown, and the location of the rest is indicated by showing the position of their centers or bv a suitable note placed on the drawing.

ABBREVIATIONS USED ON DRAWINGS

17. The most commonly used abbreviations are *D.*, *Dia.*, *Diam.*, *d.*, *dia.*, and *diam.* for "diameter," and *R.*, *Rad.*, *r.*, and *rad.* for "radius." "Wrought iron" is usually abbreviated to *Wr't Iron.* The abbreviation *Thds.* or *thds.*, with a number prefixed, stands for "threads per inch"; thus, *14 thds.* means "make 14 threads per inch." The word *tap*, with a number prefixed, always means that a hole is to be finished by tapping it with a tap of standard proportions, having the diameter given by the prefixed number. When a tap other than a standard tap is to be used, it is distinctly specified. *Drill* is always taken to mean that a hole is to be put through the object by drilling. *Bored* or *Bore* means "finish a hole by boring it." *Planed* is always understood to mean "this surface is to be finished by planing." *Cored* implies that the hole to which it is applied is to be cored out and left that way; that is, it is not to be finished by machining. *Faced* almost invariably implies that the surface to which it is applied is to be machined square with a hole in the object. *Turned* is an abbreviation for "finish by turning." *Scraped* implies that a surface is to be finished by scraping. *Tool finish* means that the surface, after machining, is not to be finished any further. *Black*, on objects formed by forging, implies that the part to which it is applied is to be left as it comes from the smith. The term *Ream* or *Reamed* means that a hole is to be finished by reaming; when applied to a bolt, it is understood that the bolt is to be fitted to a hole that has been previously reamed. The terms *Shrinking Fit*, *Forcing Fit*, and *Driving Fit* written behind a dimension always imply that, in machining the part, the workman is to make the allowance necessary for the kind of fit called for. The fact that part of an object is to be finished by machining, filing, or grinding is often indicated by marking the outlines with an *f* written across or near it, or writing *fin.* along it. In some cases, draftsmen will draw a dotted line or a full red line at a little distance from the outline and write *f* across it; it is usually understood, in that case, that the lengths of the supplementary lines denote the extent of the surfaces that are to be finished.

KINDS OF WORKING DRAWINGS

18. Working drawings are divided into two general classes, which are: *assembly*, or *general*, *drawings*, and *detail drawings*.

Assembly, or **general, drawings** show the workman the relation between, and the places or positions occupied by, the different component parts of a structure, machine, device, fixture, implement, etc. If any dimensions are given, they are usually only leading dimensions.

Detail drawings show the exact shape and size of each integral part. For this purpose they are supplied with all the dimensions required by the workman and any additional explanatory notes that the draftsman may consider necessary.

Detail drawings may be made so complete that they will answer for the patternmaker, blacksmith, and machinist, and they are usually so made in the smaller shops. In the large shops, however, separate drawings are often made for the patternmaker, blacksmith, and machinist; the detail drawing for the use of the patternmaker, then, contains only the dimensions and notes needed by him to make the pattern; that for the blacksmith contains the dimensions needed for making the forging; and, finally, that for the machinist contains all dimensions needed by him.

19. Attention is called to the fact that practice varies somewhat in different places in regard to the dimensions given on detail drawings, at least as far as drawings for the patternmaker and blacksmith are concerned. In some places, the dimensions given represent the size the object is to be when *finished;* hence, the blacksmith or patternmaker must make necessary finishing allowances himself. In other places, again, the finishing allowance has been, and usually is, made by the draftsman; the dimensions given are then those of the pattern or forging. If in doubt about the practice followed in a particular drawing office, it is a good plan to find out by inquiry what system is used in the shop under consideration. In the best modern practice, a note calling attention to the fact that the sizes given are those when finished is placed on the drawing; thus, "*All finished sizes*" or some similar note.

SCALES

20. When it is desired to make a drawing other than full size, special scales are used. Thus, suppose it is required to make the drawing ¼ size; then, 3 inches on the drawing would represent 1 foot on the object. Hence, if 3 inches are laid off and divided into 12 equal parts, each of these parts will represent 1 inch on the object. If these parts be subdivided into 2, 4, 8, etc. parts, each will represent ½, ¼, ⅛, etc. of 1 inch on the object. A scale of this kind is called a **quarter scale,** or a **scale of 3 inches to the foot.** An **eighth scale,** or a **scale of 1½ inches to the foot,** would be constructed in the same way, except that 1½ inches would be laid off instead of 3 inches. These scales are written 3" = 1 ft., 1½" = 1 ft.

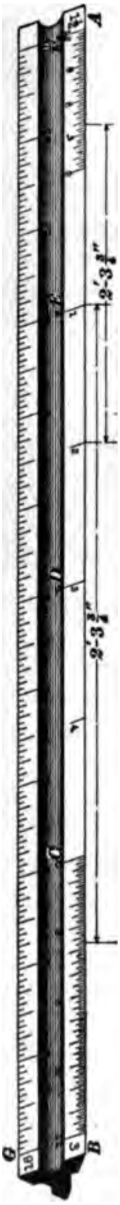


FIG. 9

21. Fig. 9 shows a scale which is convenient for the student, inasmuch as it combines eleven different systems of subdivision and may be used for all the work ordinarily done in a drafting room. This scale is triangular in section and 12 inches in length, and on each of its edges is laid off a scale, as shown at *A*, *B*, and *G*. The scale at *G* is "full size"; that is, this edge of the scale is divided into inches and fractions of an inch down to sixteenths, and is used for drawings in which an object is represented in its natural size. On its opposite side, at *B*, is shown the quarter-sized scale of 3" = 1 ft. The first 3-inch (actual size) division, from *B* to *C*, is subdivided into 12 parts representing inches, and each inch is then divided into proportional fractions of an inch, generally eighths. From *C* to *D*, *D* to *E*, and *E* to *F*, the scale is marked in its main divisions of 1 foot each, each foot being 3 inches long, actual size. From *A* to *B* the scale is independently divided into spaces of 1½ inches (actual size) to form an eighth-sized scale, or 1½″ = 1 ft., the divisions of the latter occurring on and between the marks for the 3-inch scale.

The other sides and edges of the instrument are divided into scales of 1 inch and ½ inch, ¾ inch and ⅜ inch, ¼ inch and ⅛ inch, and $\frac{3}{16}$ inch and $\frac{3}{32}$ inch to the foot. Different makers do not always arrange their scales in the same manner. Thus, instead of having a full-size scale and scales of 3″ = 1 ft. and 1½″ = 1 ft. on one side, as shown in Fig. 9, some makers have the full-size scale and $\frac{3}{16}$″ = 1 ft. and $\frac{3}{32}$″ = 1 ft. on one side. It will be observed that the numbering of the feet on these scales does not start at the end of the instrument, but at the first division from the end. Thus, on the quarter-sized scale the zero mark is placed at *C* and the first foot is measured to *D*. This is done so that the feet and inches may be laid off independently and with one reading of the scale.

The figures indicating the number of feet on this scale are placed along the extreme upper edge at *D*, *E*, and *F*, the numbers running in a direction away from the part containing the inches. The numbers indicating inches run in an opposite direction from those defining the feet.

To lay off 2 feet 3¾ inches on a scale of 3″ = 1 ft. and from a given point, place the scale on the point so that the 2-foot mark will be directly over it; then from the zero mark *C* lay off 3¾ inches, as shown, locating a second point. The length of the distance thus laid off between the two points represents 2 feet 3¾ inches. The scale of 1½″ = 1 ft. is used in a similar manner to lay off the same distance. The figures indicating feet on this scale are placed nearer the edge, in order to prevent confusion in reading.

To draw to half size, or 6 inches to the foot, use the full-size scale, and remember that every ½ inch on that scale corresponds to 1 inch on the object, that is, that every dimension is only half of the real length. To lay off 5⅞ inches, lay off 5 half inches and $\frac{7}{16}$ of an inch over; the result is a line 5⅞ inches long to a scale of 6 inches to the foot.

If it is desired to draw to a scale of ¾ of an inch to the foot, or $\frac{1}{16}$ size, the scale of 1½ inches to the foot may be used if the draftsman has no scale of ¾″ = 1 ft., halving all dimensions, as in the previous case of drawing to a scale of 6 inches to the foot with a full-size scale. It sometimes happens that a draftsman is obliged to make a scale, when the size of his plate is limited and a general drawing of some object is desired. By *general* drawing is meant a complete view of the object in plan and also one or two elevations. In such a case, one scale may be too large to enable the drawing to be made on a sheet of the required size; another scale may make it too small to show up well. For example, a ⅛ scale may be too large and a $\frac{1}{16}$ scale too small; a $\frac{1}{12}$ scale may be just right. If the draftsman has no $\frac{1}{12}$ scale (that is, a scale of 1 inch to the foot), he may make one by taking a piece of heavy drawing paper and cutting out a strip about the size of an ordinary scale and laying off the inch divisions on it. Each division or part will represent 1 foot on the object. Divide one of the end parts into 12 equal parts and each will represent 1 inch on the object. Lines indicating half and quarter inches may be drawn if considered necessary.

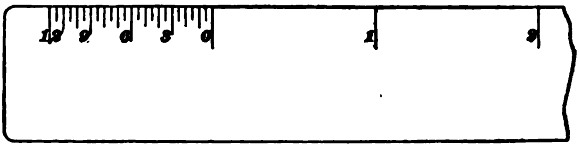


FIG. 10

Fig. 10 shows part of a scale made in this manner, giving feet, inches, and half inches—the quarters, eighths, etc. of an inch being judged by the eye.

CONVENTIONAL REPRESENTATION OF A NUT

22. Fig. 11 shows the ordinary conventional method of representing a nut. The bottom of the thread is 1⅛ inches in diameter and is represented by the dotted circle; this shows that it is intended for a screw 1⅛ inches in diameter. The height of the nut equals the diameter of the bolt or screw on which the thread is cut. The two views on the center line *m n* should be drawn without difficulty. To draw the curves *e a* and *a d*, project *b* and *c* at right angles to *t v* in the points *d*, *a*, and *e*; pass arcs of circles through *e* and *a* and through *d* and *a* tangent to *f g*, finding the centers of these arcs by trial. The best way of doing this is to draw lines parallel to *t v* midway between *e a* and between *a* and *d*. Then, by trial with the compasses, find a center on these lines such that an arc struck with the compasses from this center will pass through *e* and *a* (or *a* and *d*) and be tangent to *f g*. In the right-hand view, the radius of the arc *b c* is the same as the height of the nut; the centers of the other two arcs are found by trial in the manner just described.

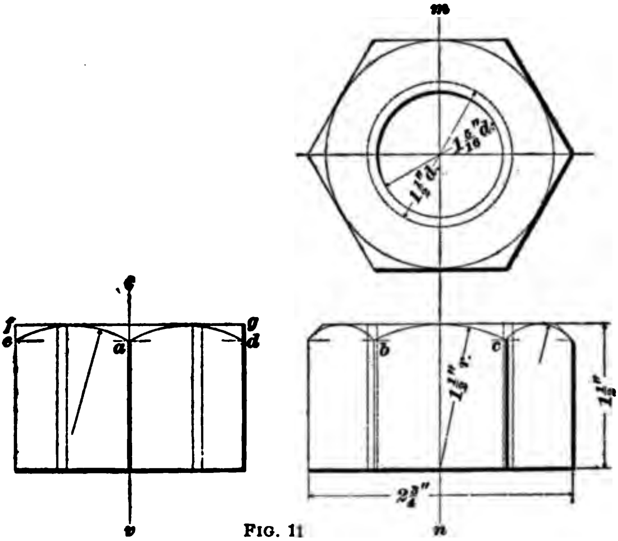


FIG. 11

DRAWING PLATE, TITLE: DETAILS

23. The first eight figures of this plate show the conventional methods of representing screws. The actual projection of a screw thread will be similar to the projection of a helix; but in order to save the time required to locate the points and trace in the curves, the following methods are universally used, except, perhaps, in the case of screws of very large diameter and pitch, drawn full size.

24. Fig. 1 represents a **single square-threaded screw** $1\frac{1}{2}$ inches in diameter and $\frac{3}{8}$ inch pitch. To draw the screw, first draw the center line $m\,n$ and a line $A\,B$ at right angles to it. Make the distance $A\,B$ equal to the diameter of the screw, or $1\frac{1}{2}$ inches, and through the points A and B draw lines $A\,D$ and $B\,E$ parallel to the center line $m\,n$. Also lay off on the line $A\,B$ distances $A\,F$ and $B\,G$ equal to one-half of the pitch, and through the points F and G draw lines $F\,H$ and $G\,I$ parallel to the center line $m\,n$. These lines show the depth of the thread. On the line $A\,D$ lay off the width of the thread and of the groove, $A\,C$, $C\,J$, $J\,K$, etc., each equal to one-half of the pitch, or $\frac{3}{8} \times \frac{1}{2} = \frac{3}{16}$ inch. Draw the line $B\,C$, and through the points J, K, L, M, etc., draw lines parallel to $B\,C$. Draw faint pencil lines through the points C and P, J and Q, K and R, etc. to represent the back edges of the threads, and make the parts that are seen full lines; then draw the lines $T\,V$, $U\,W$, etc. The method of drawing the remainder of the screw and the reason for using the heavy shade lines, as shown, should be apparent without further explanation.

It will be noticed that the width of the thread and of the groove, measured parallel to the center line $m\,n$, and the depth of the thread are all exactly the same; that is, they are each equal to one-half of the pitch. If a section were taken through the center line $m\,n$, the thread and groove would look like Fig. 12, a series of squares, hence the term *square thread.*

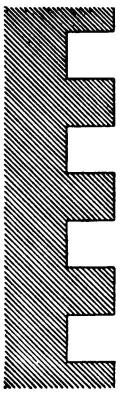


FIG. 12

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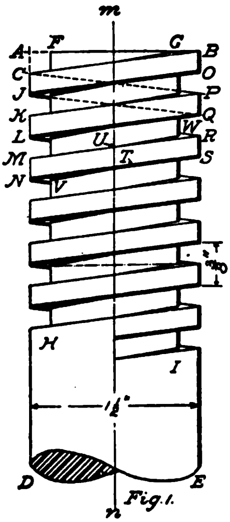


Fig. 1.

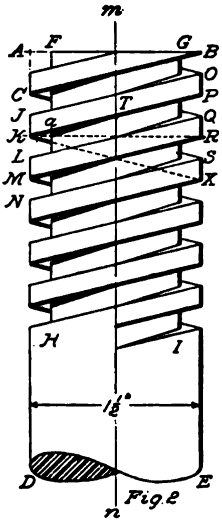


Fig. 2.

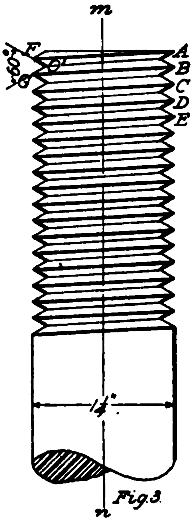


Fig. 3.

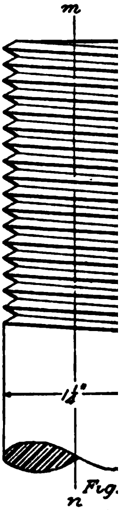
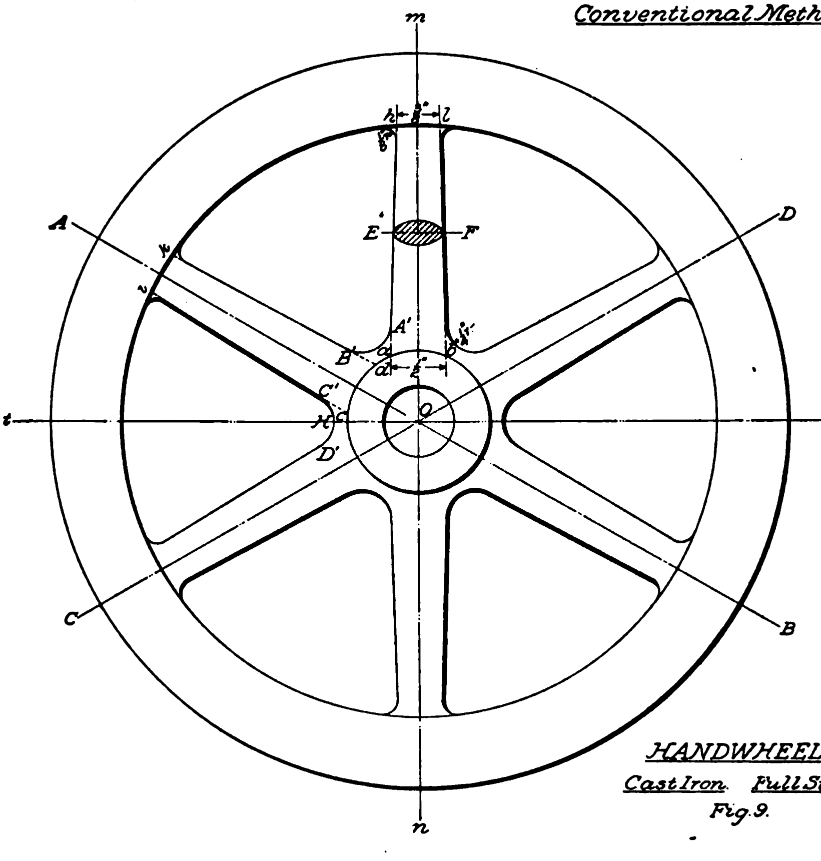


Fig.

Conventional Methods of



HANDWHEEL.

Cast Iron. Full Size.

Fig. 9.

r 25, 1893.

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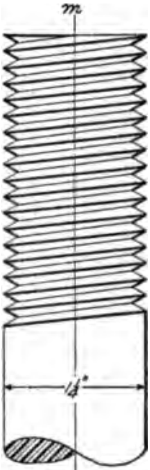


Fig.5

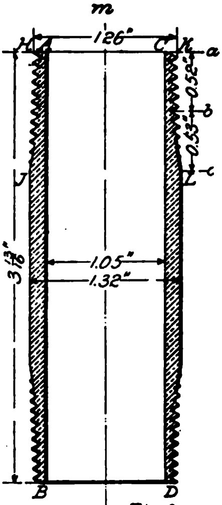


Fig.6.

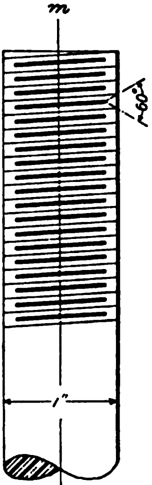


Fig 7.

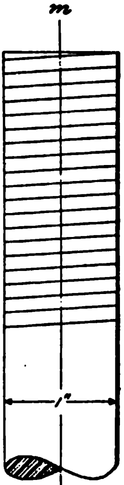
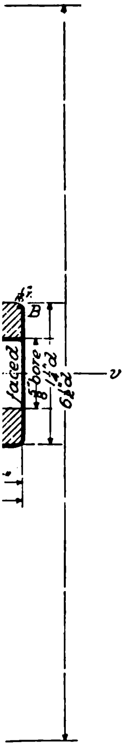
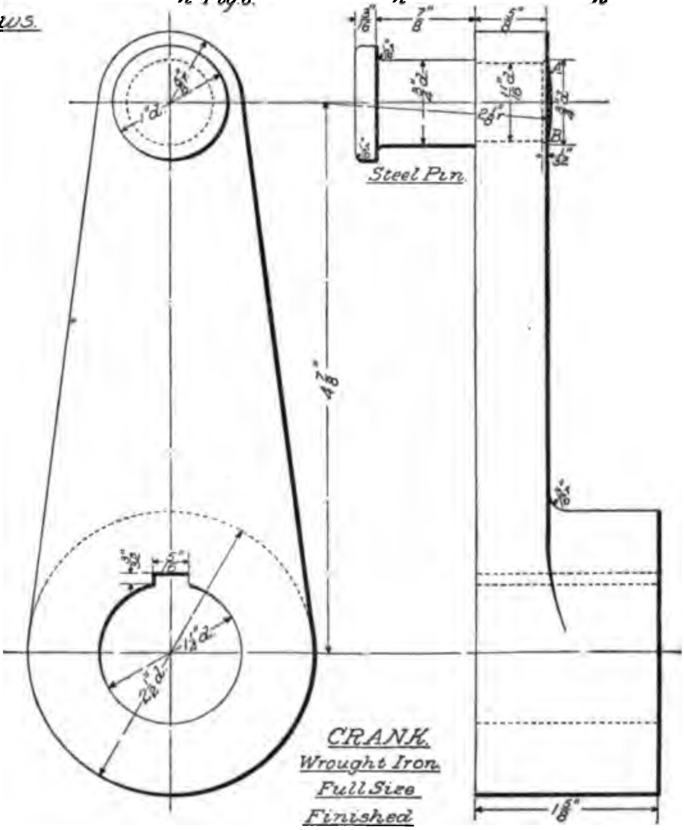


Fig.8.

esenting Screws.





CRANK.
Wrought Iron.
Full Size
Finished

Fig.10.

ately following the title page.

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25. Fig. 2 shows a **double square-threaded screw** 1½ inches in diameter and with ¾ of an inch pitch. The reason for using a double thread is that if the single square thread were used, the depth would be so great as to weaken the bolt or rod on which it was cut and render it unsafe for the purpose for which it was intended. To prevent this, either the diameter of the rod must be increased or the thread must be cut of the same depth and thickness as a thread of half the pitch, or, in this case, as if the pitch were $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ of an inch, as in the preceding problem; another thread of the same size and pitch (¾ of an inch) must be cut half way between these first threads, thus giving a double thread. The pitch, or distance that the screw would advance in one turn, would be ¾ of an inch, the same as if it were a single-threaded screw of ¾ of an inch pitch, while the depth of the thread is only half as great. To draw it, proceed exactly as in the last figure. To get the direction of the line *B C*, which in this figure represents the projection of the bottom edge of the top of the thread, lay off *A C* equal to one-half of the pitch, or $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ of an inch, and draw the line *B C*. The width of the threads and grooves, and also the depth of the threads, is one-fourth of the pitch, or $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$ inch.

Through the points *K*, *L*, *M*, etc. draw faint pencil lines *K X*, etc. to represent the back edges of the threads, and make the parts that are seen full lines. Through the point *K* draw a faint pencil line *K R* at right angles to the center line *m n*, intersecting the line *F H* in *a*, and draw the line *T a*, which represents the bottom of the thread. The remainder of the screw should now be drawn without any trouble.

26. Fig. 3 is a **single V-threaded screw** 1¼ inches in diameter and having 7 threads to the inch; that is, the pitch is 1/7 of an inch. Draw a cylinder 1¼ inches in diameter, having *m n* for the center line. Lay off *A B*, *B C*, *C D*, etc. each equal to the pitch, or 1/7 inch. Do the same on the left-hand side. By the aid of the **T** square and 60° triangle make the angles *A O B*, *F O′ G*, etc. The rest of the thread can be drawn by referring to the figure.

27. Fig. 4 represents a screw exactly like the preceding one, except that the thread is left-handed instead of right-handed, as in the previous case.

To ascertain whether a thread is left- or right-handed, hold the screw in such a position that its axis is horizontal. If the thread is right-handed, as it usually is, the angle that the edge of the thread makes with the horizontal on the right-hand side is obtuse; if left-handed, it makes an acute angle with the right-hand side of the horizontal. No further instruction should be necessary for drawing the thread.

28. Fig. 5 represents a **double V-threaded screw** 1¼ inches in diameter. It has 3½ threads per inch; that is, the pitch is 1 inch ÷ 3½ = $\frac{2}{7}$ inch. The same remarks regarding the drawing of it apply here that were used in describing Figs. 2 and 3.

29. Fig. 6 represents a section of a **brass nipple.** When the diameter of a nipple is given, the inside diameter is always meant, unless otherwise specially stated. The actual diameter of a nipple or pipe is very rarely given, but must be taken from printed tables. The *nominal diameter* of the nipple shown in the figure is 1 inch, but the actual inside diameter is 1.05 inches; from the table, the outside diameter is found to be 1.32 inches, making the thickness .135 of an inch. Owing to the thinness of the shell, pipe threads are finer than the threads on the same-sized rods. The coarsest pipe thread is 8 threads per inch. The number of threads per inch on the nipple shown is 11½. The thread is tapered to make a tight fit, and the length of the threaded part on each end is $0.52 + 0.53 = 1.05$ inches, of which length the distance between a and b represents the perfect thread, while from b to c the thread is chamfered; that is, it dies out gradually. To draw the nipple, make a sectional view as shown. Draw a cylinder 1.32 inches in diameter and having $m\,n$ as a center line; lay off the inside diameter equal to 1.05 inches and draw $A\,B$ and $C\,D$. Now lay off the diameter $H\,K$ equal to 1.26 inches. Then lay off the distances $H\,J$ and $K\,L$ equal to $0.52 + 0.53 = 1.05$ inches;

join the points *H* and *J*, and *K* and *L*, by straight lines representing the top of the threads. Now, on the center line *m n*, or on any line parallel to it, lay off a distance of 1 inch from the line *A C* downwards. Divide this distance into 11½ parts by means of the method given in *Geometrical Drawing*. Project the points just found upon *HJ*, and, by means of the **T** square and 60° triangle, draw the threads from *H* to *J* as though all the threads were perfect. Draw the threads on *K L* in the same manner, remembering that the divisions on *H J* are to be advanced half a thread, as shown; that is, the top of one thread and the bottom of the preceding thread on the other side will be on a horizontal line. Now lay off the distance *a b* equal to 0.52 inch and project it on lines drawn parallel to *K L* and *H J* and touching the bottom of the threads. From the points of intersection draw straight lines to *J* and *L*, in order to obtain the bottoms of the imperfect threads extending from *b* to *c*. Complete the rest of the drawing in the same manner.

30. Fig. 7 shows another method of representing a **V**-threaded screw. This method has the advantage of making a neat-looking drawing and of being very rapid in delineation. The pitch is laid off as in the three preceding figures. The heavily shaded lines represent the bottom of the thread and their lengths are determined by constructing an equilateral triangle on the pitch distance, as shown, and limiting the line to distances between two corresponding vertexes of the triangle. The diameter of the screw is 1 inch and the number of threads per inch is 8.

31. Fig. 8 represents the same screw shown in Fig. 7, but the lines indicating the bottom of the thread are left out altogether. This method is used on drawings where haste is necessary. Unless in very much of a hurry, the method shown in Fig. 7 is to be preferred. Ordinarily, when drawing screws as represented by Figs. 7 and 8, it is not customary to lay off the pitch and the depth of the thread as above mentioned—the distances between the lines representing the threads are simply gauged by the eye; practice will enable this to be done very quickly and accurately.

32. Fig. 9 shows two views of a small **hand wheel.** To draw it, locate the center *O*, and through *O* draw the center lines *t v* and *m n* at right angles to each other. From *O* as a center, with the compasses set to the radius of the wheel, or 3¼ inches, draw the outer circle of the rim; then, lay off the thickness of the rim, which is ⅝ of an inch, and draw the inner circle. Through *O*, with the **T** square and 60° triangle, draw the center lines *A B* and *C D*. With *O* as a center, draw two circles, one having a diameter of ⅝ inch, to represent the hole, and the other a diameter of 1¼ inches, to represent the outside of the hub. To draw the arms, make the chords of the arcs *a b*, *c d*, etc. each ½ inch long. Make the chords *h l*, *i k*, etc. each ⅝ inch long and draw *l b*, *h a*, *k d*, etc. With a radius equal to ¼ of an inch, describe the fillets, or arcs, *A' B'*, *C' D'*, etc. tangent to the arms at *A'*, *B'*, *C'*, etc. With a radius equal to ⅛ of an inch, describe the fillets or arcs tangent to the inside of the rim and to the arms. All these arcs terminate at the point of tangency. The cross-section part on the arm indicates that a cross-section taken at *E F* would look as shown; that is, that the arm is elliptical.

The other view shows a conventional method largely used in drawing rooms of indicating a section of the wheel. It is termed a conventional method because it would really be impossible to obtain a section like the one shown. Theoretically, the arm in this view should be sectioned, but, for convenience, a section is imagined to be taken through the rim and hub on the line *t v* and turned around to the position *m n*, and the two arms are shown in projection as if they were directly back of the line *m n*. Draw the center line *p q* and the sections of the rim as shown. Draw the arms from the dimensions given. Draw the hub as shown, using a radius of $\frac{5}{32}$ of an inch to draw the fillets or arcs tangent to *A B* and to the arm; make *O' H'* equal to *O H* in the other view and describe an arc through *H'* tangent

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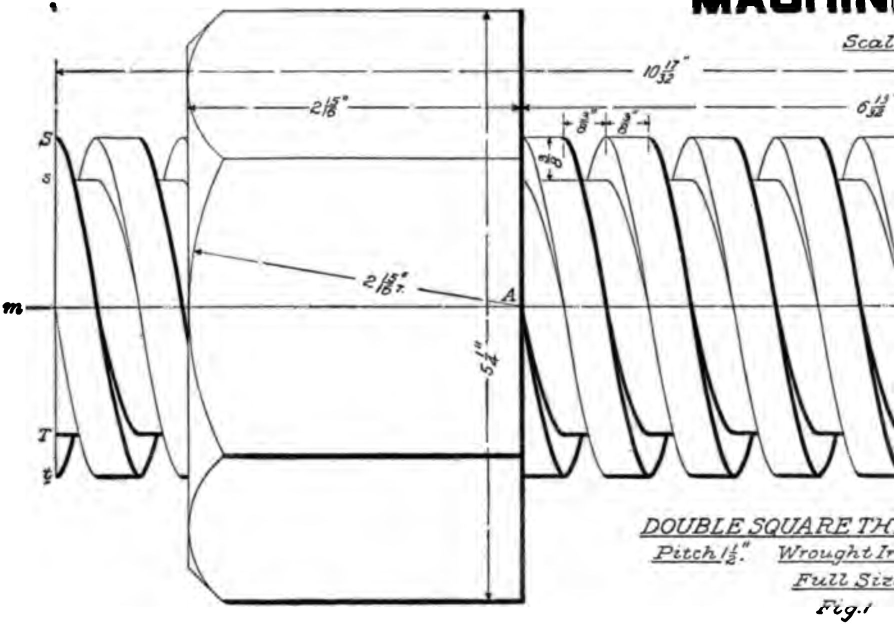
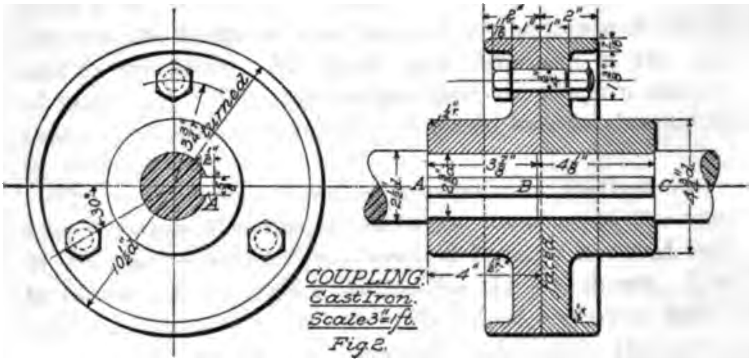


DOUBLE SQUARE TH

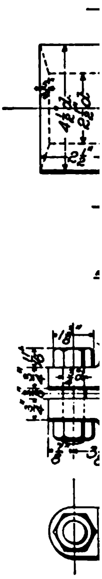
Pitch 1/2". Wrought Ir

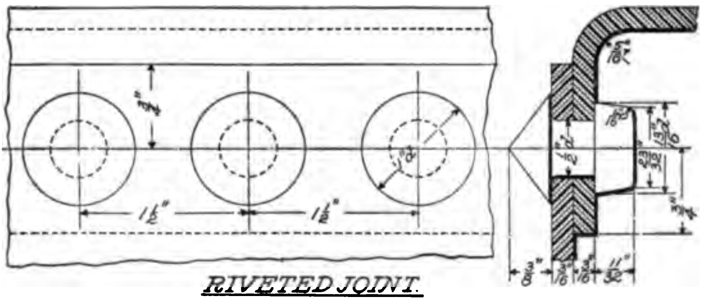
Full Siz

Fig. 1



COUPLING.
Cast Iron.
Scale 3" = 1 ft.
Fig. 2.





RIVETED JOINT.
Wrought Iron. Full Size.
Fig. 4.

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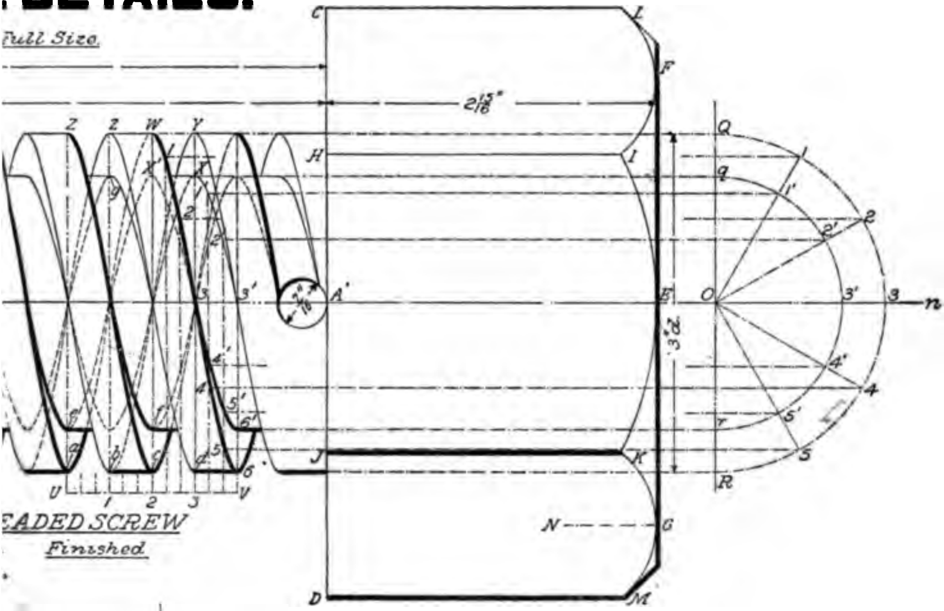
DETAILS.

ull Size.



EADED SCREW

Finished.

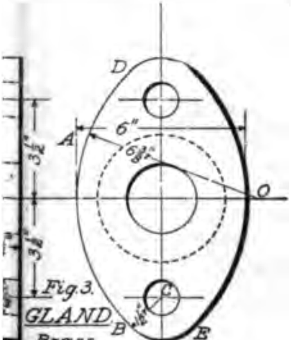
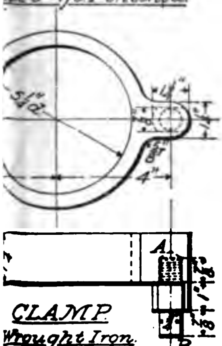


Fig. 3.

GLAND.

Brass.

le 3"=1 ft. Finished.

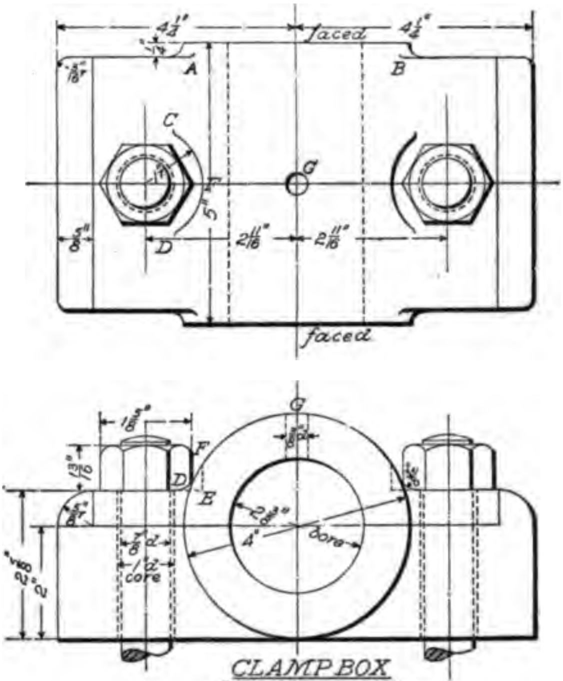


CLAMP.

Wrought Iron.

Scale 3"=1 ft.

Fig. 5.



CLAMP BOX

Cast Iron. Half Size.

Fig. 6.

immediately following the title page.

JOHN SMITH, CLASS No 24529.

to the two arcs just drawn, which are tangent to $A B$ and the arm. The rest of the drawing can then be completed without further explanation.

33. Fig. 10 shows a **crank,** which should be drawn without difficulty from the dimensions given. The pin is forced in and the end riveted over at $A B$, to prevent it from being pulled out.

DRAWING PLATE, TITLE: MACHINE DETAILS

34. Fig. 1 shows a **double square-threaded screw** 3 inches in diameter and $1\frac{1}{2}$ inches pitch, having a hexagonal head and nut. The screw is $10\frac{17}{32}$ inches long and the distance between the faces of the head and nut is $6\frac{13}{32}$ inches. The head and the nut are each $2\frac{15}{16}$ inches long, are hexagonal in shape, and their long diameters are $5\frac{1}{4}$ inches. Since this screw is drawn full size and is comparatively of large diameter, the true projection of the helix is drawn instead of the conventional method used in Fig. 2 of the plate entitled Details. Each thread terminates in a $\frac{7}{16}$-inch hole under the shoulder of the head.

Draw the center line $m n$; lay off $A A'$ equal to $6\frac{13}{32}$ inches, and through A' draw $C D$ perpendicular to $m n$. Make $A' E$ equal to $2\frac{15}{16}$ inches and draw $F G$ parallel to $C D$. Divide $C D$, which is $5\frac{1}{4}$ inches long, into 4 equal parts, and through the points C, H, J, and D draw lines $C L$, $H I$, $J K$, and $D M$ parallel to $m n$. With A' as a center and $A' E$ as a radius, describe the arc $I E K$, intersecting $H I$ and $J K$ in I and K. Project the points I and K upon $C L$ and $D M$, locating the points L and M, and pass through L and I and through K and M circular arcs tangent to $F G$. The centers of these arcs will lie on lines parallel to the center line $m n$ and midway between L and I and between K and M. Therefore, draw $N G$ parallel to the center line $m n$ through a point half the distance between $J K$ and $D M$, and find by trial a point N at such a distance from G that, with the radius $N G$, the arc will pass through K, G, and M. Draw the nut in the same manner. This is the conventional method of representing a nut.

To construct the screw, draw through some convenient point *O* a line *Q R* parallel to *C D*. From *O* as a center, describe two semicircles whose diameters are equal to the top and bottom diameters of the screw thread, intersecting the line *Q R* in the points *Q*, *q*, *r*, and *R*. From these points draw lines to *S*, *s*, *T*, and *t* parallel to the center line *m n*; evidently these lines coincide with the top and bottom of the screw thread. The depth of the thread is found as in Fig. 2 of the plate entitled Details.

Divide the semicircles into any number of equal parts, in this case 6; this is best done by first dividing the exterior semicircle into the required number of parts, as *Q*, *1*, *2*, *3*, *4*, *5*, and *R*, and then drawing radii through each point from *O*, which divides the interior semicircle similarly. Lay off on the line *R t*, or any other line, as *U V*, which is parallel to the center line *m n*, the pitch of the screw, or 1½ inches, and divide it into twice the number of equal parts that the semicircles have been divided into. Locate the point *V* at a distance of 25/32 inch from the shoulder *C D* of the head.

Now, construct the helix *6–5–4–3–2–1–W* for the top diameter; also the helix *6′–5′–4′–3–X′*, having the same pitch, for the bottom diameter. As this is a double square-threaded screw, the widths, as well as the depths, of the threads and grooves are equal to ¼ of the pitch, or 1½ × ¼ = ⅜ of an inch. (See Fig. 2, last plate.)

Consequently, to lay off the widths of the threads and grooves, divide the pitch, or line *U V*, into 4 equal parts, as *1*, *2*, *3*, and *U*, and through these points draw lines parallel to *C D*, intersecting the lines that represent the top and bottom diameters of the screw in the points *a*, *e*, and *Z′*; *b*, *g*, and *Z*; *c*, *f*, and *W*; *d*, *X*, and *Y*; draw through these points other helical curves parallel to those already drawn. The remaining threads of the screw may be drawn in the same manner, but an easier and quicker method is to cut a curve, of the same shape as those already drawn, out of a piece of bristol board or cardboard and use it as a template. The manner of constructing the curves *X–1′–2′–3′* and *3–4′–5′–6′*

should be clearly evident from the drawing. The dotted curves need not be drawn by the student, as they are merely put in to show the connections between the different threads.

35. In Fig. 2 are shown a section and end view of a **shaft flange coupling,** used for connecting the ends of two shafts. In order not to take up too much room on the plate, the coupling is drawn to a reduced scale, which is marked on the drawing as 3″ = 1 ft. Using the scale of 3″ = 1 ft. (see Art. **21**), draw the figure from the dimensions given, first drawing all of the sectional part, except the bolt. Then draw the end elevation as shown, and, lastly, the bolt in the sectional view. The part *A B C* is a key sunk into the shafts to keep them from turning within the coupling. The shafts themselves are made of wrought iron, as indicated by the sectioning.

36. Fig. 3 is a front and side elevation of a **gland.** This is also drawn to a scale of 3″ = 1 ft., and should be constructed without difficulty from the dimensions given. The line *O A*, marked 6⅜″ *r*, means that the radius of the arc *B A D* is 6⅜ inches long; so, also, *C B*, marked 1½″ *r*, means that the radius of the arc *B E* is 1½ inches long. Since no dimensions are given for the other two arcs, it is understood that their radii are the same as for the first two. Whenever a dimension is given, like *O A*, specifying the length of a radius, the letter *r*, or the abbreviation *rad.*, should always be placed after it.

37. Fig. 4 shows a **riveted joint,** or two plates riveted together. The front elevation and a cross-section through one of the rivets are given. The diameter of the rivets is ½ inch and the pitch (the distance between the centers of two consecutive rivets) is 1½ inches. These and the other dimensions are obtained from the drawing. Draw the cross-section first, as shown in the figure, and after that the elevation. The scale is full size.

38. Fig. 5 shows a **clamp, dog,** or **carrier,** as it might be termed. It can be readily drawn from the dimensions given. It will be noticed that part of the pin *A B* is flattened. This is seen more clearly in the top view, shown by the dotted lines. It is to be drawn to a scale of 3″ = 1 ft.

39. Fig. 6 shows a **clamp box** to be attached to beams for shafting to pass through. The scale to which it is drawn is half size, that is, 6″ = 1 ft. The curves at *A*, *B*, etc. are not exact projections of the curve of intersection of the round and flat surfaces shown in the other view, but are drawn as circular arcs, from which they differ but little, and the time occupied in finding the different points on the curve is saved. This answers the purpose when making shop drawings just as well as the exact method. The curve *C F D* is the projection of the part *D E F*, shown by the dotted lines in the elevation, which is removed in order that the nut may be turned round. *G* is an oil hole. It will be noticed that the bolt holes are larger than the bolts; this allows the clamp box a little play, should it be necessary. It also allows the holes to be cored in the box when the casting is made, instead of being drilled afterwards. On all drawings made for the shop or on which dimensions are given or required, the scale should invariably be specified. If more than one scale is used, as in the last plate, where three different ones were used, it should be given for each figure.

DRAWING PLATE, TITLE: FLANGE COUPLING

40. This plate shows a drawing of a flange coupling suitable for connecting two lengths of 2-inch line shafting. Fig. 1 is a section on the line *A B* (Fig. 2) and shows how the two parts *C* and *D* of the coupling are bolted together through their flanges, hence the name **flange coupling.** Each part is keyed on its shaft separately and true alinement of the shafts is insured by means of the recess in *C*, into which is fitted a raised boss on *D*. The two parts of the coupling are first bored and then faced up on the surfaces *E F G H I J*.

FLANGE

Sca

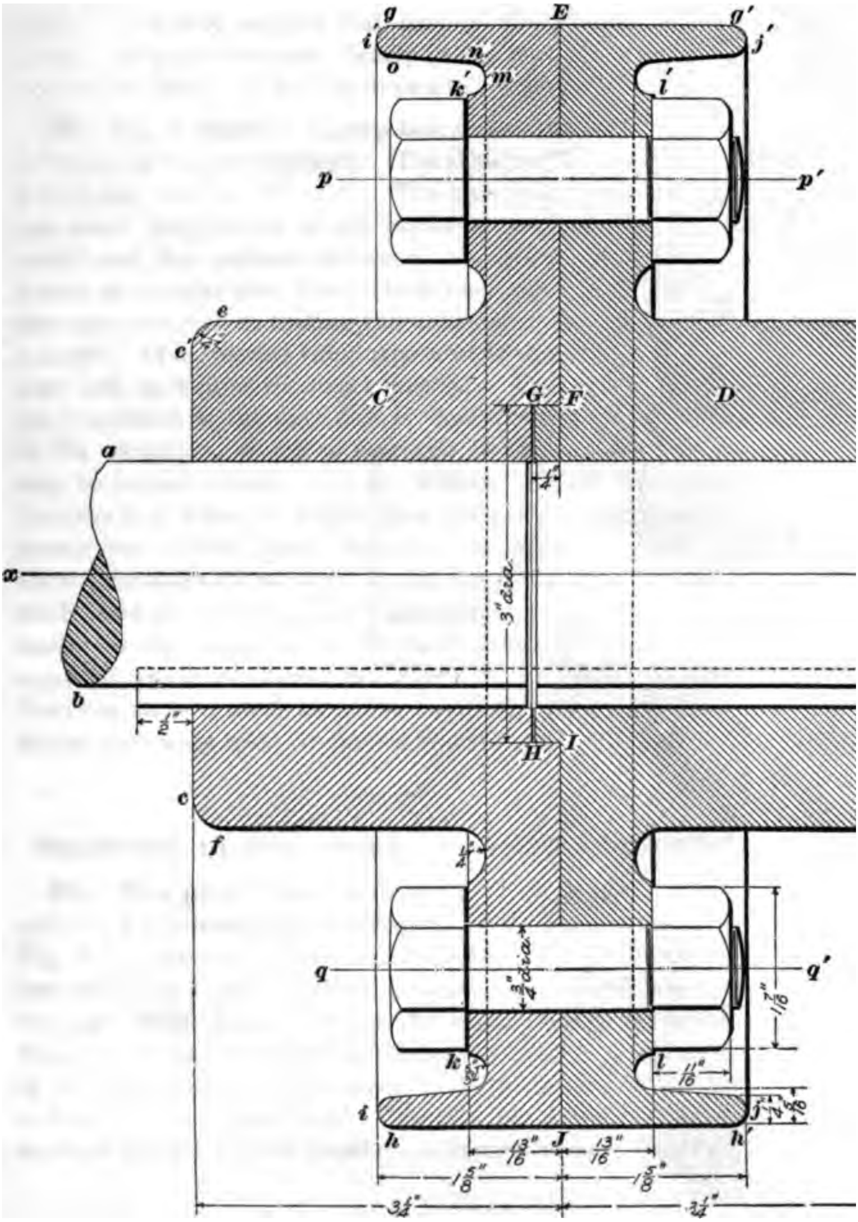


Fig. 1.

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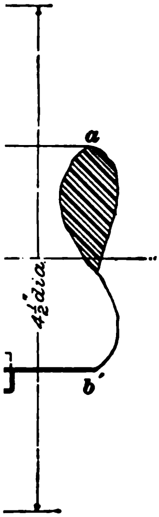

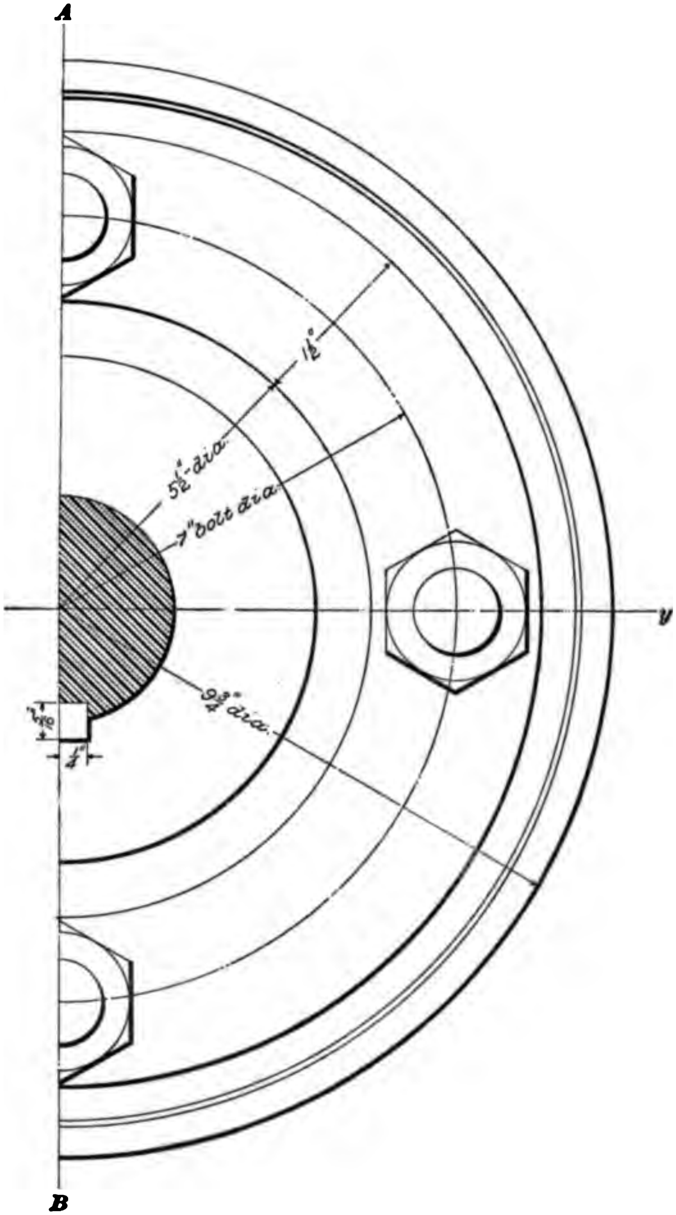

Fig. 2

tely following the title page.

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They are then clamped together and the keyway is cut. Fig. 2 is a half-end view and needs no comment.

41. To begin the drawing, draw the horizontal center line xy 6¼ inches from the lower border line. Draw Fig. 1 first, commencing with the vertical joint line EJ 5⅝ inches from the left-hand border line. It is now well to draw the shaft. This is 2 inches in diameter; therefore, lay off vertically on each side of xy, $2 \div 2 = 1$ inch, and through the points thus located draw the horizontal lines $a a'$ and $b b'$. Next draw the hub of the coupling. Lay off 3¼ inches horizontally on each side of the vertical joint line EJ and draw $c c'$ and $d d'$. The diameter of the hub is 4½ inches; therefore, lay off $4\frac{1}{2} \div 2 = 2\frac{1}{4}$ inches on each side of xy and draw $e e'$ and $f f'$; then draw in the round corners $c' e$, $e' d'$, $d f'$, and $c f$ with a radius of ¼ inch. To the left of the joint line EJ lay off a distance of ¼ inch and draw HG; lay off $3 \div 2 = 1\frac{1}{2}$ inches on each side of xy and draw GF and HI. On referring to Fig. 2, it will be found that the outside diameter of the coupling is 9¾ inches; lay off half this diameter, or 4⅞ inches, on each side of the horizontal center line xy and draw $g g'$ and $h h'$. Each flange is 1⅝ inches in width on the outer face; lay off 1⅝ inches to both right and left of EJ and draw the vertical lines $i i'$ and $j j'$, putting in the rounded corners i, i', etc. with the compasses set to ⅛ inch, in each case completing the circle faintly. Now, draw faint vertical lines $k k'$ and $l l'$ each $\frac{13}{16}$ inch from the vertical joint line EJ. Make $k k'$ and $l l'$ each equal to 8½ inches; to do this lay off 4¼ inches vertically on each side of xy and draw faint construction lines $k' l'$, $k l$. Then with the compasses set to a radius of $\frac{5}{32}$ inch and a center on the line $k k'$ produced, draw the semicircle $k' m n$. Draw $n o$ just touching this semicircle and tangent to the dotted circle mentioned above; then draw the other three similar parts of the flange in the same manner. Now, by reference to Fig. 2, locate the bolt center lines $p p'$ and $q q'$; draw the bolts and nuts and complete Fig. 1 from the dimensions given.

The reference letters printed in bold-face italics should be omitted on the drawing made by the student.

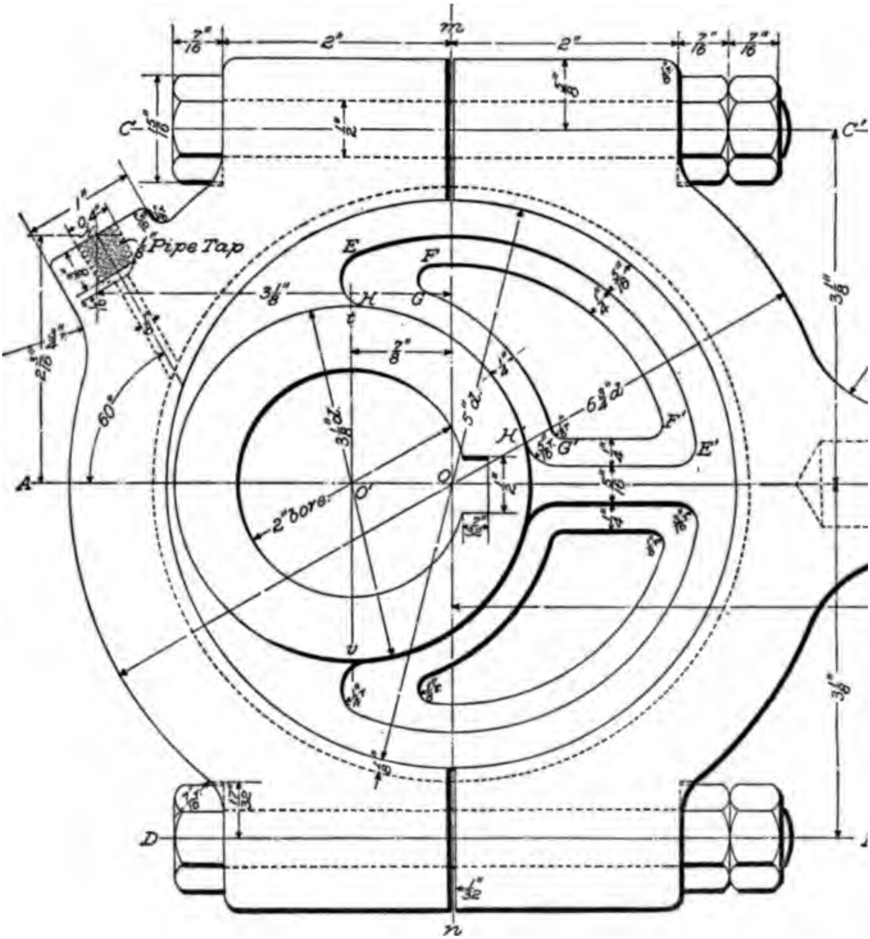
DRAWING PLATE, TITLE: ECCENTRIC AND BRAKE LEVER

42. Fig. 1 shows an elevation of an **eccentric and its strap.** The strap is made in two pieces and bolted together with a small space $\frac{1}{32}$ inch wide between them. Locate the point *O*, the center of the strap, and draw the center lines *A B*, *m n*, *C C'*, and *D D'*. Make the offset *O O'* one-half of the throw of the eccentric, or, in this case, $\frac{7}{8}$ inch, thus locating *O'*, the center of the eccentric shaft. Construct the rest of the view from the dimensions given, noting that the arcs *E E'* and *F F'* are concentric with *O*, while *G G'* and *H H'* are concentric with *O'*. The part *F F' G' G* is entirely open and is made so in order to lighten the eccentric.

43. Fig. 2 is a section of the **eccentric and strap.** The section is drawn in a conventional manner, it being all taken on the line *A B*, Fig. 1, except that part of the eccentric between *G'* and *F'*, where, instead of sectioning, the drawing shows it open from *L* to *P*, Fig. 2. This attracts attention to the open part of the eccentric and shows it more clearly.

It will be noticed that the slope of the threads in the sectional part is the same as for a left-handed screw. The thread, however, is right-handed, and the reason for showing it in this manner is that it is the bottom of the thread that is being looked at; that is, the section of a right-handed nut is the same as the projection of the top of a left-handed screw. It will also be noticed that the sectional lines on the eccentric run in opposite directions to those on the strap. This is always done when two different pieces meet and serves to indicate that they are separate pieces. Each piece should be sectioned entirely in the same direction, no matter if there is a break, as in the present case, between *L* and *P*. This shows that *A B C D E* is one piece. The dotted hole at *K*, Fig. 1, is an oil hole.

ECCENTRIC A



ECCENTRIC AND STRAP.

Cast Iron. Full Size.

Fig 1.

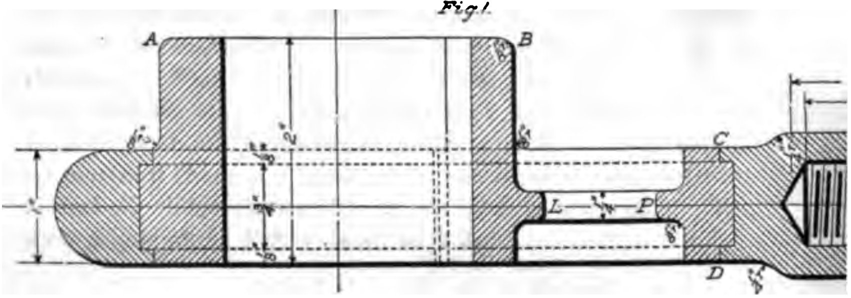


Fig 2.

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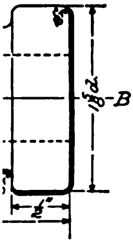
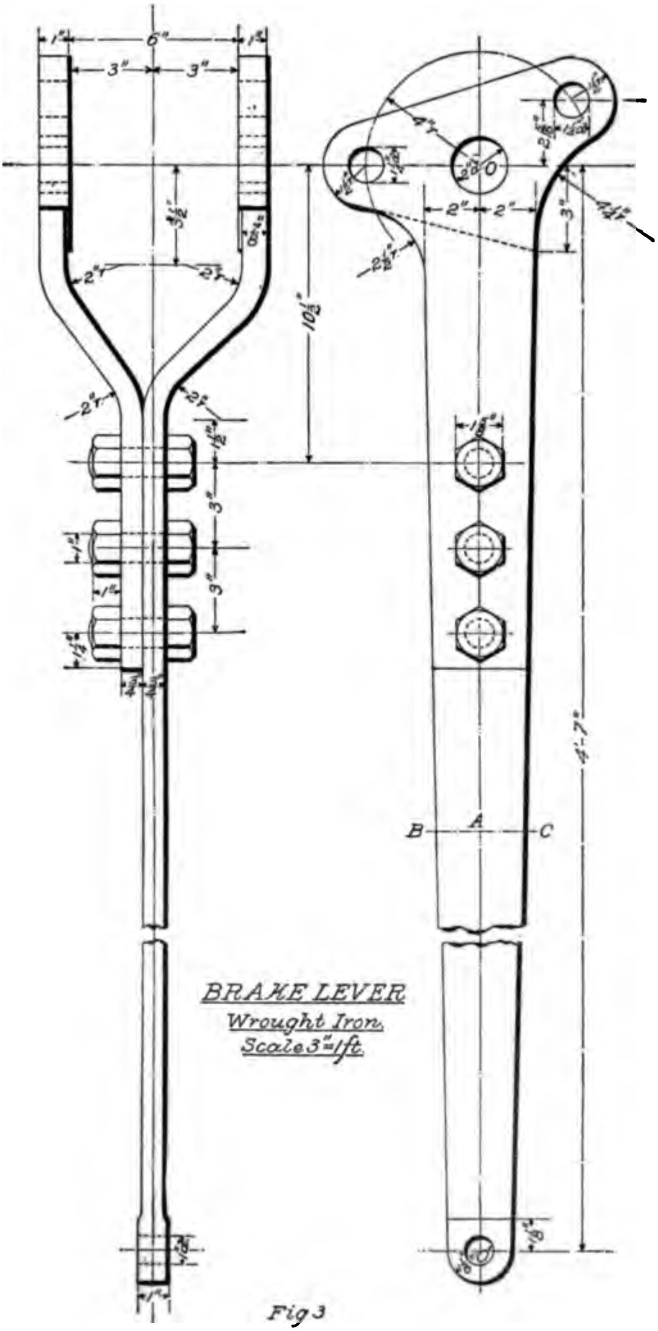
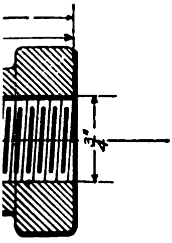
BRAKE LEVER.





BRAKE LEVER
Wrought Iron.
Scale 3"=1ft.

Fig 3



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44. Fig. 3 shows a **brake lever** drawn to a scale of 3 inches = 1 foot. Owing to its length being too great to be shown entirely on the drawing to this scale, the handle is shown as if a piece had been broken out, the dimension line, 4 feet 7 inches, giving the distance between the two centers O and O'. The lever should be readily drawn from the dimensions given. To proportion it properly, where the size of the paper does not permit the whole of it to be drawn, proceed as follows: The length between the centers is 4 feet 7 inches = 55 inches. The width through O' is $2\frac{1}{4}$ inches and through O 4 inches. Hence, $4'' - 2\frac{1}{4}'' = 1\frac{3}{4}''$; $1.75''$ = the taper in 55 inches. Measure off OA = say, 2 feet. The width at A may be found as follows: $\frac{1.75}{55}$ = taper in 1 inch. $\frac{1.75}{55} \times 24 = .76$ inch, nearly, the taper in 2 feet. $4'' - .76'' = 3.24'' = BC$, or the width at A. Now locate the point O' and from it as a center describe a curve $2\frac{1}{4}$ inches in diameter. Draw lines tangent to this curve and parallel to the edges between A and O already found.

It should be noticed that the center of the brake lever in the left-hand view is not situated at the joint where the two parts come together, but coincides with the center of the handle, as it should do.

DRAWING PLATE, TITLE: TIMBER TRESTLE

45. This plate shows a drawing of a standard framed **timber trestle** as used on the Cleveland and Canton Railroad. It is called a **compound trestle,** its various members being built up of comparatively small timbers. The advantages of this style of construction are the reduction of the cost of timber and the facility and safety with which repairs can be made. As the parts are bolted together, a defective piece may be readily replaced; and the pieces being separated from each other, there is a thorough circulation of air throughout the structure, which seasons and preserves the timber. It is desirable in a structure of this kind to show three views—end view, side view, and top view or plan. It is found that the limits of the drawing will admit of the figures being drawn to a scale of ⅜ inch = 1 foot; therefore, this scale will be adopted.

The drawing should be made from the written dimensions given on the plate; and when drawing Fig. 1, reference must frequently be made to Fig. 2 for some of the dimensions required.

46. To begin the drawing, draw the vertical center line $a a'$ at a distance of 4¼ inches from the right-hand border line. Draw the horizontal base line $b b'$ at a distance of 1⅛ inches above the lower border line. From and above the line $b b'$ lay off a distance of 12 inches on the vertical center line $a a'$, and through the point thus laid off draw the horizontal line $c c'$. On each side of the center line $a a'$ lay off on the horizontal base line a distance of 9 feet 6 inches and draw the short vertical lines $b c$ and $b' c'$ representing the ends of the ground sill. From and above the line $c c'$ lay off on the vertical center line $a a'$ a distance of 12 feet, which is the distance from the upper edge of the ground sill to the lower edge of the cap, and through the point thus laid off draw the horizontal line $d d'$; at a distance of 12 inches above this line draw the line $e e'$ representing the upper edge of the cap. On each side of the center line $a a'$ lay off on the horizontal line $d d'$ one-half the length of the cap, or $12 \div 2 = 6$ feet, and draw the vertical lines $d e$ and $d' e'$ representing the ends of the cap. Next draw the upright or *plumb* posts between the ground sill and the cap; on each side of the center line $a a'$ lay off on the horizontal line $b b'$ a distance of 2 feet 6 inches, and through the points thus laid off draw the vertical center lines $f f'$ and $g g'$ of the plumb posts. On each side of the center line $f f'$ lay off on the horizontal line $b b'$ one-half the width of the post, or $12 \div 2 = 6$ inches, and draw the vertical lines $h h'$ and $j j'$. Draw the other plumb post in a similar manner. From the point c on the ground sill and towards the center line $a a'$ lay off on the line $c c'$ a distance $c k$ of 18 inches; next, from the

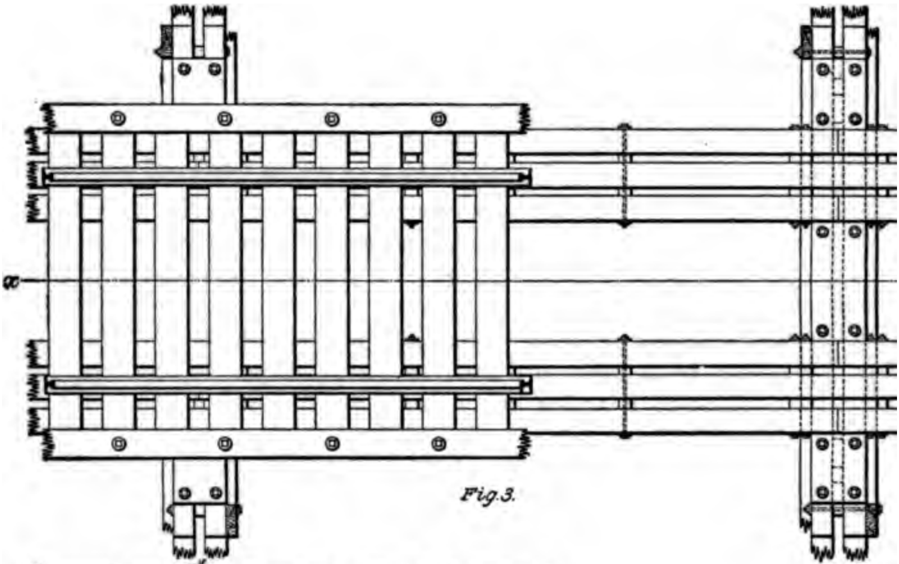

Fig. 3.

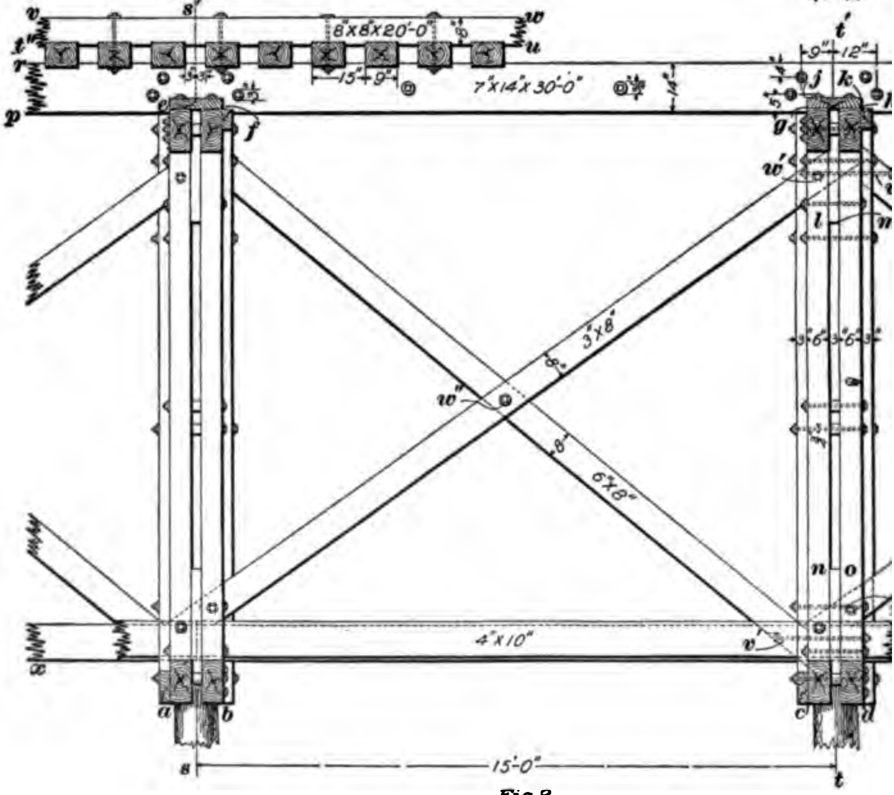

Fig. 2.

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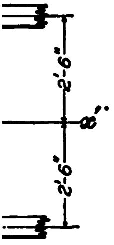

TIMBER TRESTLE.

Scale $\frac{3}{8}$" = 1 ft.

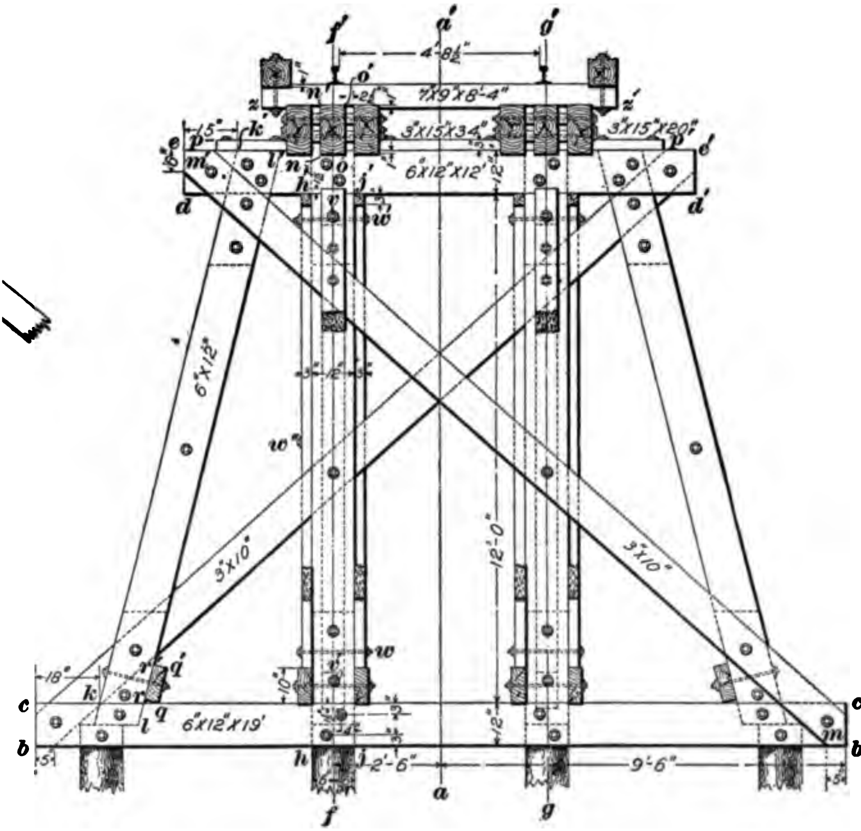

Fig. 1.

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point *e* on the cap and towards the center line *a a'* lay off a distance *e k'* of 15 inches; join the points *k* and *k'*. Now, with the aid of a triangle draw a line in lead pencil intersecting and at right angles to the line *k k'*, and on this line lay off towards the center line *a a'* and from the line *k k'* a distance equal to the width of the post, or 12 inches; through the point thus laid off draw the line *l l'* parallel to *k k'*. This will complete the left-hand batter post. Draw the right-hand batter post in a similar manner. We are now ready to draw the 3″ × 10″ sway-braces which connect the opposite ends of the ground sill and cap. From and to the left of the lower right-hand corner *b'* of the ground sill lay off a distance *b' m* of 5 inches; from and below the upper left-hand corner *e* of the cap lay off a distance *e m'* of 6 inches; join the points *m* and *m'*; then draw a line parallel to *m m'* at a distance from it equal to the width of the sway-brace—that is, 10 inches. Draw the brace connecting the other ends of the sill and cap in a similar manner. The frame so far drawn is called the **bent.** There is a space of 15 feet from the center of one bent to the center of the next, as shown in Fig. 2, and they are connected from cap to cap by **stringers.** In Fig. 1 these stringers are shown in section, three being set side by side on the cap over each plumb post. Each stringer is notched 1 inch over the cap. From and below the line *e e'* lay off a distance of 1 inch and draw a horizontal line representing the position of the lower edges of the stringers; from and above this line lay off a distance equal to the depth of the stringers, or 14 inches, and draw another horizontal line representing the upper edges of the stringers. On each side of the plumb post center line *f f'* lay off on the line *e e'* one-half the thickness of a stringer, or $7 \div 2 = 3\frac{1}{2}$ inches; then, through the points thus laid off draw the vertical lines *n n'* and *o o'*, completing the middle stringer. To the left and right of this stringer, at a distance of 2½ inches from the faces *n n'* and *o o'*, respectively, draw vertical lines representing the inner faces of the other two stringers. Lay off 7 inches to the left and right, respectively, of these lines, and through the points thus laid off draw vertical lines completing the two outer stringers. In a similar manner draw the stringers over the right-hand plumb post. The stringers are kept in position sideways by three **brace-blocks** bolted to the cap; to draw these lay off from and above the top edge *c c'* of the cap a distance of 3 inches and draw the horizontal line *p p'*. The center brace-block fits between the two inner stringers, as shown; the outer ones are 20 inches long; lay off this length to the right and left of the right- and left-hand outer stringers and draw the short vertical lines completing the brace-blocks. Across the stringers are laid the ties, which are notched 1 inch over the stringers. From and below the top edge *n' o'* of one of the stringers lay off a distance of 1 inch and draw the horizontal line *z z'* representing the lower edge of the tie notched over the stringers, as shown; from and above this line lay off the depth of the tie, or 7 inches, and draw the line representing the top edge of the tie. On each side of the center line *a a'* lay off on the line *z z'* one-half the length of the tie, or 8′ 4″ ÷ 2 = 4 feet 2 inches, and draw the vertical lines representing the ends of the tie, continuing these lines upwards for a short distance. Now, connecting the ends of the ties, in a direction parallel to the rails, are wooden guard rails to prevent the cars from jumping the trestle, if, by any chance, they should leave the tracks. These guard rails are notched 1 inch over the ties. From and below the top edge of the tie lay off a distance of 1 inch and draw a short horizontal line at each end of the tie; from and above these lines lay off a distance of 8 inches and draw horizontal lines representing the top edges of the guard rails. From each end of the tie lay off towards the center line *a a'* a distance of 8 inches and draw the short vertical lines to complete the guard rails, as shown. The distance between the inner flanges of the steel track rails is 4 feet 8½ inches; on each side of the center line *a a'* lay off on the top edge of the tie one-half this distance, or 4′ 8½″ ÷ 2 = 2 feet 4¼ inches, and draw vertical lines in lead pencil. The rails may now be drawn in and should be 5 inches high, 5 inches wide on the lower flange, and 2 inches wide on the top flange. By referring to Fig. 2, it will be seen that the bents are held together at their bases by longitudinal ties; these are called **girts** and extend from post to post, being attached to the lower ends of both batter and plumb posts. All the girts are of 10″ × 4″ section. From and to the right of the line *l l′* lay off a distance of 4 inches and draw the line *q q′* parallel to the line *l l′*. From the point of intersection of the line *q q′* and the line *c c′* draw a line *q r* at right angles to the line *l l′* of the batter post. From and above the point *r* lay off along the line *l l′* a distance of 10 inches and draw *q′ r′* parallel to the line *q r*. In a similar manner draw the girt at the base of the right-hand batter post and then draw two girts at the base of each plumb post. The bents are still further secured by means of diagonal braces which connect the top of a plumb post in one bent to the bottom of a plumb post in the next bent, as shown in Fig. 2; to each 6″ × 8″ central brace, which is bolted to the post by means of the bolts *v*, *v′*, there are two 3″ × 8″ braces, one on each side of the central brace, secured to the posts by bolts *w′* and to the central brace by a bolt *w″*. We will first draw the two 3″ × 8″ braces. From each side of the left-hand plumb post lay off a distance of 3 inches and draw vertical lines extending from the upper edge of the girts to the lower edge of the cap; on either of these lines lay off from and below the line *d d′* a distance of 3 inches and draw the short horizontal lines representing the ends of the diagonal braces seen in this view. Now draw the 6″ × 8″ brace by laying off on each side of the center line *f f′* one-half the thickness of the brace, or 6 ÷ 2 = 3 inches, and drawing vertical lines. This brace extends 1½ inches below the top of the sill and 1½ inches above the lower edge *d d′* of the cap; lay off these distances and draw the short horizontal lines representing the ends of the brace. Each bent is supported upon four piles, two of which are located immediately under the plumb posts, the centers of the other two being 18 inches from the ends of the sill. The tops of the piles are tenoned and enter for a depth of 6 inches between the two timbers forming the sill.

The student should be able to draw these piles without special instructions. Next locate the bolt centers; begin by drawing the center lines of the batter posts and sway-braces in lead pencil, and from the drawing locate all bolts on these lines. Where the timbers cross each other and are bolted together, draw in lead pencil the short diagonal of the diamond formed by the crossing of the two timbers; the point where these diagonals meet is the center of the bolt. Now locate all bolts on the center lines ff' and gg' and then the bolts on the center lines of the two outside piles. Dimensions are given which will enable the student to locate all other bolts. All the bolts are ¾ inch diameter, with heads and nuts ⅝ inch thick and 1¼ inches square. The washers are all 3 inches in diameter, with the exception of those marked 3½ inches in Fig. 2, and are all ¾ inch thick. In many places washers are introduced between timbers to act as separators; these are all 3 inches in diameter, with the exception of those between the stringers, which are 3½ inches in diameter.

47. We are now ready to proceed with Fig. 2. Draw the vertical center line ss' at a distance of 2½ inches from the left-hand border line. From and to the right of ss' lay off a distance of 15 feet and draw the center line tt'. Projecting from the lower edge of the sill in Fig. 1 draw short horizontal lines ab and cd and also draw the lines ef and gh by projecting from the top edge of the cap. On each side of the center line tt' lay off one-half the distance between the two timbers that form the sill, or $3'' \div 2 = 1\frac{1}{2}$ inches, and draw vertical lines between cd and gh. To the right and left of cg and dh, respectively, lay off a distance of 6 inches and draw vertical lines as before. Projecting from the top edge of the brace-block in Fig. 1 draw the horizontal line jk and complete the end view of the brace-block by drawing the short vertical lines gj and hk. The ends of the pieces forming the posts are separated by tenon blocks 3 inches thick and 2 feet 8 inches long. From and below the line gh lay off a distance of 2 feet 8 inches and draw the line *l m*. From and above the line *c d* lay off a distance of 6 inches and draw a dotted horizontal line; above this line lay off a distance of 2 feet 8 inches and draw the line *n o*. In a similar manner draw the sill, post, and cap, etc. about the center line *s s'*. Projecting from Fig. 1 draw the horizontal lines *p q* and *r s''* representing the upper and lower edges of the stringers. These stringers are 30 feet long and their joints are staggered; the joint on the center line *t t'* is represented by a full line, while that on the center line *s s'* is a dotted line. Projecting from Fig. 1 draw in lead pencil horizontal lines representing the upper and lower edges of the ends of the ties. From and to the right of the center line *s s'* on the line *r s''* lay off a distance of 3 inches and draw a vertical line, as shown; on each side of this vertical line and along the line *r s''* space off distances of 15 inches and draw vertical lines through each of the points; to the right of each of these vertical lines lay off a distance of 9 inches and draw vertical lines to complete the end views of the ties, as shown. Projecting from Fig. 1 draw the upper edges *v w* and lower edges *t'' u* of the guard rails, which are notched 1 inch over the ties, and next draw the horizontal lines *x y*, etc. representing the longitudinal girts, and then draw the diagonal braces, as shown. Now, to the left of the vertical line *c g* and to the right of *d h* lay off a distance of 3 inches and draw vertical lines defining the sway-braces. Then draw in the bolts from dimensions given and by projecting from Fig. 1.

48. Next draw Fig. 3. Draw the horizontal center line *x x'* at a distance of 3⅛ inches below the top border line. The student should experience no difficulty in drawing this figure; vertical lines should be projected from corresponding lines in Fig. 2, and all measurements may be found by reference to Figs. 1 and 2.

The reference letters printed in bold-face italics should be omitted on the drawing made by the student.

DRAWING PLATE, TITLE: STEEL COLUMNS AND CONNECTIONS

49. In this plate are shown two designs of columns used in the construction of modern office buildings, and a common method of connecting the floorbeams to the columns is shown. A side view and a front view of each design is given, and as it would manifestly be impossible to draw the columns in their full length, and since, besides, it would not serve any useful purpose to do so, a part of the column between the base and the floorbeam connection is broken away. When this is done, it is understood that the part broken away is similar to the ends of the column next to the break, which in this case is indicated by drawing a line consisting of a long dash and two dots across the column. Among mechanical draftsmen it is customary to indicate a break by a wavy freehand line, but many architectural draftsmen indicate a break in the manner shown in this plate.

50. In the design shown by Fig. 1, the column consists of two channels placed back to back and tied together by cover-plates. A cross-section of this column is shown in Fig. 13 (*a*). In the design shown by Fig. 2, the column is built up of four **Z** bars riveted to a web-plate, the column having the cross-section shown in Fig. 13 (*b*). The columns in both designs are made in sections, the lower section being 22′ 8″ long. The upper sections are spliced to the lower sections by means of horizontal plates, angle irons, and splice plates, the joint *m n* being a short distance above the floorbeams.

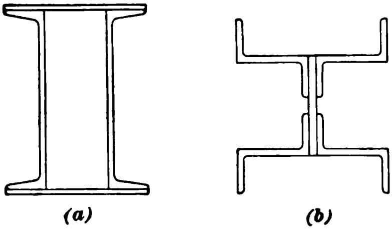
(*a*) (*b*)

FIG. 13

51. In the drawings there will be noticed several conventionalities. There are a number of rivet holes shown black in the side views (the views having the center lines *c d*

STEEL COLUMN

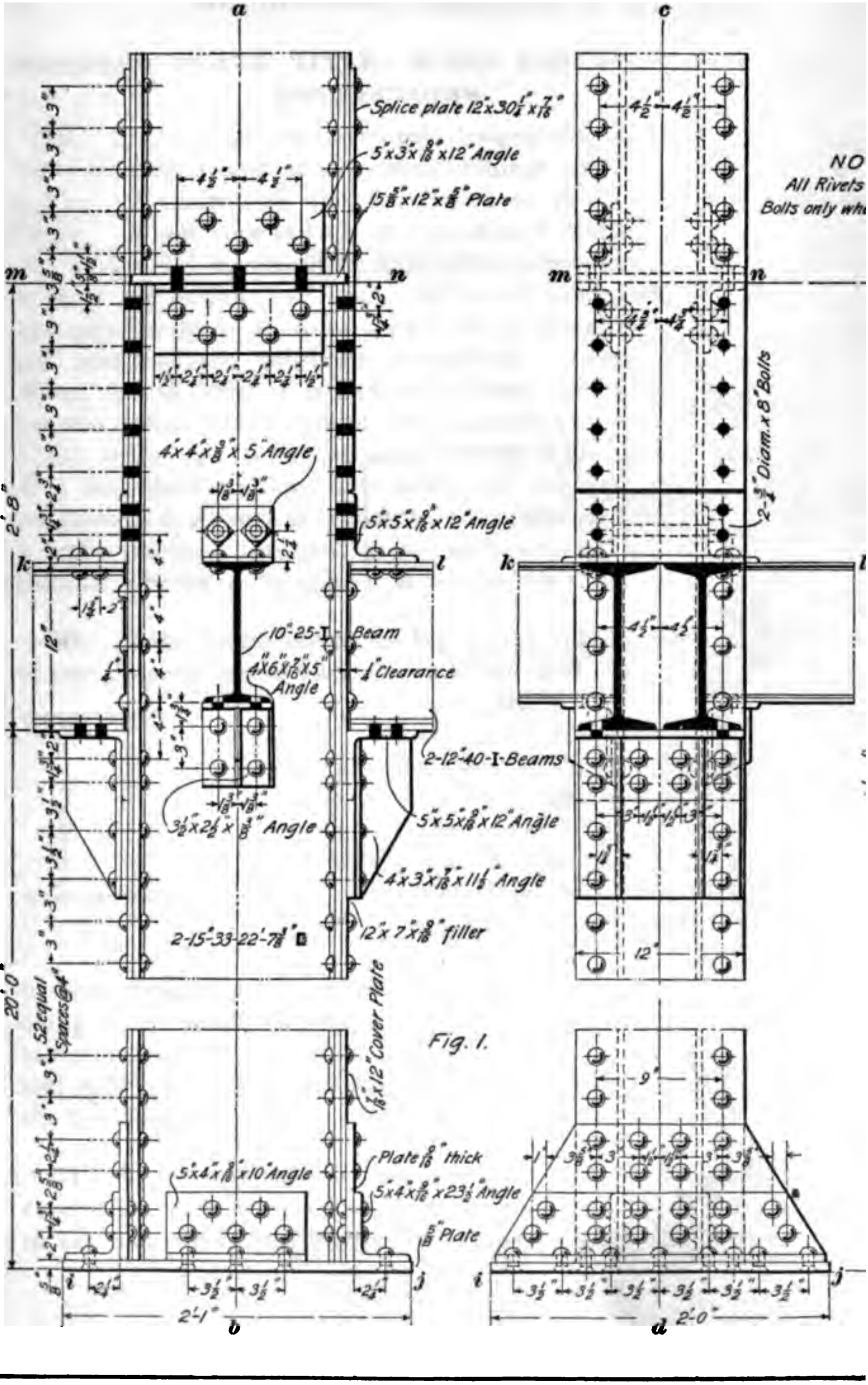

Fig. 1.

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ND CONNECTIONS.

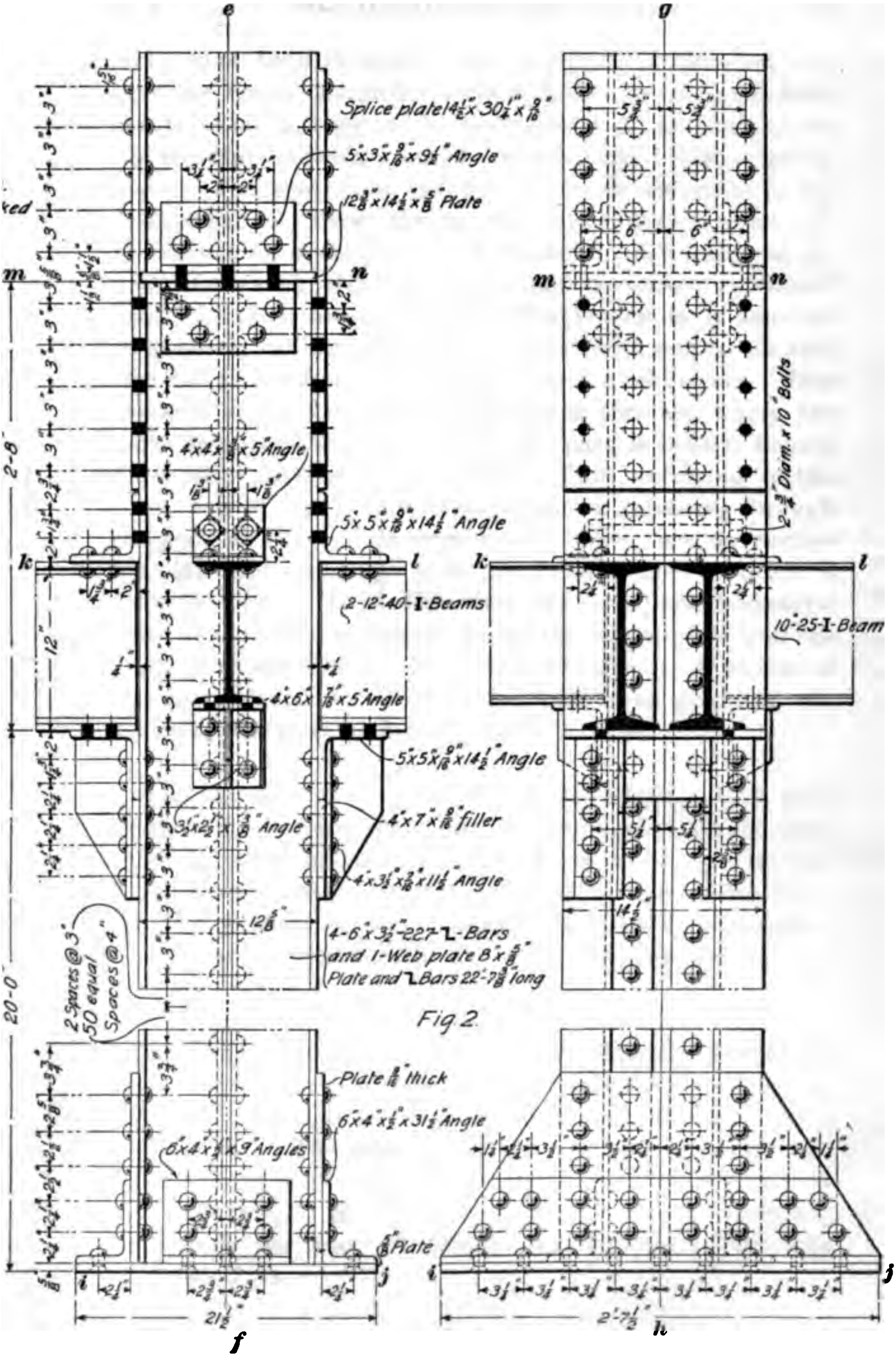

Fig. 2.

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and *g h*) of both designs. This indicates to the shop men that the rivets destined for these holes are so-called **field rivets**; that is, they are to be driven in at the place where the structure is being erected and after the columns are in place. The position of the field rivets is indicated in the front views (the views having the center lines *a b* and *e f*) by blackening in the *position* of the holes. According to the rules of drawing, their position would be shown by dotted lines, but the conventionalism of blackening in is resorted to as better calling the attention of the workmen to the fact that the rivets destined for these holes are field rivets. This common usage is departed from only in the case where the parts through which the field rivets pass is hidden behind some other parts of the structure. Thus, referring to the front view of Fig. 1, field rivets are shown as passing through the plate and two angle irons at the splice, and as neither the plate nor the angle irons are hidden, their position is shown in black. In the side view, however, the horizontal plate at the splice is hidden behind the splice plate and the angle irons are hidden behind the splice plate and the legs of the channels; consequently, the field rivet positions are shown in this view by dotted lines.

52. All the rivets that are to be driven in the shop where the column is made are shown in position; in some drawing rooms, in order to save time, the rivets are omitted, their position being merely indicated by two center lines. In order to show the rivets clearly, their heads are shaded a little in the manner shown, using the bow-pen for the shading.

53. The dimensions of the channels, **I** beams, and **Z** bars are not given in detail, this not being necessary on a working drawing. All that is required is to give the size of the channel, etc. The notes giving the sizes are read as follows: On the front view of the left-hand design we find a note 2–15″–33–22′–7⅜″ ⊏; this means two 15-inch channels, 33 pounds per foot of length, 22′ 7⅜″ long. The note on the front view of the right-hand design "4–6″ × 3¼″ –22.7–**Z** Bars" means that four **Z** bars having a depth of 6 inches, legs 3¼ inches wide, and weighing 22.7 pounds per foot of length are to be used. The note "2–12″–40–**I** Beams" means that two 12-inch **I** beams weighing 40 pounds per foot of length are to be used. The note "4″ × 3¼″ × $\frac{7}{16}$″ × 11¼″ Angle" means that an angle iron with legs measuring 4 inches and 3¼ inches, $\frac{7}{16}$ inch thick, and 11¼ inches long is to be used.

54. In order to aid the student in drawing the various parts the dimensions of which are not fully given on the plate, sectional views of them, fully dimensioned, are given in Fig. 14.

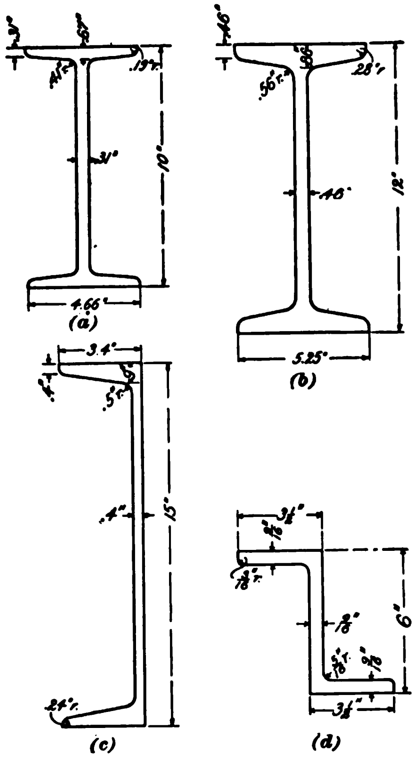


FIG. 14

The section of the 10-inch **I** beam is given in Fig. 14 (*a*), of the 12-inch **I** beam in Fig. 14 (*b*), of the 15-inch channel in Fig. 14 (*c*), and of the **Z** bar in Fig. 14 (*d*).

In Fig. 14 (*a*), (*b*), and (*c*), the dimensions are given in decimals, or as they appear in the catalogue of the rolling mill. Since it is practically impossible to lay off these decimal dimensions to a scale of 1½″ = 1 ft., it is recommended to use the nearest sixteenths of an inch, taking .19 = $\frac{3}{16}$, .24 = ¼, .28 = ¼, .31 = $\frac{5}{16}$, .40 = ⅜, .41 = ⅜, .46 = $\frac{7}{16}$, .56 = $\frac{9}{16}$, .66 = $\frac{11}{16}$, .67 = $\frac{11}{16}$, .86 = ⅞, .90 = ⅞.

55. All rivets in these two designs are $\frac{3}{4}$ inch diameter, having a head $1\frac{1}{4}$ inches in diameter and $\frac{9}{16}$ inch high. The holes to receive the rivets are generally punched $\frac{1}{16}$ inch larger than the shank of the rivet (the size of a rivet is denoted by the diameter of the shank *before* the rivet is driven), so that the rivet holes for $\frac{3}{4}$-inch rivets are punched $\frac{13}{16}$ inch diameter. As a matter of course, the rivet when driven fills the hole, so that the *driven* size of a rivet = nominal size + $\frac{1}{16}$ inch.

56. To draw the plate, begin by drawing the center lines, locating *a b*, *c d*, *e f*, and *g h* respectively $2\frac{7}{16}''$, $6\frac{1}{4}''$, $10\frac{11}{16}''$, and $14\frac{9}{16}''$ from the left-hand border line. The top line *K L* of the floorbeams is to be located $7\frac{5}{16}''$ and the bottom *i j* of the bases $\frac{13}{16}''$ above the lower border line. The floorbeams are shown broken off and should be made about the same length in proportion to their depth as they appear on the plate.

The reference letters printed in bold-face italics are to be omitted on the drawing made by the student.

DRAWING PLATE, TITLE: TURRET-LATHE TOOLS

57. This plate represents a working drawing of three tools used in connection with a turret lathe, viz., a **roughing box tool**, a **hollow mill holder**, and a **releasing die holder.**

58. The roughing box tool has a body having a shank to fit a hole in the turret of the machine; a tool post and adjusting washer receives the turning tool (not shown), which is clamped by means of the tool-post screw; two backrest jaws, which are clamped in position by means of the back-rest clamping bolt and adjusted by the back-rest adjusting studs and nuts, serve to steady the work being turned.

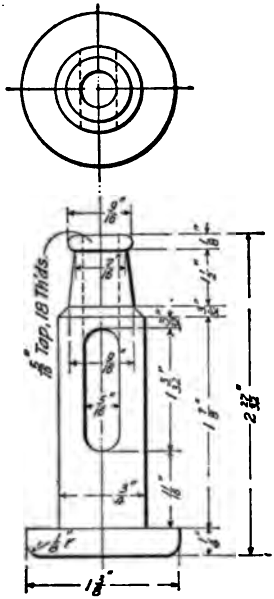
59. The hollow mill holder has a shank to fit the turret and a head recessed to take a hollow mill, which is held from turning by a flat-pointed $\frac{5}{16}$-inch setscrew.

60. The releasing die holder is composed of a sleeve and a die holder; the latter is recessed to receive a die for thread cutting, the die being held by four pointed $\frac{1}{4}$-inch setscrews with fillister heads. (The head of the screws marked "Releasing Die Screws" is called a **fillister head** by the trade.) The releasing die holder permits a thread to be cut up close to a shoulder or to any predetermined length in a turret lathe, and when this length has been reached, the die is left free to revolve with the work. To back the die off the work, the lathe is reversed and the turret moved backwards. This causes the $\frac{3}{16}$-inch pin in the end of the shank of the die holder to drop into the recess at the end of the sleeve, it being guided into it by the helical surface shown. The die now being held stationary and the work revolving backwards, the die backs off the work.

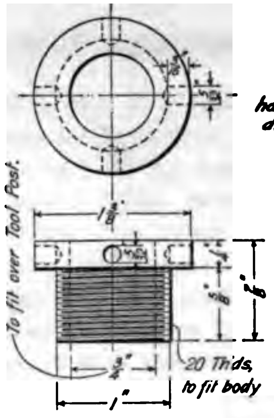
61. To begin, draw the center line $a\,b$ of the front and side view of the box tool at a distance of $4\frac{15}{16}$ inches from the upper border line. Locate the center c of the end view at a distance of $7\frac{3}{4}$ inches from the left-hand border line and draw the end view from the dimensions given. To the lower left of the end view is shown a partial view of the back-rest part of the body, which is given in order to show clearly the location and depth of the slot intended to receive the back-rest jaws and also in order to show the location of the adjusting studs in reference to the slot. This view is taken in the direction of the center line $c\,d$, looking towards c, and can readily be drawn from the dimensions given and projecting from the end view parallel to the center line $c\,d$.

62. The side view can be drawn partially by projecting over from the end view and partially from the dimensions given on the side view. The thickness of the lug for the back-rest jaws is given on the view to the lower left of the end view. The center line $e\,g$ of the tool-post boss may be located at a distance of 6 inches from the right-hand border line.

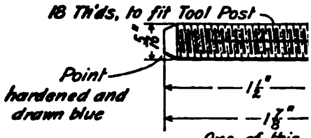
63. To draw the top view, begin by locating the center line $h\,i$ at a distance of $2\frac{3}{16}$ inches from the upper border line. Locate the center of the boss, which reference to the end



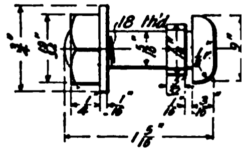
One of this,
Machinery Steel.
TOOL POST.



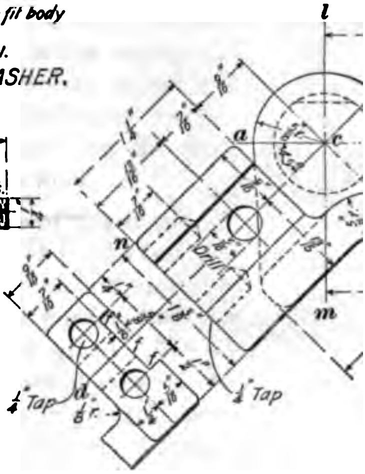
One of this, Tool Steel.
ADJUSTING WASHER.

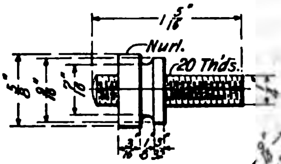


One of this,
TOOL POS

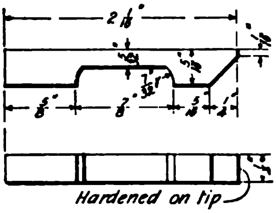


One of this, Tool Steel.
BACK REST CLAMPI

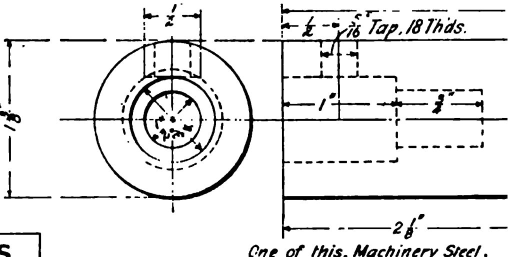




2 of this,
Machinery Steel.
ADJUSTING NUT
AND STUD FOR
BACK REST.

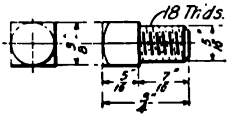


2 of this, Machinery Steel.
BACK REST JAWS.

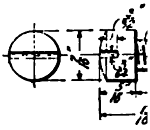


One of this, Machinery Steel.
HOLLOW MILL HOLDER.

<table>
<tr><td colspan="2">THE SCRANTON TOOL WORKS
SCRANTON, PA.</td></tr>
<tr><td colspan="2">TURRET LATHE TOOLS
For No. 1 Turret Lathe</td></tr>
<tr><td colspan="2">MATERIAL AS SPECIFIED</td></tr>
<tr><td>Scale Full Size.</td><td>Date Oct. 25, 1901.</td></tr>
<tr><td>Drawn by W.W.C.</td><td>Drawer 157.</td></tr>
<tr><td>Checked by J.A.G.</td><td>Order No. 184.</td></tr>
</table>



One of this,
Tool Steel, Hardened.
MILL HOLDER SCREW.



Four of thi
Tool Steel, Ha
RELEASING L

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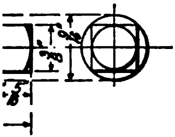
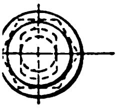
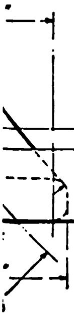
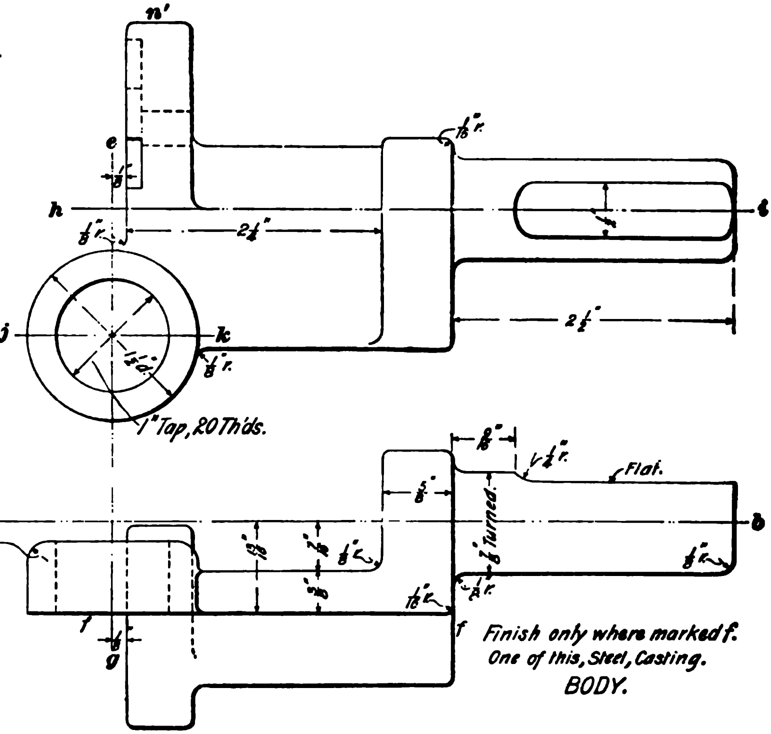


Steel.
;CREW.



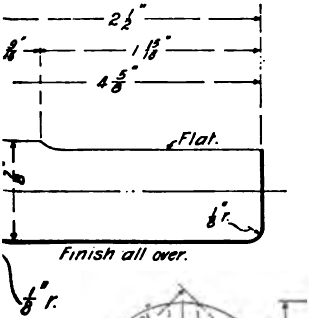
3OLT.



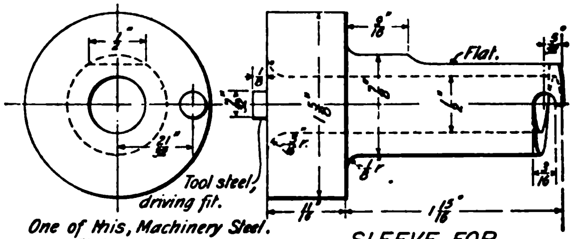


Finish only where marked f.
One of this, Steel, Casting.
BODY.

ROUGHING BOX TOOL



Finish all over.



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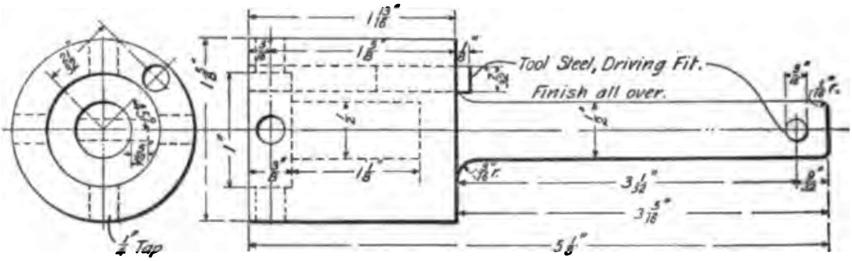
Finish all over.

SLEEVE FOR
RELEASING DIE HOLDER.

d.
SCREWS.



One of this, Machinery Steel.
DIE HOLDER, FOR RELEASING DIE HOLDER.

NO. 1984

mmediately following the title page.

JOHN SMITH. CLASS No 4529.

view shows to be at a distance of 1⅛ inches from the center line of the shank, by drawing the center line *j k* to intersect *e g*. While all the vertical lines can be projected up from the side view and some of the horizontal lines can be drawn from the dimensions given, the location of other horizontal lines must be determined from the end view. Since it often occurs in making drawings that a third view must be constructed by reference to one of the other two views from which points or lines cannot be projected, the operation will be explained in detail. The center line *h i* of the top view is imagined to be the top edge of a plane at right angles to the paper; and in the end view, the front edge of this imaginary plane is given by the center line *l m* passing through the center *c* of the shank. Suppose we want to locate the edge *n* of the end view in the top view. Then, we take the perpendicular distance of *n* from the plane *l m* with the dividers and lay it off in the top view at right angles from the plane *h i* and upwards, drawing the line *n'* through the point thus laid off. In case of doubt as to which side of the imaginary plane of the view being drawn a certain point or line to be transferred from a view at right angles to it is located on, simply imagine the center lines representing the edges of the imaginary plane to be produced until they intersect. Then, everything within the 90° angle in one view is within the same angle in the other view; likewise, everything within the 270° angle in one view is within the same angle in the other view. Thus, if we imagine the lines *h i* and *l m* to be produced until they intersect, the edge *n* will lay within the 270° angle and must, consequently, be laid off from *h i upwards*, in order to be located in the same angle in the top view. With this explanation of transferring points and lines from one view to another, the student will experience no difficulty in completing the top view.

64. The body having been drawn, draw the tool post, locating its center line 1 3/16 inches from the left-hand border line. The center of the top view should be about 1 3/16 inches and the lower end of the tool post $5\frac{3}{8}$ inches below the upper border line. It will be noticed that the hole for the tool-post screw has no thread shown in it, the information that it is threaded being conveyed by the note "$\frac{5}{16}''$ Tap, 18 Thds." On working drawings a thread is rarely shown in a hole which is to be tapped; the general rule is to draw two lines a distance apart equal to the outside diameter of the thread and to place a note on the drawing stating the size of tap to be used. This practice has been followed on the plate "Turret-Lathe Tools."

65. The center line of the adjusting washer may be located $4\frac{3}{16}$ inches from the left-hand border line; the center of the top view may be $1\frac{3}{16}$ inches and the lower end of the front view $3\frac{3}{8}$ inches below the upper border line. Locate the center line of the tool-post screw $\frac{13}{16}$ inch below the upper border line and the right-hand end of the side view $8\frac{11}{16}$ inches from the left-hand border line. The center of the end view may be $\frac{3}{4}$ inch to the right of the right-hand end. The center line of the back-rest clamping bolt should be drawn $2\frac{7}{16}$ inches below the upper border line; the right-hand end of the side view may be placed $7\frac{7}{8}$ inches from the left-hand border line and the center of the end view $\frac{15}{16}$ inch from the right-hand end of the bolt. Locate the center line of the adjusting stud $5\frac{9}{16}$ inches below the upper border line and the left-hand end $3\frac{5}{8}$ inches from the left-hand border line. The adjusting nut being movable on the stud, it may be shown in any convenient position, say about in the position shown. The top line of the back-rest jaw may be drawn $7\frac{5}{16}$ inches below the upper border line and its left-hand end may be located $\frac{1}{8}$ inch from the left-hand border line. A distance of $\frac{5}{8}$ inch may be left between the two views of the jaw.

66. The center line of the hollow mill holder may be located $3\frac{3}{4}$ inches above the lower border line and the center of the end view $5\frac{1}{8}$ inches from the left-hand border line. A space of $\frac{1}{4}$ inch mav be left between the front view and the side view.

67. The center line of the releasing-die-holder sleeve should be located $4\frac{3}{4}$ inches above the lower border line and the center of the end view $4\frac{7}{16}$ inches from the right-hand border line. A space of $\frac{3}{8}$ inch should be left between the end view and side view. The right-hand end is a helical surface having a pitch of $\frac{1}{4}$ inch; it can be laid out by means of the method given in *Geometrical Drawing*, laying off helixes of $\frac{1}{4}$ inch pitch and $\frac{7}{8}$ inch diameter and $\frac{1}{4}$ inch pitch and $\frac{1}{8}$ inch diameter. In the machine shop this helix would be cut by gearing the lathe to cut a thread of 4 to the inch and using a flat side tool set with its cutting edge at right angles to the center line of the sleeve. The center line of the die holder should be located 2 inches above the lower border line and the center of the end view $6\frac{7}{8}$ inches from the right-hand border line. A space of $\frac{7}{16}$ inch should be left between the end view and the side view.

68. The releasing-die screw and mill-holder screw should have their center lines drawn $1\frac{11}{16}$ inches above the lower border line. The center of the end view of the mill-holder screw may be placed $4\frac{11}{16}$ inches and the center of the end view of the releasing-die screw $7\frac{1}{16}$ inches from the left-hand border line. A space of $\frac{3}{8}$ inch should be left between the end views and side views of the screws.

69. All the notes on the drawing, in accordance with the most general practice, are written in capital and small letters, the former being $\frac{3}{32}$ inch and the latter $\frac{1}{16}$ inch high. Detailed instruction for this style of lettering have been given in *Geometrical Drawing*. The names of the different parts are written in capitals $\frac{1}{8}$ inch high.

70. This drawing plate differs from those previously given in that the title is placed in the lower left-hand corner. Different drawing offices have different rules in regard to titles for a drawing; some insist on having all titles placed in the lower left-hand corner and others place it in the lower right-hand corner; more rarely one of the upper corners is selected. In the arrangement of the title and the information contained therein, practice also greatly varies; this plate simply shows the practice of one drawing office, giving the name and place of the firm, the title of the drawing, the material, the scale, the initials of the persons who made the drawing and checked it, the date, the number of the drawer in which the tracing is kept, and the number of the drawing. When a number is given to a drawing, it generally, but not always by any means, gives the total number of drawings that have been made in a particular drawing office. Since practice varies so much, it can only be found by inquiry at the office where the drawing was made what a number on a drawing signifies.

71. To draw the title, construct a rectangle $3\frac{7}{16} \times 2\frac{1}{4}$ inches, placing it $\frac{1}{2}$ inch from the left-hand and the lower border line. Divide it vertically into four equal parts and subdivide each of the two lower parts into two equal parts again; then draw the horizontal lines shown. Divide the three lowest divisions horizontally into two equal parts and draw the vertical line shown. The firm name is printed in a style of letter very similar to a block letter, the letters being $\frac{5}{32}$ inch high. The location of the firm is printed in block letters $\frac{1}{8}$ inch high, excepting the first letter of each word, which is $\frac{5}{32}$ inch in height. The line "Turret-Lathe Tools" is a light-face block letter $\frac{5}{32}$ inch high, except the first letter of each word, which is $\frac{3}{16}$ inch high. The line "For No. 1 Turret Lathe" is printed in italics, the capitals and numeral being $\frac{1}{8}$ inch high and the small letters $\frac{3}{32}$ inch high. The next line is printed in light-face block letters $\frac{9}{64}$ inch high, and the succeeding lines are written freehand in the same style used in all the notes, the capitals and numerals being $\frac{5}{32}$ inch and the small letters $\frac{7}{64}$ inch high.

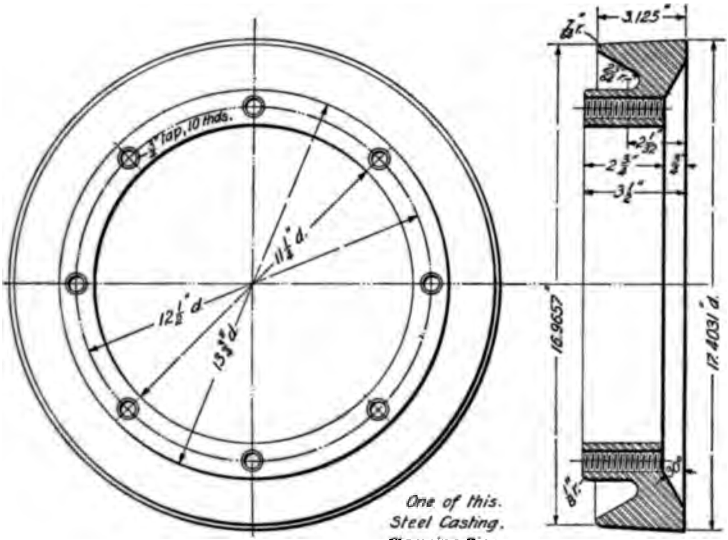
It is optional with the student, as far as the first five lines of the title is concerned, whether to write them in the freehand style used for the last three lines or in the different styles shown on the plate.

72. The number appearing in the right-hand lower corner is to be made $\frac{5}{32}$ inch high.

The reference letters printed in bold-face italics should be omitted on the drawing made by the student.

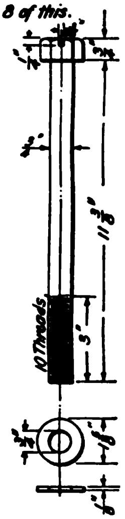
COMM

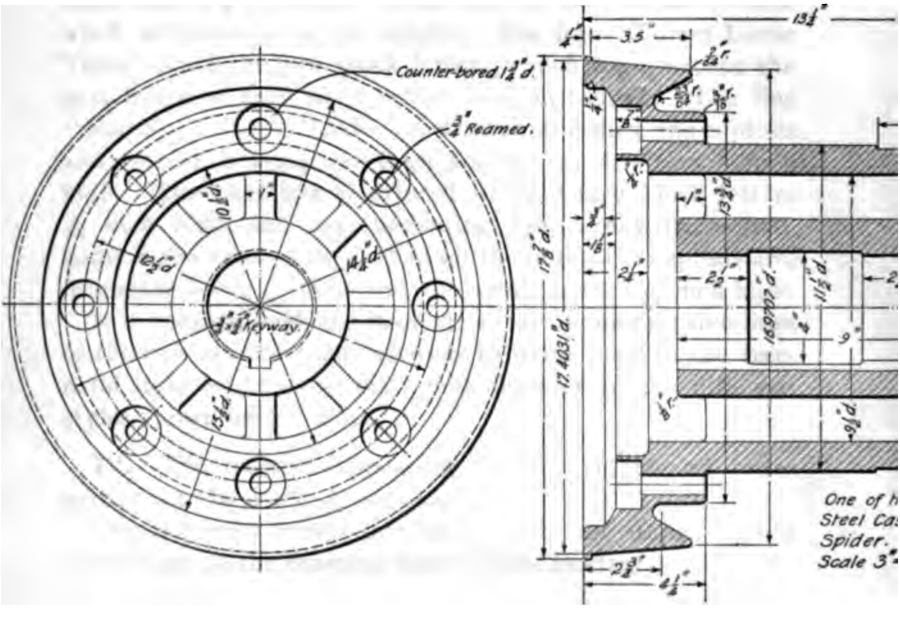
Scale



One of this.
Steel Casting.
Clamping Ring.
Scale 3"=1ft.

Clamping Bolt
and Washer.
8 of this.





One of h
Steel Ca
Spider.
Scale 3"

JUNE 25, 1893

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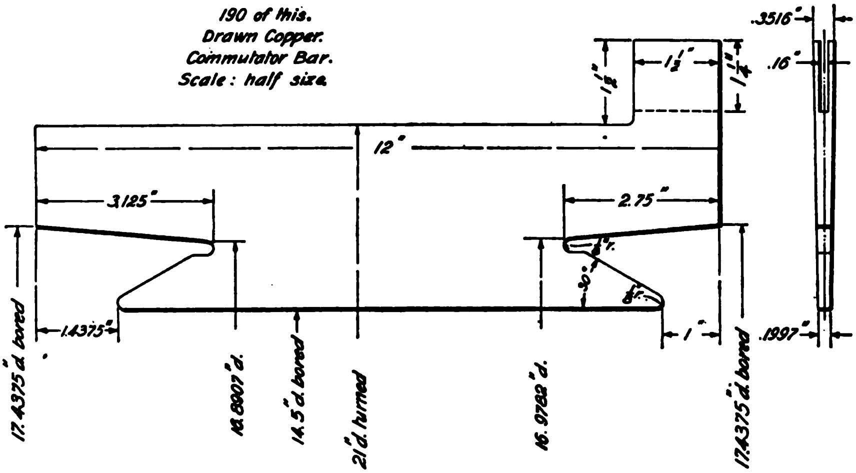
TATOR.

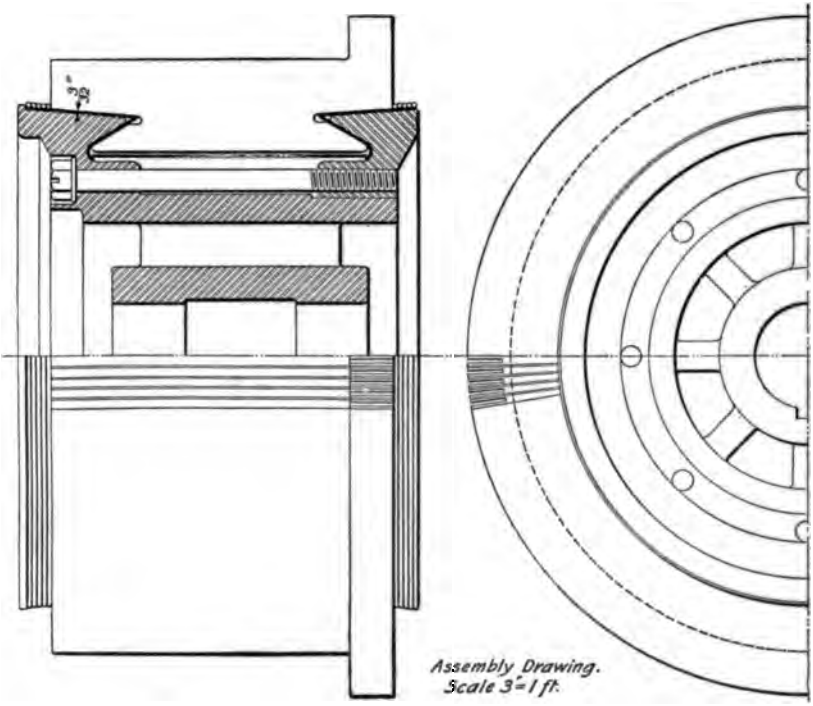
nd 6"=1 ft.



190 of this.
Drawn Copper.
Commutator Bar.
Scale: half size.

7.

:



Assembly Drawing.
Scale 3"=1 ft.

DRAWING PLATE, TITLE: COMMUTATOR

73. This plate shows the detail drawings and an assembly drawing of a dynamo commutator. It is composed of a spider and clamping ring, which are drawn together by eight $\frac{3}{4}$-inch bolts and clamp 190 commutator bars separated by mica insulation .04 inch thick. The commutator bars are insulated from the spider and clamping ring by mica rings $\frac{3}{32}$ inch thick.

74. Begin by drawing the clamping ring, locating its horizontal center line $4\frac{1}{8}$ inches below the upper border line and its vertical center line $2\frac{3}{4}$ inches from the left-hand border line. Leave a space of $\frac{11}{16}$ inch between the two views. In the sectional view, which is a section taken on the vertical center line, it will be noticed that the two largest diameters are given to ten-thousandths of an inch. In a shop where many commutators are made from the same drawing, the clamping surfaces (those in contact with the insulation) of the clamping ring, spider, and commutator bars are turned to fit gauges prepared by the toolmaker, who lays out the gauges and makes them to suit the dimensions given. While it cannot be expected that he will get such large gauges as are here required correct to $\frac{1}{10000}$ inch, still the giving of the dimensions in such a small subdivision of the inch calls his attention to the fact that great accuracy is required. This practice of giving *accurate* dimensions in decimals and *approximate* dimensions of a machine part in halves, quarters, eighths, sixteenths, etc. of an inch, is now largely adopted in the better class of drafting rooms, the purpose of its adoption being to show the workman at a glance which parts of a machine part require to be very accurate and which do not require it, thus tending to prevent the waste of time incidental to needless accuracy.

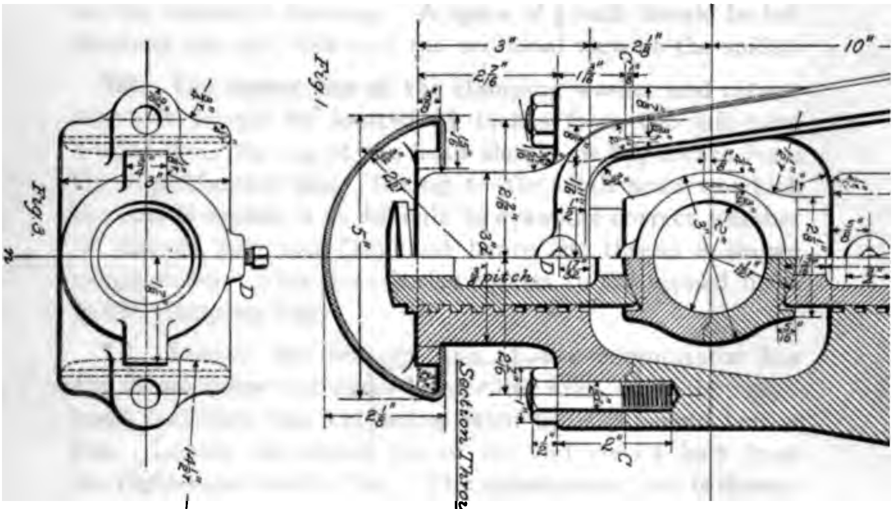
75. The spider consists of an outer shell and a hub joined together by eight arms. The hub has a keyway for a $\frac{3}{4}'' \times \frac{3}{4}''$ key, which is let half into the shaft and half into the hub, making the depth of the keyway in the hub $\frac{3}{8}$ inch.

Locate the vertical center line in line with the vertical center line of the clamping ring and locate the horizontal center line $3\frac{1}{2}$ inches above the lower border line. Draw this horizontal center line clear across the sheet, as it also serves for the assembly drawing. A space of $\frac{5}{8}$ inch should be left between the end view and the sectional view of the spider.

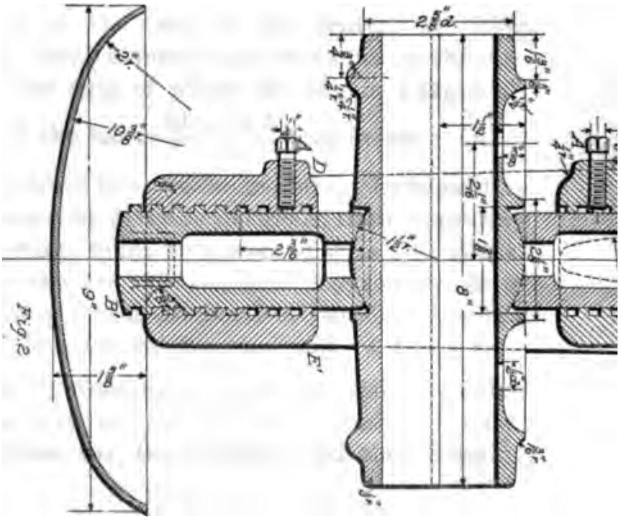
76. The center line of the clamping washer and clamping bolt should be located $7\frac{3}{4}$ inches from the left-hand border line; the top of the head should be $2\frac{7}{16}$ inches from the upper border line. Owing to the small scale to which the bolt is drawn, it is difficult to draw the correct number of threads per inch (10), and hence the thread is shown exaggerated. This remark also applies to the tapped holes in the clamping ring.

77. Locate the bottom line of the commutator bar $4\frac{13}{16}$ inches below the upper border line and locate the right-hand boundary line $1\frac{1}{16}$ inches from the right-hand border line. Locate the center line of the end view $\frac{3}{4}$ inch from the right-hand border line. The commutator bar is dimensioned with the dimensions required by the toolmaker for making the gauges and by the machinist for turning it to size; consequently, some of the dimensions required for drawing it must be obtained by calculation. Thus, to obtain the depth of the bar at the center, we have 14.5 inches as the inside diameter and 21 inches as the outside diameter of the ring of which the bar is a segment. Then, the depth of the bar is $\frac{21 - 14.5}{2} = 3\frac{1}{4}$ inches. Other dimensions are obtained in a similar manner. In regard to laying off the dimensions given in decimals, those appearing on the detail drawings made to a scale of $3'' = 1$ ft. should be laid off to the nearest thirty-second inch (on a scale of $3'' = 1$ ft.), while those on the commutator bar should be laid off to the nearest sixty-fourth inch on a scale of $6'' = 1$ ft.

78. The assembly drawing is made up from the detail drawings, drawing first the spider in the half-sectional view; then the commutator bar, the follower, and the clamping



Section Through Y-Z.



Longitudinal Section Through m-n

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SHAFT HANGER.

Cast Iron *Half Size*

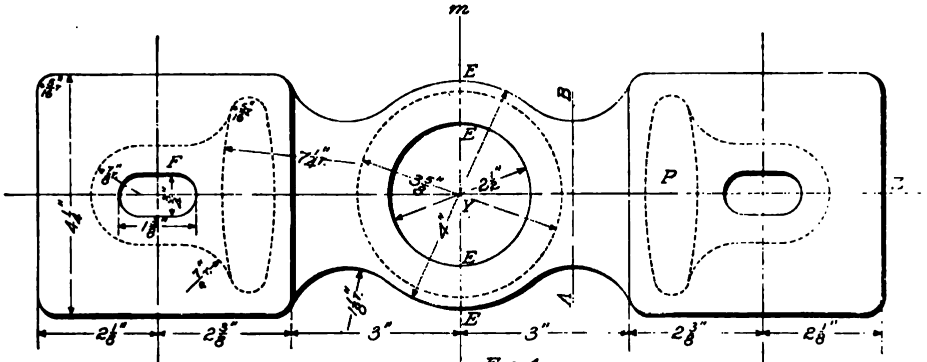


Fig 4

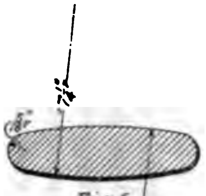


Fig 5
Section Through C-D

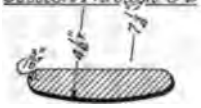
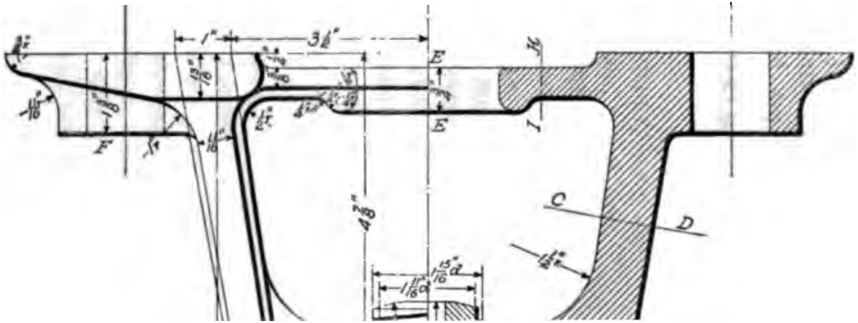
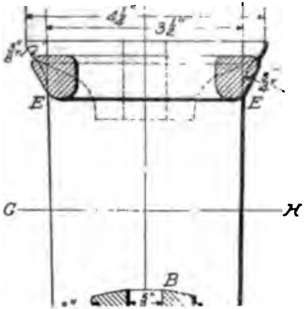


Fig 6
Section Through A-B





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bolt should be drawn. The lower half of the side view can now be drawn. In order to get the assembly drawing within the space available, only half of the end view is shown, it being understood that the omitted part is similar to the part shown. In a working drawing, in order to save time, it is not customary to show all the commutator bars on the assembly drawing, only a few being indicated, as shown. The insulation between the bars being only .04 inch thick, it is not feasible to draw it to a scale of 3″ = 1 ft. and define its thickness correctly by two lines; consequently the thickness is somewhat exaggerated in the drawing, which under the circumstances is a permissible liberty. Part of the mica insulation between the commutator bars and the spider and clamping ring projects beyond the end surfaces of the bars; it is wrapped securely with a layer of heavy twine well shellacked. In the assembly drawing, the vertical center line of the end view is to be located ½ inch from the right-hand border line and the left-hand surface of the spider $7\frac{1}{16}$ inches from the same border line.

DRAWING PLATE, TITLE: SHAFT HANGER

79. This plate shows a drawing of a **hanger** used to support shafting. There are shown a side view, half in elevation and half in section on the line YZ, part of a top view, part of a bottom view, a complete longitudinal section on the center line mn at right angles to the plane of the paper, and two small sections through the frame. For convenience in describing, the different views will be numbered.

Draw Fig. 1 first, which shows a half elevation and a half section. No great difficulty should be experienced in drawing it. It will be noticed that the journal-box through which the shaft passes has a spherical bearing; this is shown by the fact that the radius of the curve is the same, or 1¼ inches, in Fig. 1 and Fig. 2, the latter being a view at right angles to Fig. 1. The object to be attained by making the bearing partly spherical is to provide for self-adjustment in any direction, in case the shaft is not perfectly straight.

Fig. 2 shows very plainly how the shaft may be adjusted vertically. The setscrews *A*, *A* are released, and the large screws *B* and *B'* are screwed up or down, according as the shaft is to be raised or lowered, the spherical bearing allowing of adjustment at every point. The setscrews are then screwed down or set, so as to prevent *B* and *B'* from changing their positions. It will be noticed that the journal-box is made in two parts. Fig. 2 is a section on a line *m n* and shows the whole length of the hanger. The part sectioned at *E* and *E* is shown in Figs. 1 and 4 by *E E*. The dotted lines between them show the projection of the boss drawn at *F*, in Figs. 1 and 4, but not seen in this view. The lines forming the outline *E D E' E* show the projections of the hanger frame. The screws *B* and *B'* are made hollow to lighten them. The dotted ellipse in *B* is the projection of the rib shown at *L* in Fig. 1.

Fig. 3 is a bottom view as far as the line *C C*, after the oil pan and screw *B'* have been removed. *D* is the projection of the boss *D* in Figs. 1 and 2, through which the setscrew *A* passes.

Fig. 4 is a partial top view. This can be drawn from the dimensions given without any special instructions.

Fig. 5 is a section through *C D*, Fig. 1, and *G H*, Fig. 2. It shows the shape of the frame, which is also shown by the dotted outline *P*, in Fig. 4. This section is drawn more particularly to show that the radius of the curve on each side of any section of the vertical part of the frame is the same.

Fig. 6 is a section through *A B*, Fig. 4, and *I K*, Fig. 1.

DRAWING PLATE, TITLE: BENCH VISE

80. This plate shows a **bench vise and its details.** A drawing of this kind is called a **detail drawing.** Fig. 1 is a complete top view and Fig. 2 is a section through the center line *C D*. The remaining figures are drawings of the different parts, or details. The actual practice in the drawing room would be to draw Figs. 1 and 2 first and then the

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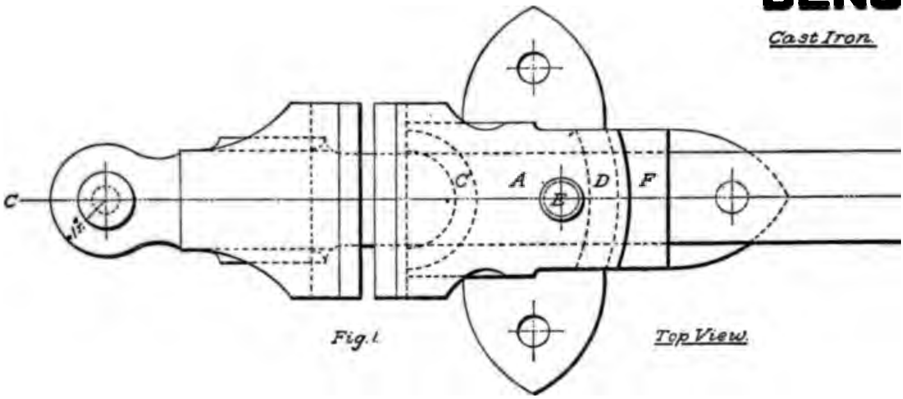

Fig. 1.

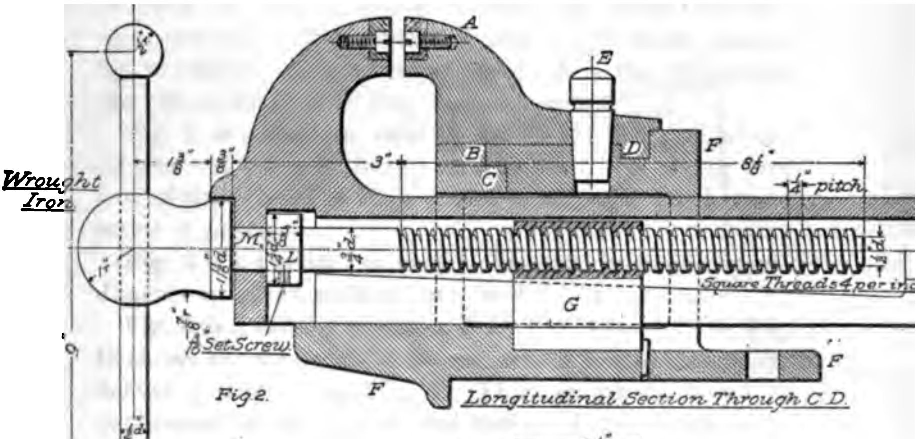

Fig. 2.

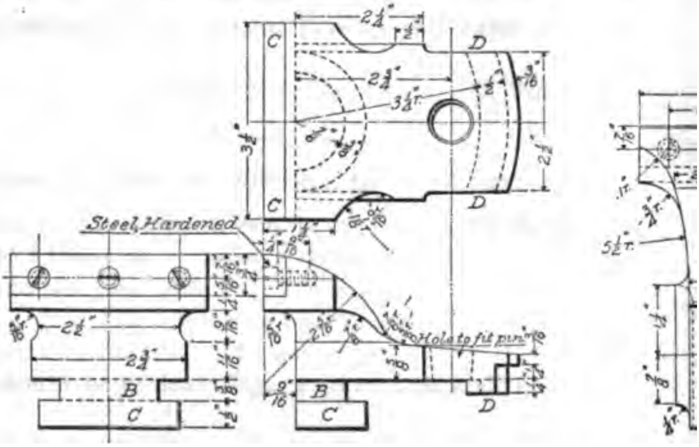

Fig. 3.

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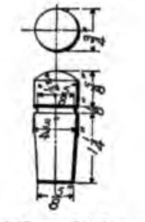
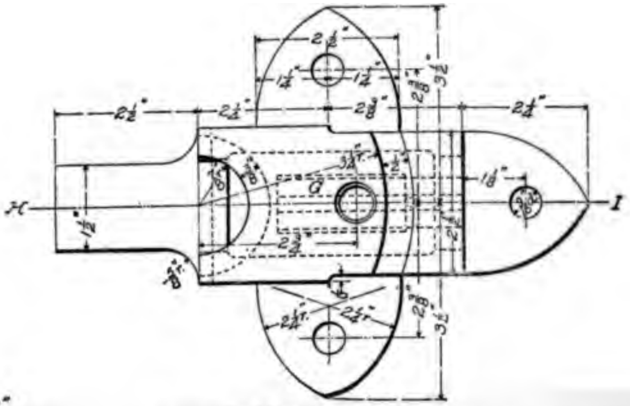
VISE

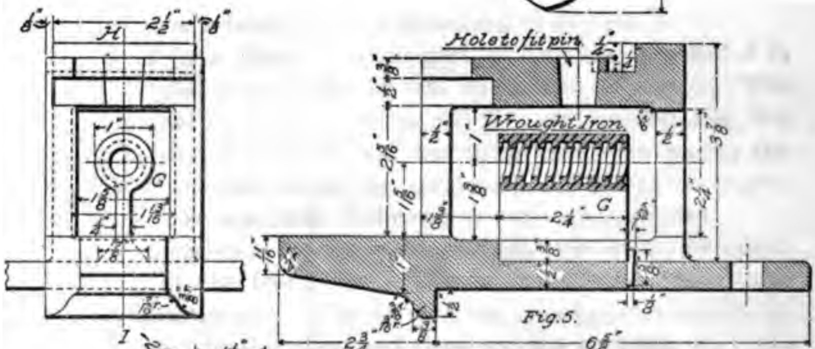
Half Size.



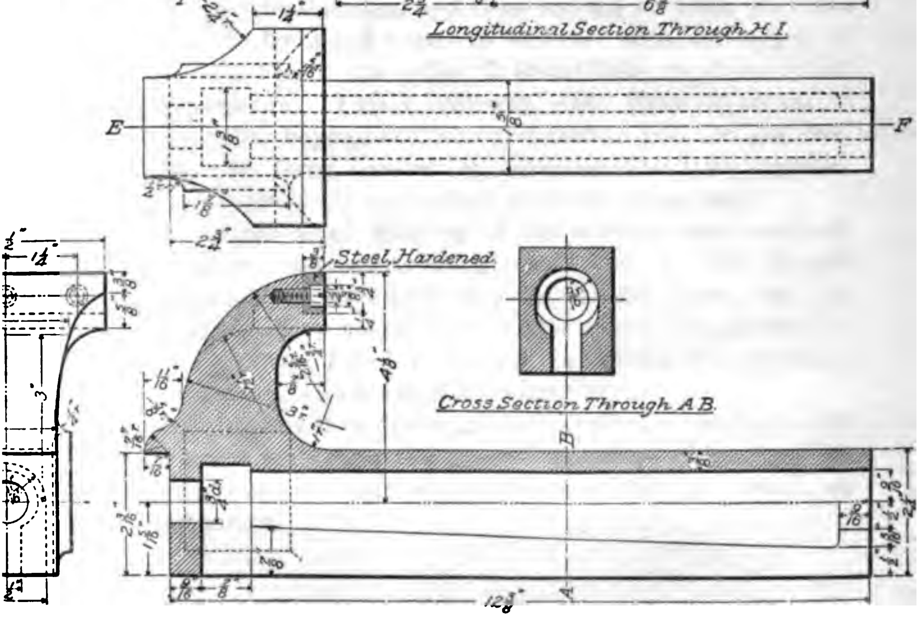
Wrought Iron.

Fig.4.





Longitudinal Section Through H I.



Cross Section Through A B.

Fig.6.

Longitudinal Section Through E F.

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details, but in the present case the student will do well to draw the details first, as it will help him in drawing the other two views, particularly since nearly all the dimensions are given on the details. Following out this plan, leave space for Figs. 1 and 2 and begin by drawing Fig. 3. This consists of three views of the jaw, the part marked *A* in Figs. 1 and 2. The parts marked *B*, *C*, and *D* in Figs. 1, 2, and 3 are circular in shape, so as to permit the back jaw *A* of the vise to swing when the pin *E* is removed. This allows the vise to hold tapered pieces with as much firmness as straight ones.

Fig. 4 is a detail, with dimensions, of the pin *E*.

Fig. 5 is a detail of that part of the vise marked *F* in Figs. 1 and 2, and also of the wrought-iron nut *G*. The reason that only a part of the nut *G* is sectioned in Fig. 2 is that a conventional method has been used. In reality the whole of the nut should be sectioned. It would be impossible to take a section of the nut in the manner shown.

Fig. 6 shows a top view, longitudinal section, and cross-section of the front jaw. This is cored out to admit the screw and the nut. The form of the cored part is shown by the longitudinal section and cross-section through *E F* and *A B*. The front jaw moves in and out with the screw, as shown by Fig. 2, the collar *L* being held in place by the small setscrew. This is sufficient, since there is no stress on the collar beyond the force required to pull the jaw outwards, the force exerted by screwing the jaws together coming entirely on the surface *M* of the screw head.

A separate detail drawing of the screw is not required, since all the dimensions are given in Fig. 2. The threads of the screw are shown the way they really appear, but the student can draw them in the conventional way explained in Art. **24.** Figs. 1 and 2 can now be drawn, the necessary dimensions being obtained from the details.

For ordinary shop drawings, the details are usually drawn to a larger scale than the general drawings, Figs. 1 and 2. In this case the size of the plate would not permit an enlargement.

DRAWING PLATE, TITLE: PROFILES OF GEAR-TEETH

81. If a circle is rolled on a straight line without sliding, a point on the circumference of the circle will describe a curve called the **cycloid.** The circle is called the **generating circle.** The shape of the curve and the manner of drawing it are shown in Fig. 1. Let *O* be the center of the generating circle, which is 1¾ inches in diameter, *P* the point on the circumference of the generating circle, and *A B* the straight line on which the generating circle is rolled and which is equal in length to the circumference of the generating circle, or 1¾" × 3.1416 = 5.4978, say 5½ inches. The generating circle should be so placed that its center *O* lies over the center of the line *A B*, as shown. Divide the generating circle into any number of equal parts, in this case 12, or *P 1*, *1–2*, *2–3*, *3–4*, etc., and through these points draw lines *C D*, *E F*, *G H*, etc. parallel to the line *A B*. Through the center *O* of the generating circle draw the radius *O 6*. Divide each half of the line *A B* into half the number of equal parts that the generating circle is divided into, as *A 1*, *1–2*, *2–3*, etc., and through these points draw lines perpendicular to *A B* terminating in the line *G H*, as *A G*, *1–1'*, *2–2'*, *3–3'*, etc. From the point *1'*, with a radius equal to the radius of the generating circle, as *O 6* or *1'–1*, describe an arc intersecting the line *K L* in the point P^1; from the point *2'*, with the same radius, intersect the line *I J* in the point P^2; from the point *3'*, with the same radius, intersect the line *G H*; continue in a similar manner with the remaining points *4'*, *5'*, *7'*, *8'*, etc., intersecting the lines *E F* and *C D* in the points P^4, P^5, P^7, P^8, etc. The points *A*, P^1, P^2, P^3, etc. are points in the curve through which the cycloid may be drawn. It will be noticed that when the center *O* of the generating circle coincides with the point *G*, the point *P* on the circumference of the generating circle coincides with the point *A*; and that when the generating circle is revolved towards the right, without sliding, until the center *O* coincides with the point *1'*, the point *P* will coincide with the point P^1. Thus it is seen how

PROFILES O

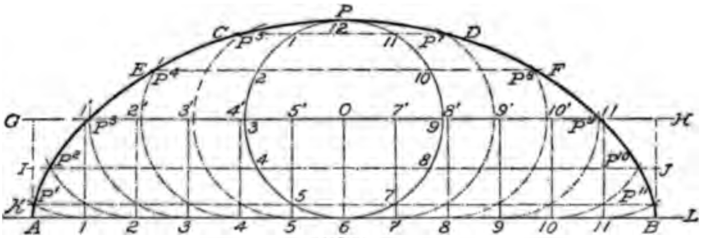


Fig.1.

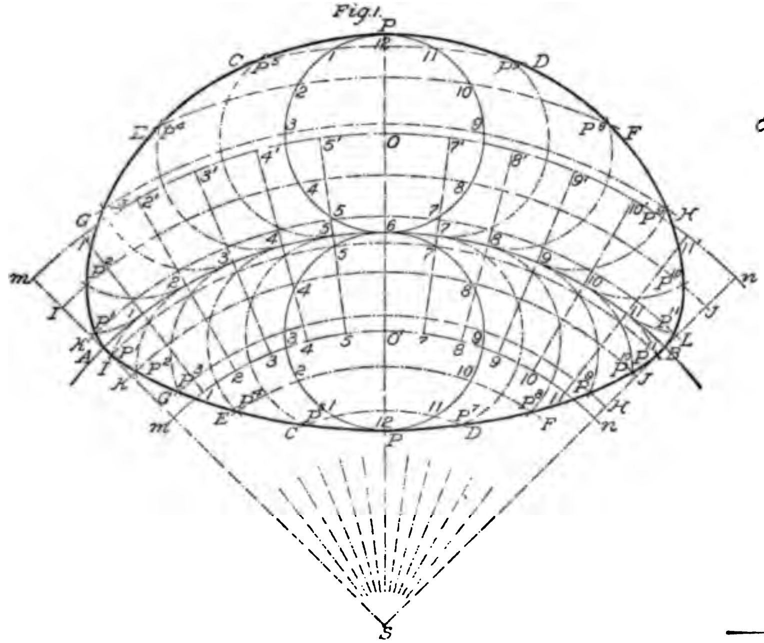


Fig.2.

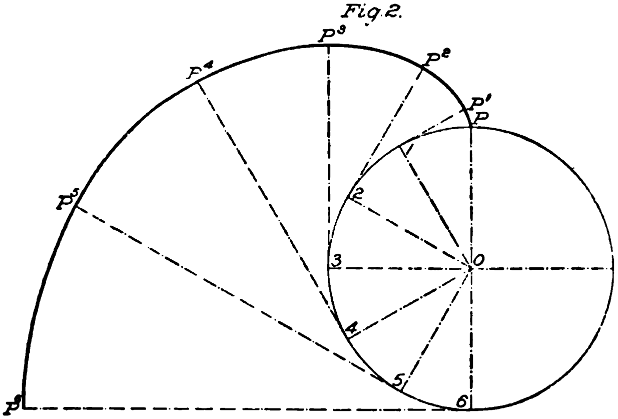
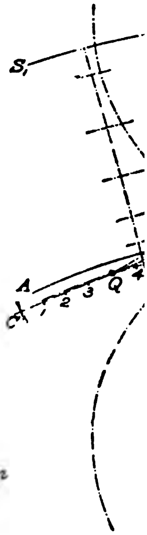
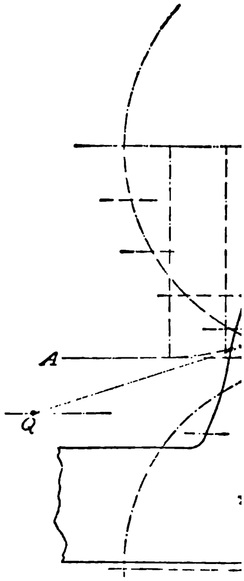


Fig.3.





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;EAR TEETH.

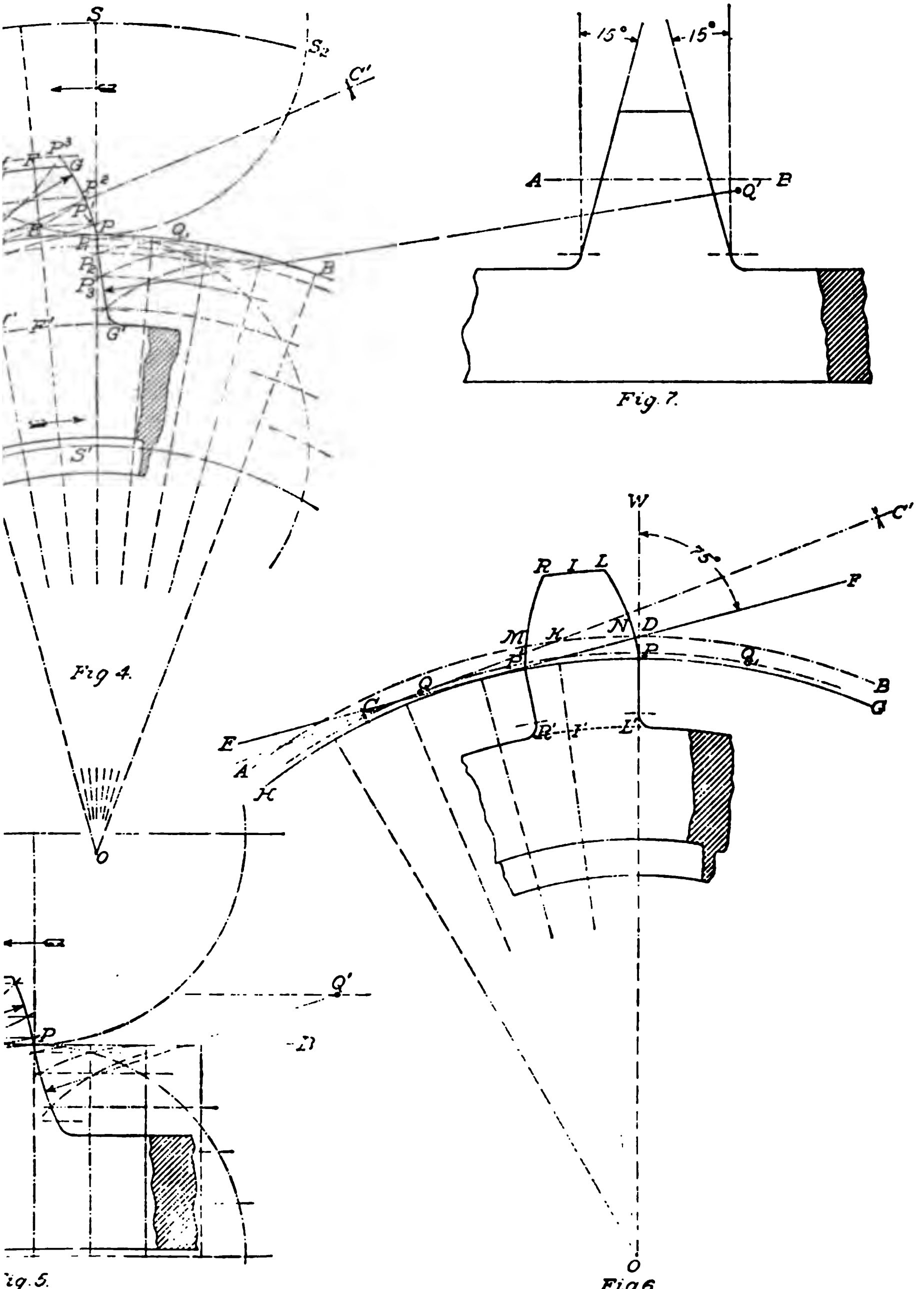

Fig 4.

Fig. 5.

Fig 6.

Fig. 7.

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the point *P* passes through all the points from *A* to *B*, namely, *A*, P^1, P^2, P^3, etc., when started at *A* and revolved towards the right to *B*.

82. If the generating circle is rolled, without sliding, on the outside of the circumference of an arc of a circle supposed to be at rest, instead of being rolled on a straight line, the curve described by a point *P* of the generating circle will be an **epicycloid.**

The manner of drawing such a curve is shown in Fig. 2. *A B* is the arc upon which the generating circle is rolled, its center being at *S* and its radius being 3½ inches. The diameter of the generating circle is in this case the same as in Fig. 1, or 1¾ inches. Make the lengths of the arcs *6 A* and *6 B* equal to half the length of the circumference of the generating circle, by first calculating the length of half the circumference of the generating circle and drawing a straight line tangent to the arc *A 6 B* at *6*, making it equal in length to half the circumference of the generating circle. Then make the arc *6 A* equal to this line by means of the approximate method given in *Geometrical Drawing.* Divide the arc *A 6 B* and also the generating circle into the same number of equal parts, in this case 12, as *A 1*, *1–2*, *2–3*, etc. and *P 1*, *1–2*, *2–3*, etc., and draw radii from the center *S* to the points of division on the arc *A 6 B*. During the revolution of the generating circle, the center *O* will describe an arc *m O n* concentric with the arc *A 6 B* and having the same number of degrees in it as *A 6 B*. Produce the radii just drawn to the arc of center positions *m O n*, intersecting this arc in the points *m*, *1′*, *2′*, *3′*, *4′*, etc. Through the points of equal divisions, *1*, *2*, *3*, etc., of the generating circle pass concentric arcs having the center *S*, as *C D*, *E F*, *G H*, *I J*, and *K L*. With the points *1′*, *2′*, *3′*, *4′*, etc. as centers and radii equal to the radius of the generating circle describe arcs cutting the arcs *K L*, *I J*, *G H*, etc. in the points P^1, P^2, P^3, etc., which are points on the epicycloid.

83. When the generating circle rolls on the inside of the arc, the curve described by a point on the circumference is called a **hypocycloid.** The method of drawing it is similar in all respects to that just given for the epicycloid. The student should be able to construct it from the drawing without further explanation. The diameter of the generating circle is 1¾ inches, as before.

84. Suppose that a string is wound on a cylinder and that the end of the string is at the point *P* in Fig. 3. If this string is unwound from the cylinder, keeping it constantly tight, the end *P* will describe a curve known as the **involute of the circle,** or, more simply, the **involute.** To construct it geometrically, let *O* be the center of the given circle representing the cylinder, which, in Fig. 3, is 2½ inches in diameter, and *P* the free end of the string when wound on the cylinder. Divide one-half of the given circle representing the cylinder into any number of equal parts, in this case 6, as *P 1*, *1–2*, *2–3*, etc., and through each of these points draw tangents to the circle, as P^1, P^2, P^3, etc. To draw these tangents, first draw the radii *O 1*, *O 2*, *O 3*, etc. and then draw the tangents $1 P^1$, $2 P^2$, $3 P^3$, etc. at right angles to them. By means of the approximate method given in *Geometrical Drawing*, find the length of the arc *1 P* and make the length of the tangent $1 P^1$ equal to this length; of the tangent $2 P^2$ equal to twice this length; of the tangent $3 P^3$ equal to three times this length, and so on. The curve drawn through the points P^1, P^2, P^3, P^4, etc. will be the required involute. The use of these curves will now be explained.

85. On the plate entitled Spur Gear-Wheels, Fig. 1, is shown one-half of two **spur gear-wheels** in mesh. The two dotted circles tangent to each other at *P* are concentric to the centers of the gear-wheels and are called the **pitch circles.** The diameter of any gear-wheel is always understood to be the diameter of its pitch circle unless it is specified as **diameter at root** or **diameter over all.** The length of that part of the pitch circle between the centers of any two consecutive teeth is called the **circular pitch,** or simply the **pitch.** Thus, in the last-mentioned figure, the length of the arc $a\ b$ is equal to the pitch of either gear-wheel. When the gear-wheels are cut in a gear cutter, the width of the tooth $c\ d$ on the pitch line is equal to the space $d f$; that is, the arc $c\ d$ is equal to the arc $d f$, and each is equal to half the pitch. When the gear-wheels are cast, that is, when they are not cut in a gear cutter, clearance is given between the back of one tooth and the front of the tooth following, to allow for inequalities in casting. This clearance, or **backlash**, as it is usually termed, is generally made equal to 4% of the pitch. This is done by making the thickness of the teeth $c\ d$ equal to .48 of the pitch.

The part $C\ C_1$ of the tooth that lies beyond the pitch circle is called the **addendum**, and the part $C\ C_2$ that lies below it is called the **root.** The **face** of the tooth is the part $C\ C_1\ C'\ C$, Fig. 2, of the tooth above the pitch circle, extending the whole width of the tooth. The **flank** is the part $C\ C_2 C''\ C$, Fig. 2, of the tooth below the pitch circle, extending the whole width of the tooth. The terms addendum and root mean distances only, while face and flank mean surfaces.

The usual practice is to make the addendum equal to $.3\ P$, and the root equal to $.4\ P$. $P =$ the circular pitch. The distance $C_1\ C_2$ is called the whole depth of the tooth. The method of describing the curves of teeth shown on the plate entitled Profiles of Gear-Teeth, Fig. 4, is a convenient way of drawing the **cycloidal, or double-curved teeth.** Cycloidal teeth are constructed by making the outline of the face a part of an epicycloid and the flanks a part of the hypocycloid, hence the name double-curved teeth.

86. In Fig. 4 of the plate entitled Profiles of Gear-Teeth, let $A\ B$ be part of a pitch circle struck with a radius of, say, $5\frac{1}{2}$ inches. For convenience in drawing the tooth, let the pitch be 2 inches. With O as a center, which is the center of the gear-wheel, and a radius equal to $5\frac{1}{2}$ inches describe the arc $A\ B$, part of the pitch circle. Through O draw a straight line $O\ S$, cutting $A\ B$ in P. Take the radius of the generating circles $S\ P$ and $S'\ P$ equal to $1\frac{3}{8}$ inches for this case and describe arcs having centers at S and S' on the line $O S$. With O as center and $O S$ as radius describe the arc $S_2 S_1$. In connection with the gear-wheel teeth, the generating circles are frequently called **describing circles.** Roll the outer describing circle upon $A B$ in such a manner that the center S will move in the direction of the arrow along the arc $S_2 S_1$. By means of the method given in Fig. 2, find the points P^1, P^2, P^3, etc. on the epicycloid described by the point P. Trace a faint curve through the points just found and measure off on the pitch circle the thickness of the tooth.

$$PD = .48p = .48 \times 2'' = .96''.$$

Make $E F =$ the addendum $= .3 \times p = .3 \times 2'' = .6''$.

With O as a center and $O F$ as a radius describe an arc cutting the epicycloid in G. Now roll the inner describing circle on $A B$, so that its center S' moves in the direction of the arrow, and find the points P_1, P_2, P_3, etc. of the hypocycloid described by the point P, through which trace a faint curve. Make EF' equal the flank of the tooth $= .4p = .4 \times 2'' = .8''$, and with O as a center and OF' as a radius describe an arc cutting the hypocycloid in G'. PG' is the outline of the flank of the tooth and $P G$ that of the face. Since it would be a tedious operation to draw all the tooth curves in this manner, it is usual to approximate the curves by means of circular arcs; that is, to find by trial a center Q and a radius $Q P$ such that an arc described from this center and with this radius will pass through the points on the curve $G P$ and coincide with that curve as closely as possible; also, to do the same with regard to the curve $P G'$, using the center Q' and the radius $Q' P$. To find the center Q or Q' of these circular arcs proceed as follows: With P and G as centers and any radius describe arcs intersecting in C and C'. Draw a straight line through C and C'; the center Q must line on $C C'$ to the left of $G P$. Try different points *1*, *2*, *3*, *4*, etc. on this line as centers and $1 G$, $2 G$, etc. as radii, and see if one of the arcs struck with either one of these centers and radii will coincide with the epicycloidal curve $G P$. Make this circular arc fit the curve for a short distance beyond G—as far as P^3, for example; this will insure the arc being more nearly correct. This should be done in every case when finding an approximate radius of this kind. Continue in this manner until the point Q is found such that an arc struck with Q as a center and QG or QP as a radius will coincide as closely as possible with GP. If a circle were drawn with O as a center and OQ as a radius, the centers of all the circular arcs of the faces of the teeth would lie in this circle, and the radii of these arcs would be equal in length to QP. Hence, to find the center Q_1 of the arc DH forming the back of the tooth, take D as a center and QP as a radius and describe a short arc cutting, in Q_1, the circle passing through Q. Then, with Q_1 as a center and the same radius describe the arc DH. In a similar manner find the center Q' and describe PG', also DH'. Instead of letting the flank form a sharp corner at the bottom of the tooth, as shown dotted at G', it is usual to put a small fillet there, as shown by the full line. This makes the tooth stronger and less liable to break or to crack in casting. The entire tooth outline or curve GPG' or HDH' is called the **profile** of the tooth.

87. A **rack** is a part of a gear-wheel whose pitch circle is a *straight line;* the tops of the teeth all lie in the same plane.* A portion of a rack and one tooth are shown in Fig. 5. Take the pitch the same as before, then the addendum and root are also the same, that is, .6 of an inch and .8 of an inch. Take the radius of the describing circles 1⅞ inches, as before. It is evident that the tooth profile will be formed of parts of cycloids formed by rolling the describing (generating) circle upon the pitch line AB. Draw a small part of the cycloidal curves, as shown in the figure, by the method given in Fig. 1; lay off the addendum and root and find the approximate radius in the same manner as in the last figure. The centers of the curves for the faces and flanks of all the teeth of the rack will evidently lie on the straight lines passing through Q and Q', respectively, and parallel to the pitch line AB.

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* As the radius of a circle is increased indefinitely, any arc of the circle approaches more and more to a straight line; and when the radius becomes infinite, the arc becomes a straight line.

88. In Fig. 6 is shown the manner of drawing the **involute, or single-curve tooth.** The profile in this case is formed of a portion of an involute curve and a portion of the radius of the pitch circle. The circle from which the involute is constructed is called, in this case, the **base circle.** To find it draw the pitch circle, of which the arc AB is a part, with a radius equal to 5½ inches and having its center at O. Draw any radius OW cutting the arc AB in D. Through D draw the straight line EF, making an angle of 75° with OW. With O as a center and a radius to be found by trial, draw a circle tangent to EF. This circle, of which the arc HG is a part, is the base circle, and cuts OW in P. Upon this circle construct, in exactly the same manner as was shown in Fig. 3, a portion of an involute curve passing through P. Lay off the addendum $IK = .6$ inch, and with O as a center and OI as a radius describe an arc to form the top of the tooth, intersecting the involute in L. That part of the flank below the base circle is straight and is a part of the radius drawn to the point P. KI' is the root. The tooth has a fillet at L' and R', as in cycloidal teeth. A circular arc is passed through the points L and P, coinciding as nearly as possible with the involute curve LP. Its center Q is found in the same manner as in Fig. 4. For involute teeth it is only necessary to find the one center Q; the centers for all the remaining teeth lie on a circle having O as a center and passing through Q. To draw the other side of the tooth, lay off on the pitch circle $MN = .96$ inch, as before. With M as a center and $QN = QP$ as a radius draw an arc cutting, at Q_1, the circle passing through Q; with Q_1 as a center and the same radius describe the part $P'R$ of the tooth profile above the base circle. The part $P'R$ below the base circle is a part of the radius OP'.

89. In drawing any of the curves previously described, the greater the number of parts into which the describing or base circles are divided the greater will be the accuracy obtained. The profile of the rack tooth used for involute gears is a straight line making an angle of 15° with a line drawn perpendicular to the pitch line. Its construction is shown in Fig. 7.

DEFINITIONS AND CALCULATIONS

90. When a revolving shaft transmits motion to another shaft parallel to it by means of gear- or tooth-wheels in such a manner that two corresponding points, one on each gear-wheel, always lie in the same plane, the two gears are called **spur gear-wheels.** When the shafts are not parallel, but their axes intersect in a point, as O in the plate entitled Bevel Gears, they are called **bevel gear-wheels.** If two bevel gear-wheels that work together have pitch diameters of the same size, they are called **miter gear-wheels.** From what has preceded, it is evident that *the circular pitch multiplied by the number of teeth equals the circumference of the pitch circle.*

Let p = circular pitch of gear-wheel;
n = number of teeth;
d = pitch diameter;
$\pi = 3.1416$ (π is pronounced **pi**).

Then,
$$d = \frac{pn}{\pi}, \qquad (1.)$$

or, *the diameter of the pitch circle equals the circular pitch multiplied by the number of teeth divided by 3.1416.*

$$p = \frac{d\pi}{n}, \qquad (2.)$$

or, *the circular pitch equals the pitch diameter multiplied by 3.1416 divided by the number of teeth.*

$$n = \frac{d\pi}{p}, \qquad (3.)$$

or, *the number of teeth equals the pitch diameter multiplied by 3.1416 divided by the circular pitch.*

When constructing cycloidal teeth for gear-wheels, the diameters of the describing circles are usually made equal to one-half the diameter of the pitch circle of a gear-wheel having 12 teeth of the same pitch as those of the gear-wheel about to be made.

Let d' be the diameter of the describing circle; then,

$$d' = \frac{12p}{\pi} \times \tfrac{1}{2}, \text{ or } d' = \frac{6p}{\pi}. \qquad (4.)$$

Addendum $= .3p$; root $= .4p$; thickness of teeth for cast gears is $.48p$, and for cut gears $\frac{1}{2}p$.

DRAWING PLATE, TITLE: SPUR GEAR-WHEELS

91. This plate shows the halves of two cast **gear-wheels** having cycloidal teeth, which work together, a cross-section of each gear being also given. The drawing is full size, the wheels not being shown entire for want of room; to have done so it would have been necessary to make the drawing to a reduced scale. The pitch is 1 inch, the number of teeth in the large gear is 36, and in the small one 18. The pitch diameter of the large wheel is found by formula **1** to be

$$d = \frac{1 \times 36}{3.1416} = 11.46 \text{ inches, nearly.}$$

The pitch diameter of the small gear =

$$\frac{1 \times 18}{3.1416} = 5.73 \text{ inches, nearly.}$$

The diameter of the describing circle is found by formula **4** to be

$$d' = \frac{6 \times 1}{3.1416} = 1.91 \text{ inches.}$$

For all practical purposes, the diameter of the describing circle may be taken to the nearest 16th of an inch. For circular pitches under $\frac{1}{2}$ inch, approximate the diameter of

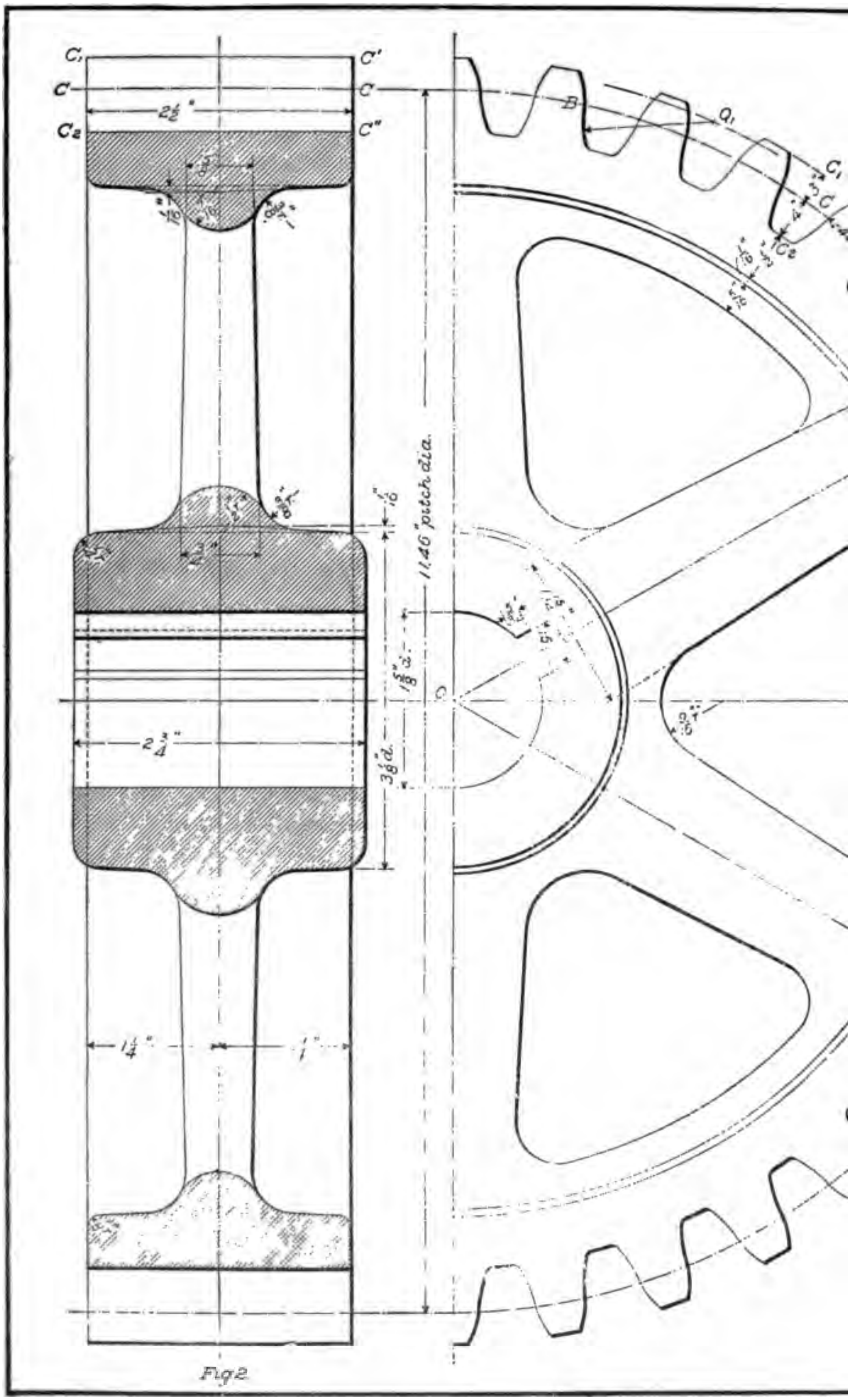

Fig 2.

JUNE 25, 1893.

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SPUR GEAR WHEELS.

Cast Iron. *Full Size.*

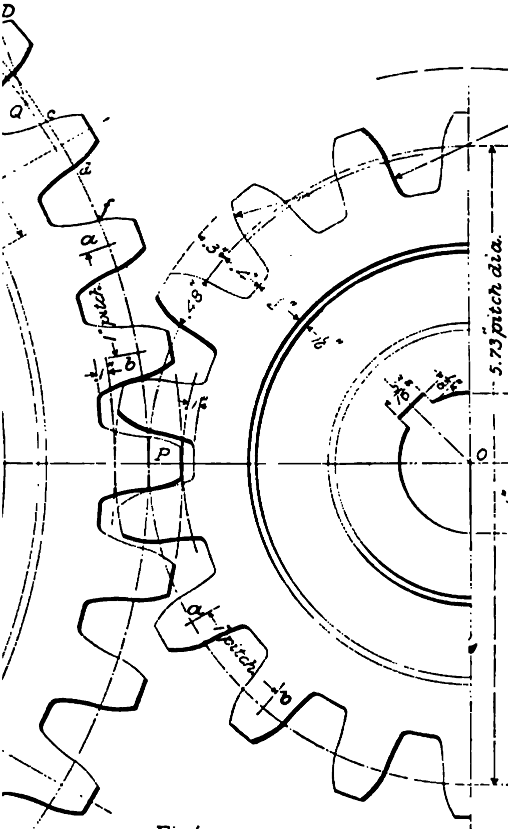

Fig. 1.

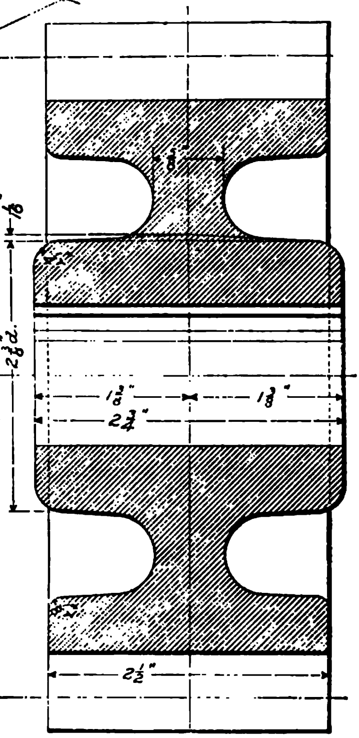

Fig. 3.

ediately following the title page. JOHN SMITH, CLASS Nº 4529.

the describing circle to the nearest 32d of an inch. To find the nearest 16th or 32d, multiply the decimal part of the diameter by 16 or 32 and take the nearest whole number of the product as the number of 16ths or 32ds that the decimal represents. Thus, in the above, the decimal part of the diameter is .91 inch; $.91 \times 16 = 14.56$. The nearest whole number to 14.56 is 15; hence, the diameter of the describing circle is $1\frac{15}{16}$ inches. If the diameter had been required to the nearest 32d, $.91 \times 32 = 29.12$; 29 is the nearest whole number; hence, the diameter would be $1\frac{29}{32}$ inches. In this case, take the diameter as $1\frac{15}{16}$ inches, approximating to the nearest 16th for all circular pitches above $\frac{1}{2}$ of an inch. The addendum will be $.3 \times 1$ inch $= .3$ inch; the root $= .4 \times 1$ inch $= .4$ inch; and the thickness of the tooth on the pitch circle $= .48 \times 1$ inch $= .48$ inch.

Draw the line of centers $O\,O'$ between the two axes. With O as a center describe a semicircle having a diameter of 11.46 inches, cutting $O\,O'$ in P. With O' as a center describe a semicircle having a diameter of 5.73 inches which shall be tangent to the first circle; this semicircle also cuts $O\,O'$ in P. These are the pitch circles. Divide the larger pitch circle into $\frac{36}{2}$, or 18 equal parts by using the protractor. This is accomplished by laying the protractor on the drawing in such a manner that the center of the protractor coincides with the center O of the gear-wheel and then laying off on the drawing 18 divisions, each equal to 10°. The reason for this will be clear when it is remembered that there are 360° in a circle, or $\frac{360^\circ}{2} = 180^\circ$ in a semicircle, and as there are 18 teeth in the semicircle, $180^\circ \div 18 = 10^\circ$, which is the angle between two lines drawn from the centers of any two consecutive teeth to the center O. In a like manner, any circle may be divided into parts by using the protractor. Make $C\,C_1 = .3$ inch = addendum, and $C\,C_2 = .4$ inch = root. With O as a center and $O\,C_1$ and $O\,C_2$ as radii, describe light circles, called **addendum** and **root circles,** to represent the tops and bottoms of the teeth.

Consider the points of division just laid off on the pitch circle as the centers of the teeth, and lay off on each side one-half of the thickness *cd* of the tooth, or $.48 \times \frac{1}{2} = .24$ inch. Upon another sheet of paper strike a short arc of the pitch circle and construct the profile of the tooth as described in the plate entitled Profiles of Gear-Teeth, using describing circles $1\frac{5}{16}$ inches in diameter. Having found the centers Q and Q_1 of the circular arcs used for the profiles of the teeth, draw circular arcs through these centers, as previously described; then, with O (see plate entitled Spur Gear-Wheels) as a center and the same radii describe circles; these circles will contain the centers of the circular arcs which form the teeth profiles. With the point A as a center and a radius equal to the radius of the face of the tooth (found on the other sheet of paper before mentioned), describe an arc intersecting the circle of face centers at Q. With Q as a center and the same radius, describe the arc $A D$ for the face of the tooth, the point D being the point of intersection of $A D$ with the addendum circle. In the same way draw the remaining faces of the teeth. To draw the flanks take a point representing the intersection of a tooth profile with the pitch circle (the point B, for example) as a center and a radius equal to the radius of the flank found on the other paper on which the tooth profile was drawn, and describe an arc intersecting the circle of the flank centers in Q_1. With Q_1 as a center and the same radius, describe the flank curve, stopping at the root circle. Draw the flanks of the remaining teeth in the same way and then put in the fillets. The remaining part of Fig. 1 can be easily drawn from the dimensions given.

Fig. 2 is a conventional method of drawing cross-sections of gears. The hubs and rims are sectioned, but the teeth and arms are not. This is similar to the wheel shown on the plate entitled Details. This method of sectioning makes the views clearer and saves the time spent in sectioning.

In Fig. 3 the entire gear is sectioned, except the teeth. The student should now be able to finish the plate without further instructions.

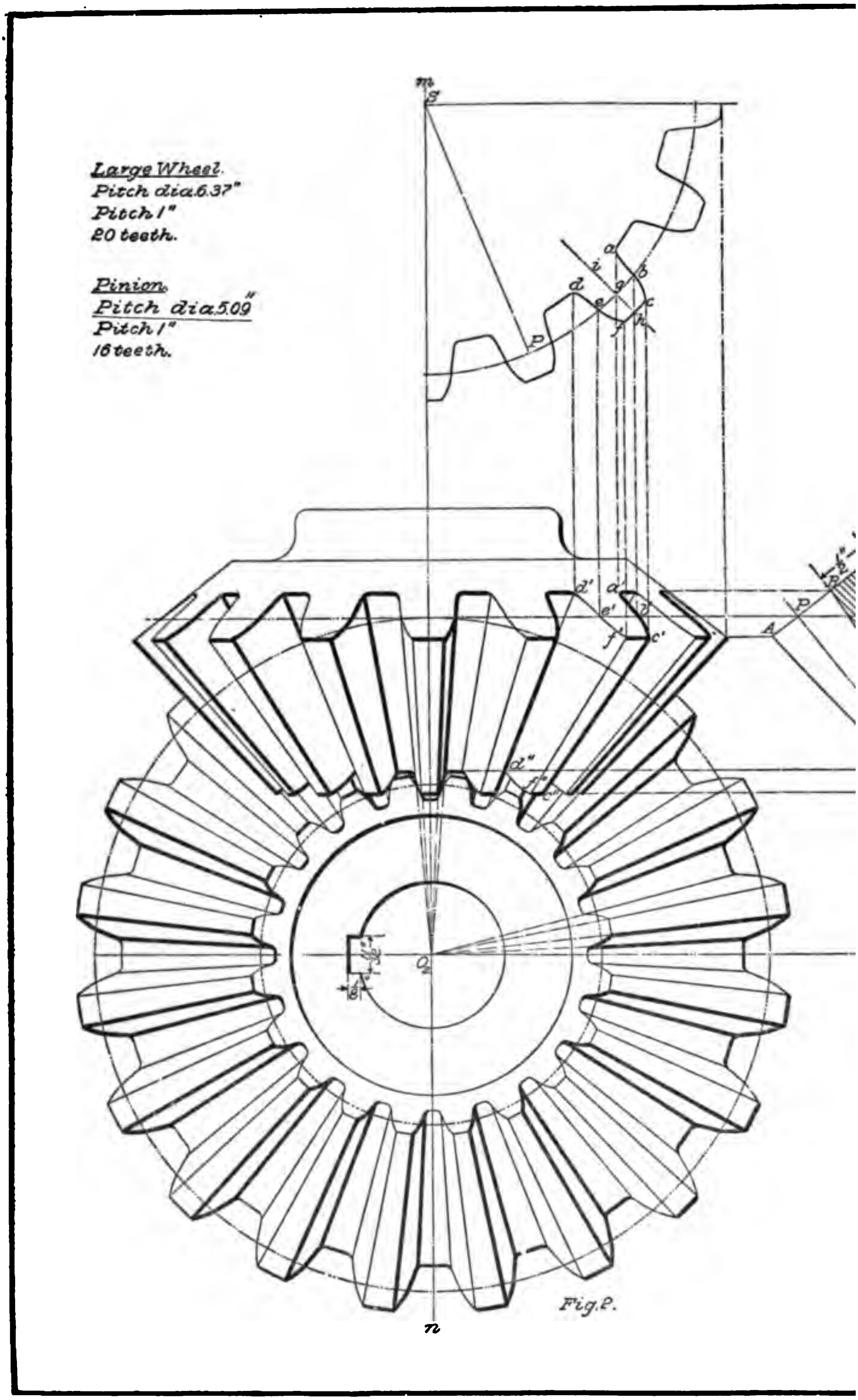

Fig. 2.

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BEVEL GEARS.

Cast Iron. *Full Size.*

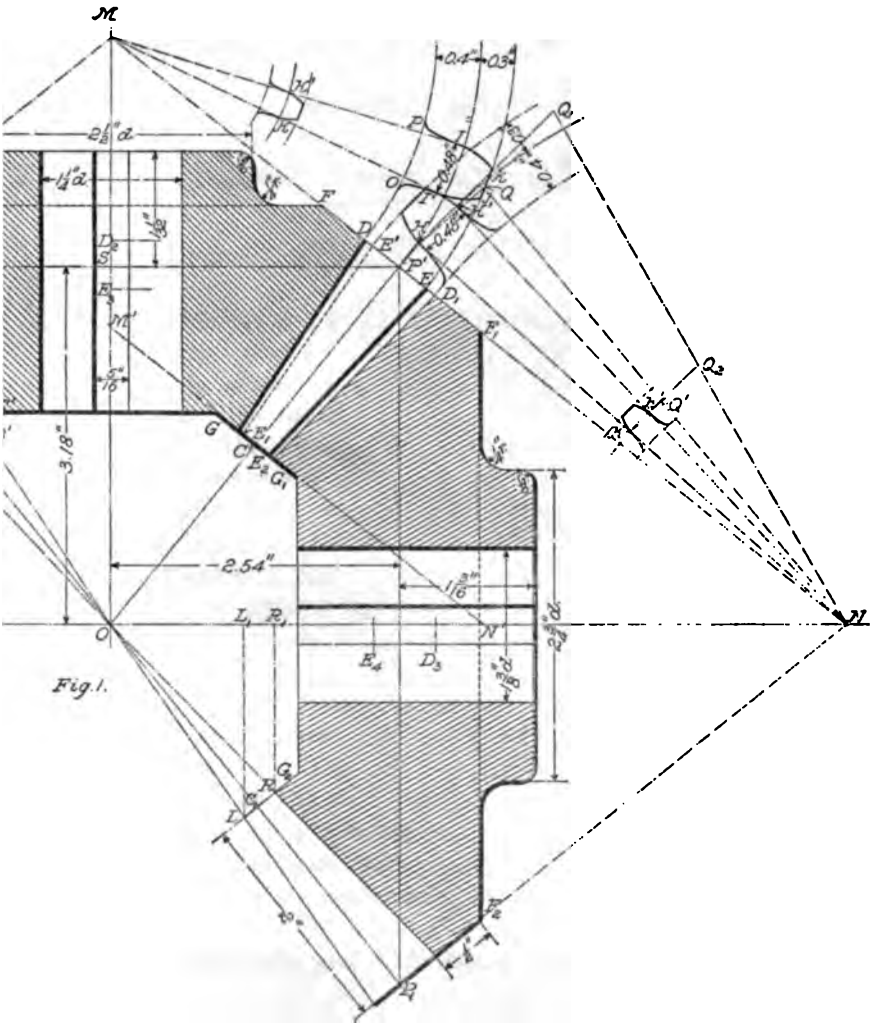

Fig. 1.

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JOHN SMITH, CLASS No 4529.

DRAWING PLATE, TITLE: BEVEL GEARS

92. To draw in section and projection two cast **bevel gears** whose axes intersect at right angles: The number of teeth in the large gear is 20, in the pinion 16. The circular pitch is 1 inch; the teeth are to be of the cycloidal form, having a face 2 inches wide. In any kind of gearing, whether spur, bevel, or spiral, the smaller wheel is called the **pinion.**

Calculate the pitch diameters, addenda, roots, and describing circles by the same rules that were given for spur gears.

$$\text{Diameter of pinion} = \frac{16 \times 1 \text{ inch}}{3.1416} = 5.09''.$$

$$\text{Diameter of the large gear} = \frac{20 \times 1 \text{ inch}}{3.1416} = 6.37''.$$

$$\text{Diameter of describing circle} = \frac{6 \times 1 \text{ inch}}{3.1416} = 1.91''$$

Take this as $1\frac{15}{16}$ inches, as in the last plate.

Addendum = .3 inch; root = .4 inch. The sectional view must be drawn first. Draw PP' and through some point P' on this line draw $P'P_1$ perpendicular to it. Lay off PP' equal to the diameter of the pinion = 5.09 inches; also $P'P_1$ equal to the diameter of large gear = 6.37 inches. Bisect PP' and $P'P_1$, and draw OM and ON perpendicular to those lines at the point of bisection; they intersect in O. OM and ON are the axes of the two gears and intersect at right angles as required. Draw POP_1 and $P'O$. Through P draw APM perpendicular to OP. Through P' draw $MP'N$ perpendicular to OP' and through P_1 draw P_1N perpendicular to OP_1. PM and $P'M$ intersect at M on the line OM; $P'N$ and P_1N intersect at N, on the line ON. Lay off $P'C'$, PC, and P_1C_1, each equal to 2 inches, or the width of the face of the teeth; these lines are called the **pitch lines,** and the width of the face of the teeth is always measured on these lines. Lay off PA equal to .3 inch = the addendum, and PB equal to .4 inch = the root. Lay off $P'E$ and $P'D$ for the addendum and root of the other side, and $P'E'$ and $P'D_1$ for the addendum and root of the large gear. All these addenda and roots are each equal to .3 inch and .4 inch, respectively. In bevel gears, all straight lines of the tooth profiles pass through the point of intersection O of the axes; hence, draw AO, and AA' will be the projection of the top of the tooth. Draw BO, and BB' will represent the bottom of the tooth, the line $A'CB'$ being perpendicular to OP. Make BF', DF, D_1F_1, etc. each equal to $\frac{1}{4}$ inch, according to dimensions. Join F', F, F_1, and F_2 with O, intersecting the perpendiculars through C, C', and C_1 (namely, the lines $A'CB'$, etc. produced) at G', G, G_1, and G_2. $G'G$ and G_1G_2 will represent the bottom of the gears. The rest of the sectional part can be drawn from the dimensions.

93. To show the shape of the teeth, proceed as follows: For the large gear, take N as a center, NP' as a radius, and describe an arc. Choose a point H and lay off $HH' = .48$ times the pitch $= .48$ inch, or the width of the tooth. With NE' and ND_1 as radii, describe the addendum and root circles. Roll the describing circles upon the arc whose radius is NP' and construct the tooth profile in exactly the same manner as in Fig. 4 of the plate entitled Profiles of Gear-Teeth, QH and Q_1H' being the radii of the faces and flanks. To show the shape of the same tooth at C', draw $C'N'$ perpendicular to OP', or, what is the same thing, parallel to NP'. With $N'C'$ as a radius and N as a center, describe an arc. Draw NH and NH', and the distance between the points of intersection on the arc just drawn, measured on that arc, will be the pitch of the gear at the bottom of the tooth. With the same center and $N'E_1$ and $N'E_2$ as radii, describe arcs representing the addendum and root circles. Draw NQ and NQ_1, also QH and Q_1H'. Through K draw KQ' parallel to HQ, and through K' draw $K'Q_2$ parallel to $H'Q_1$; the points of intersection Q' and Q_2 of these lines with NQ and NQ_1 are the centers for the face and flank of the tooth at K and K'. Circles passing through these points concentric with N contain the centers of all the circular arcs forming the tooth profiles that may be laid off upon the arc whose radius is NK. The whole process is called **developing the teeth of bevel gears.**

In the same manner construct the tooth curves for the pinion, using the same describing circles, $1\frac{15}{16}$ inches in diameter, and MP', $M'C'$ as radii, instead of NP' and $N'C'$.

94. To construct the other view, draw first the projection of the pinion. Draw the center line mn. Produce the lines FF', DB, $P'P$, and EA across the drawing, as shown. Choose a point S on mn as a center and draw a quadrant with a radius equal to the radius of the pinion, as SP. Project the points D and E upon MO in D_2 and E_2. With S as a center and the distances E_2E and D_2D as radii, describe quadrants to represent the tops and bottoms of the teeth, that is, the projection of the addendum and root circles of the pinion in Fig. 2. Since the whole pinion contains 16 teeth, the quadrant will contain 4 teeth; hence, divide the quadrant into 4 equal parts on the pitch circle to represent the centers of the teeth. Lay off on each side of the points of division distances ge and gb, each equal to one-half the thickness of the tooth. On each side of the points of division on the addendum circle lay off hf and hc, each equal to one-half the thickness of the top of the tooth JK, Fig. 1, measured on the addendum circle. On each side of the points of division on the root circle lay off id and ia, each equal to one-half the thickness of the tooth at the root, as OP, Fig. 1, measured on the root circle. Having now three points on each side of all the teeth to the right of the center line mn, project them upon the lines EA, $P'P$, and DB, produced as shown. For example, project f and c upon EA in f' and c'; e and b upon $P'P$ in e' and b'; d and a upon DB in d' and a'. Draw a curve through these points, either by using an irregular curve or by circular arcs. This remark also applies to the other curves shown in the quadrant.

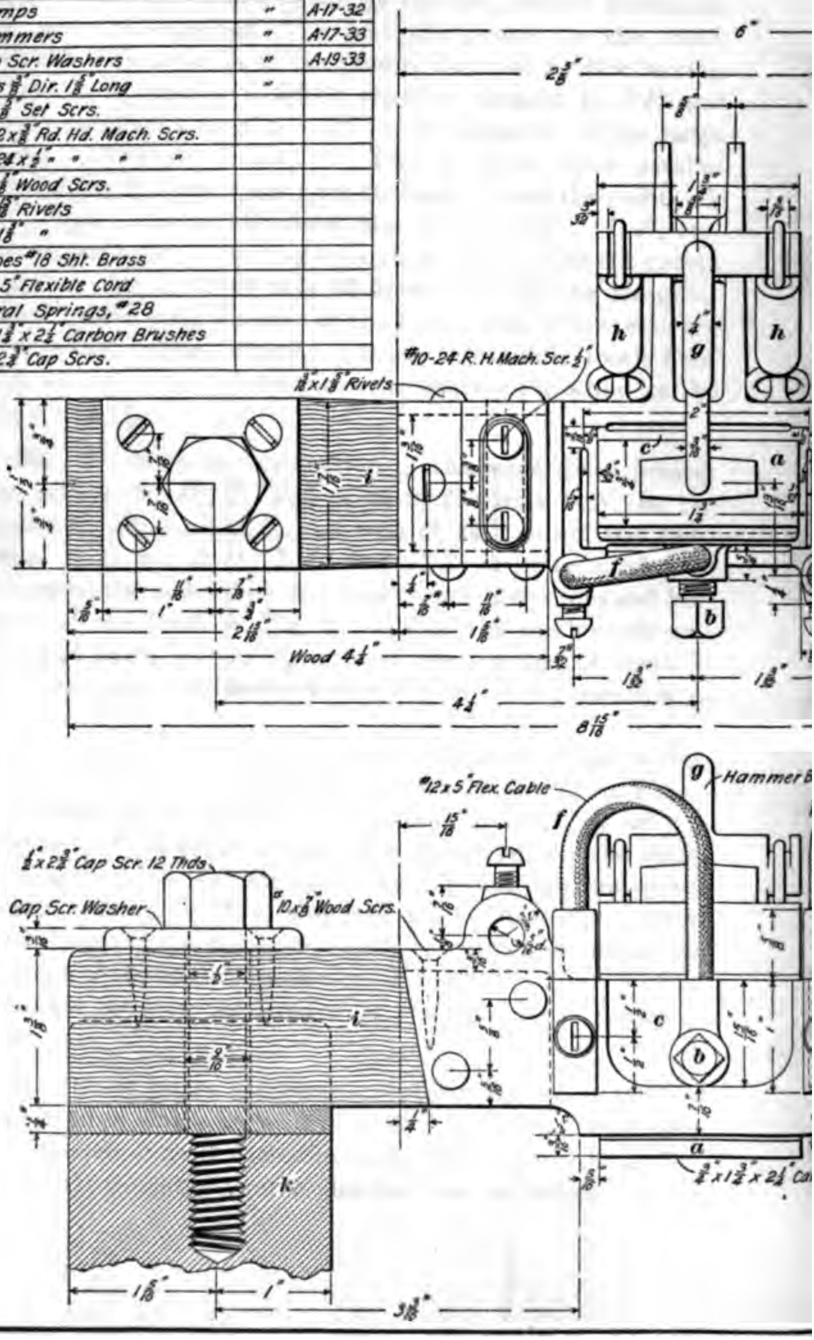
95. The tooth curves in Fig. 1 must be drawn as accurately as possible, but those shown in Fig. 2, being oblique projections, are drawn to satisfy the eye, and no particular accuracy is required. To find the points on the tooth curves at the bottom of the pinion, describe a circle having a center O_2 upon $m\,n$, which shall be tangent to PP' and have a diameter equal to 6.37" = the diameter of the large gear. Through B' and A', Fig. 1, draw lines parallel to $O\,O_2$; also draw other lines through O_2 and the points d', f', c', etc., cutting the lines first drawn in d'', f'', c'', etc. Two points are considered enough in this case, as the curves are very short. They may be drawn in with the irregular curve in the same manner as the tops. The other teeth are drawn in a similar manner. Draw the middle tooth first. The left-hand half of the pinion is exactly the same as the right-hand half.

96. To draw the projection of the large gear, project the points E', D_1, L, and R upon the axis $O\,N$, in the points E_4, D_2, L_1, and R_1, and with O_2 as a center and radii equal to E_4E', D_2D_1, L_1L, and R_1R, describe circles to represent the addendum and root circles of the tops and bottoms of the teeth in Fig. 2. Divide the pitch circle into 20 equal parts, to correspond with the number of teeth in the large gear, beginning with the point of intersection of the pitch circle with the center line $m\,n$. Lay off on each side of these pitch-circle divisions, distances equal to one-half the thickness of the teeth = one-half of $H\,H'$ in Fig. 1. By exactly the same method that was used to lay off the thickness of the teeth at the top and bottom on the quadrant, lay off the thickness of the top and bottom of the teeth on the addendum and root circles in Fig. 2. Draw the bottoms of the teeth in exactly the same manner as the bottoms of the pinion teeth were drawn.

All the teeth of the large gear are alike in the projected view.

97. Bevel gears are always measured according to their largest pitch diameter, as PP' and $P'\,P_1$. If a bevel gear were spoken of as 12 inches in diameter, it would be understood that the largest pitch diameter was 12 inches.

120 V.	240 V.	SPECIFICATION D 247	MATERIAL	PATTERN NUMBER
	1	Brush Holder R. Hand	Brass	A-17-30
	1	" " L. "	"	A-17-31
	4	Clamps	"	A-17-32
	4	Hammers	"	A-17-33
	2	Cap Scr. Washers	"	A-19-33
	4	Pins 3/8" Dir. 1 5/8" Long	"	
	8	5/16" x 3/8" Set Scrs.		
	4	#8-32 x 3/8" Rd. Hd. Mach. Scrs.		
	4	#10-24 x 1/2" " " "		
	10	#10 x 5/8" Wood Scrs.		
	4	3/16" x 15/16" Rivets		
	4	3/16" x 1 5/8" "		
	4	Shoes #18 Sht. Brass		
	4	#12 x 5" Flexible Cord		
	6	Spiral Springs, #28		
	4	3/4" x 1 3/4" x 2 1/4" Carbon Brushes		
	2	1/2" x 2 3/4" Cap Scrs.		



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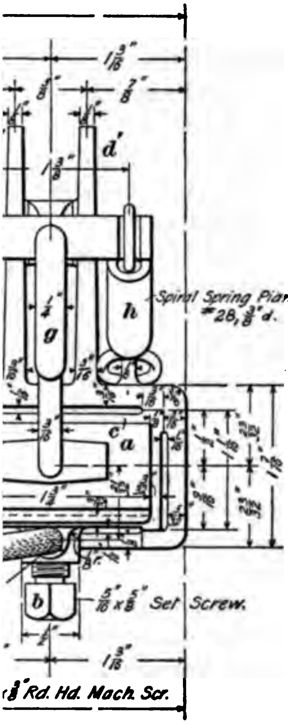
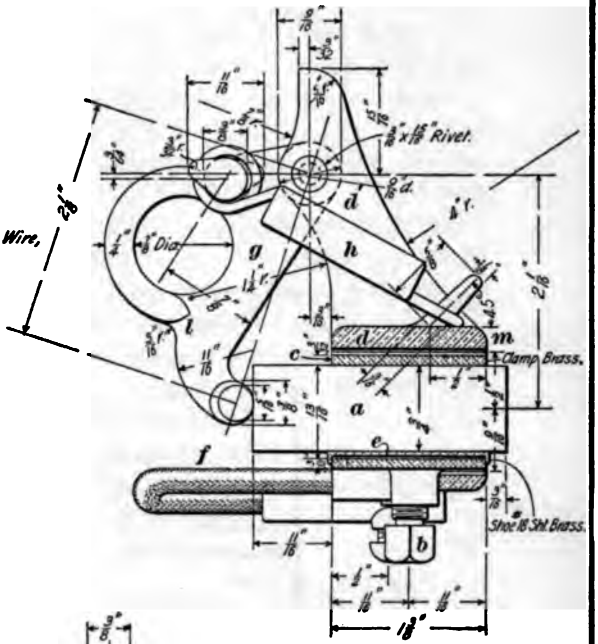
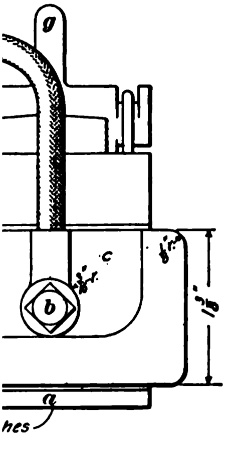
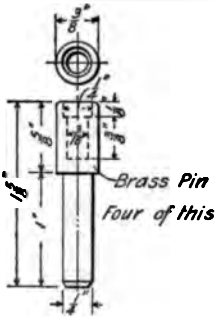








BRUSH HOLDER
FOR
15 H. P. MOTOR.

Two of this, One right handed, One left handed.
Scale Full Size.

THE SCRANTON ELECTRIC CO.
SCRANTON, PA.,
Drawn by R. B. W. Nov. 6-01.

ly following the title page.

JOHN SMITH, CLASS No. 4529.

DRAWING PLATE, TITLE: BRUSH HOLDER

98. This drawing plate is a complete working drawing of the left-hand brush holder for a 15-horsepower motor and is designed for the use of carbon brushes, there being four brushes, two in each holder. The carbon brushes *a*, *a* are clamped by means of the setscrews *b*, *b* to the clamps *c*, *c*, which are free to slide in rectangular holes in the body *d* of the brush holder. The setscrews *b*, *b* do not bear directly against the brushes, but against a brass shoe *e*. A thorough electric connection between the carbon brushes and the body of the holder is insured by flexible No. 12 cables *f*, *f*, which are composed of strands of copper wire covered with insulating material. The outside diameter of these cables is $\frac{7}{32}$ inch, about. The carbon brushes are held against the commutator by hammers *g*, *g* operated by springs *h*, *h*. The hammers are pivoted to brackets *d'* cast in one with the body, and the springs are so hung that when the hammers are rotated away from the brushes, the springs will come to the other side of the center around which the hammers turn and thus hold the hammers away from the brushes. The springs are hooked over lugs on the body at one end and over arms projecting from the hammers on their other end. In order to insulate the brush holder from the frame of the machine, it is fastened to a piece of hardwood *i* by two $\frac{3}{16}$-inch rivets and one No. 10 wood screw $\frac{7}{8}$ inch long. The piece of hardwood is fastened to the frame *k* of the machine by a $\frac{1}{2}$-inch capscrew as shown.

99. Begin drawing the plate by drawing the end view, locating the horizontal center line passing through the center of the rivet serving as a fulcrum for the hammer at a distance of $3\frac{7}{16}$ inches below the upper border line and locating the vertical center line $2\frac{1}{2}$ inches from the right-hand border line. The end view is to be drawn first because it is the only view in this particular instance in which everything can be drawn without having to project from another view. For several of the dimensions it is necessary to refer to the top view. The main part of the hammer is flat and has joined to it a handle having a circular cross-section. The flat and round part coming together cause the intersection curve shown at *l*. The distance that the flexible cord *f* projects from the left-hand face of the body is not given on the drawing, as this information would be useless on a working drawing. For the information of the student it is here given, being $1\frac{3}{4}$ inches. It is a general and a good rule with draftsmen not to give any dimensions on a drawing unless they serve a useful purpose; everything superfluous is to be left off. The shoe *e* is marked No. 18 sheet brass; the corresponding thickness is .04 inch, nearly.

It will be observed that the spiral spring *h* is not drawn the way it actually appears, but that it is drawn conventionally. This is merely done in order to save the draftsman's time, as the note "Spiral Spring Piano Wire, No. 28, $\frac{3}{8}''$ *d*," appearing in the top view supplies the necessary information to the mechanic.

100. The end view having been completed, the top view should be drawn next, locating the center of the $\frac{3}{8}$-inch capscrew fastening the hardwood strip *i* to the frame $3\frac{1}{16}$ inches from the left-hand border line. This view is drawn partially from the dimensions given and partially by projecting over from the end view. The springs *h* being at an inclination, they appear foreshortened in the top view, and their length in the top view must be determined by projecting over from the end view. Each spring is hooked over a horn cast on the body *d*, a cylindrical ring being formed on the ends of the springs for this purpose. Owing to this ring being at an inclination, it will show elliptical in the top view. The outside diameter of this ring is $\frac{9}{16}$ inch. The heads of the $\frac{3}{16}'' \times 1\frac{5}{8}''$ rivets have a radius of $\frac{5}{32}$ inch and are $\frac{1}{8}$ inch high. The head of the No. 10–24 round-headed machine screws is $\frac{21}{64}$ inch diameter and $\frac{5}{32}$ inch high; the diameter of a No. 10 machine screw is .189 inch, say $\frac{3}{16}$ inch. The head of the No. 8–32 round-headed machine screws is $\frac{5}{16}$ inch diameter and $\frac{9}{64}$ inch high; the diameter is .163 inch, say $\frac{11}{64}$ inch. Machine screws are made in accordance with the standard American screw gauge adopted by all manufacturers of screws; for this reason it is only necessary on a drawing to specify the gauge number of the screw, the number of threads per inch, and the length, which latter is always measured under the head, except in flat-headed screws, where the length is measured over all. Wood screws are measured by the same gauge as machine screws and bear the same number. The heads of the No. 10 wood screws are $\frac{21}{64}$ inch diameter. The diameter across the flats of the head of the $\frac{1}{2}$-inch capscrew is $\frac{13}{16}$ inch; the height of the head is $\frac{1}{2}$ inch. The $\frac{5}{16}$-inch setscrews have a head $\frac{5}{16}$ inch high and measuring $\frac{7}{16}$ inch across the corners.

101. To draw the side view, begin by locating the edge *m*, which corresponds to the surface *m* in the end view, at a distance of $1\frac{3}{4}$ inches above the lower border line. The view is to be drawn partially by projecting down from the top view and partially by transferring measurements from the end view, as explained in connection with the drawing plate, title: Turret-Lathe Tools. In order to have a line in the end view from which to measure, the line representing the surface *m* should be produced in lead pencil; in the side view the edge *m* will then represent an edge of the same plane defined in the end view by the lead-pencil line just drawn. The shading of the flexible cords is done freehand.

102. The center line of the brass pin is to be located $4\frac{1}{4}$ inches from the right-hand border line; the center of the top view is to be placed $5\frac{3}{16}$ inches above the lower border line and a distance of $\frac{3}{16}$ inch is to be left between the two views.

103. The first line of the title is a block letter $\frac{3}{16}$ inch high, the second line is $\frac{3}{32}$ inch high, and the third $\frac{3}{16}$ inch high. The fourth and fifth line is written freehand, the capitals being $\frac{3}{16}$ inch and the small letters $\frac{1}{8}$ inch high. The sixth line is composed of capitals $\frac{3}{16}$ inch high, except the first letter of each word, which is $\frac{1}{4}$ inch high. The seventh line is composed of capitals $\frac{1}{8}$ inch high and the eighth line is the same as the fourth and fifth. The student is advised to practice the block lettering shown in this title, but if he desires it he may substitute the freehand letter shown in the fourth line.

104. In the upper left-hand corner will be found a list of the different parts needed. It is the practice in many shops to have such a list on the drawing for convenience of reference. Draw the bottom line of the list $4\frac{1}{2}$ inches below the upper border line and draw the upper line $3\frac{3}{4}$ inches above the bottom line and divide the space between the two into 17 equal spaces. Draw the left-hand line of the list $\frac{1}{4}$ inch from the left-hand border line and the right-hand line $4\frac{11}{16}$ inches from the border line. To the right of the left-hand border line of the list draw vertical lines at a distance of $\frac{1}{4}$ inch, $\frac{1}{2}$ inch, $3\frac{1}{16}$ inches, and $3\frac{5}{8}$ inches, respectively. Then write in freehand, in the style of lettering shown, all the contents of the list.

The student must omit on his drawing the reference letters printed in bold-face italics.

READING A WORKING DRAWING

105. The following general method of procedure has, by experience, been shown to be conducive to the accurate and rapid reading of a drawing made in projection. *First*, if the drawing is dimensioned, ignore the existence of the dimension lines and dimensions entirely until after the general shape of the object is fixed on the mind. *Second*, by referring to the several views, form an idea of the shape of the main body of the object; that is, observe if its outline shows it to be a cube, a sphere, a cylinder, a cone, a pyramid, etc., or a combination of several of these elementary forms. The shape of the main body having been impressed on the mind, observe how it is modified by details, determining, by reference to the several views, whether they project from the main body or are recesses, or holes. *Finally*, by referring to the dimensions, form an idea of the relative sizes of the component parts. Pay due regard to all conventional representations that may have been used; for instance, do not become confused if the arm of a pulley, or a rib, which, truly speaking, should have been in section, is shown in full. If two half sections are placed on either side of a common center line, remember that each half must usually be viewed independently of the other and must be mentally completed.

106. When reading a drawing in which the views are correctly placed, it is often a great aid to project points or edges of some part the shape of which is doubtful over to another view by the aid of a straightedge, in order to find the location of the doubtful part in another view. When the views are not placed in their correct relative positions, this cannot be done. An example of a case of this kind is given in the plate Compound Rest, and in reading a drawing with the view thus placed, the reader is supposed to constantly imagine that the views are in their correct relative positions; with a little practice this will be found to be quite easy.

107. In a case of this kind, it is manifestly impossible to project points or lines from one view to the other by means of a straightedge, and a different method must be followed. Select some surface whose projection appears in both views, or a center line; now place a pair of dividers so that one point rests on the projection of the surface or center line selected, and open them until the other point reaches the point or line whose projection it is desired to find in the other view. Then place one point of the dividers on the line representing the selected surface in the second view, and move the dividers along this line until a line, or the projection of a line, is found to coincide with the other point of the dividers. Examples of this will appear later on.

In order to aid the student to read a drawing, we have selected the plate Compound Rest and will show in detail by what process of reasoning this drawing is read.

108. To find the shape of the different parts and also to discover, if possible, the relation between them, we must commence our investigation somewhere. Let us choose the bottom of the front view. Looking at this it is noticed that a partial section is shown, from which, by reason of the section lining running in opposite directions, we conclude that *A* and *B* are separate parts. At the right and left of the front view, the full lines *c*, *c* show that some part of *A* is higher than the bottom of *B*, but we do not know whether these lines denote the top surfaces of projecting parts between which *B* is fitted, or if *c* is the top surface of a raised strip of some kind that extends clear through the inside of *B*. In order to settle this question, we note whether the top surface is continued somewhere. Looking at the front view it is seen that the line *c* is dotted clear through *B*, which settles conclusively that the part whose top surface is shown by the line *c* is a raised strip extending clear through *B*; this fact immediately implies that *B* has a groove of some kind running through it longitudinally in order to admit the raised strip.

Referring now to the sectional view, which, as previously stated, is a view taken on the line *a b* of the front view, and everything to the right of this line being removed, we may choose the bottom line *d'* of the sectional view as a base from which to make measurements. From the fact that the section is taken on the line *a b*, we know that the line just chosen is the projection of the intersection *d* of the plane represented by *a b* with the bottom of *A*. Measuring from *d'* upwards to the highest line *c'* of *A* in the sectional view, and placing one point of the dividers on *d* in the front view, it will be seen that the other point coincides with the dotted line forming a continuation of *c c*; this shows that *c'* is the projection of *c*. In a similar manner, we determine that *c'* is the projection of *c*, and tracing the outlines of *A* in the sectional view, we notice that the raised strip on *A* has inclined sides. We also notice that *B* is cut out to suit the profile of *A*, except that on one side a steel part *L* is interposed between the inclined sides of *A* and *B*; it is also seen that a screw rests with its point against *L*.

Referring now to the front view, and knowing from inspection of the sectional view that the upper and lower surfaces of the steel part *L* are flush with *c'* and *e'*, or *c* and *e* in the front view, to determine the length of this part, notice if any dotted or full lines showing its length are shown anywhere at a right angle to *c* and *e*. None being found, the conclusion to be drawn is that either the steel part *L* is as long as *A* or has the same length as *B*. A person without any practical experience might conclude that the length is the same as that of *A*; but any one having engineering instinct or practical knowledge would immediately notice that, as the steel strip has setscrews which evidently serve to push it against the inclined side of *A*, it would be unnecessary to make the strip the length of *A*, and hence would immediately conclude that its length is the same as that of *B*. This latter conclusion is the one the draftsman desired to convey.

109. Looking at the sectional view of *A* again, we notice that a groove, open on top, is cut into *A*. To find its length we must find lines corresponding to it in the front view. Measuring from *d'* upwards to the bottom of the groove and transferring the measurement to the front view, we find that the dotted line *g g* represents the bottom and end of the groove, which at the left is also shown to be open at the bottom, since the dotted line *g* curves around and continues to the bottom of *A*. This is also indicated by the dotted lines *g'* that form an extension of the sides of the groove in the sectional view; measuring from *c'* downwards to the horizontal dotted line joining the ends of *g' g'*, and then passing along *c* in the front view, the point of the dividers will be found to coincide with the point where the dotted line *g* meets the bottom of *A*. From this the conclusion is drawn that the dotted horizontal line joining *g' g'* in the sectional view is the bottom edge of the opening.

110. Looking at the front view we notice a left-handed screw *C* that is placed within the slot just investigated. Knowing that, in a view in line with its axis, the outline of a screw will be a circle, and knowing that this circle will be found inside of the slot, the screw is readily found in the sectional view. Now, experience teaches us that when a screw is shown in place in a machine drawing, there must also be somewhere along its axis a threaded hole (or a nut) to receive it. Looking at the sectional view we see the outline of something (marked "*Bronze*") that surrounds the screw. Now, this part, at first glance, appears to be a continuation of the pin *E* directly above it; there are two reasons, however, why this is not the case. In the first place, the part *E* is sectioned for steel; this immediately shows that *E* and the part under investigation are separate parts. Furthermore, when tracing out the shape and positions of the objects in the front view, they will be seen neither to be in line nor to have any connection with each other. To find the part under investigation in the front view, we may take, in the sectional view, a measurement from the center of the screw downwards to the lowest point of the part we are investigating, and then, referring to the front view, proceed along the center line of the screw *C* until we strike the dotted line *h*. Since the sectional view shows only things to the left of the line *a b*, we know that as the part being investigated shows in the sectional view, we must look for it to the left of *a b* in the front view. At the ends of the horizontal dotted line *h* we notice two vertical dotted lines that show the length of the part under investigation; since these lines terminate against the line *c*, we know that the part butts against the surface of *B*.

This latter conclusion is further confirmed by examining, in the sectional view, the full outline of the part. Referring again to the front view, we see a screw thread indicated in *B* right above the part we are discussing, and in the absence of any indication to the contrary may justly assume that it is a threaded shank by means of which the part is attached to *B*. By this time we are probably convinced that the part we have been investigating is the nut we are looking for, but are not sure of it. To find out, let us try to investigate the whole of the screw. In the front view, the dotted lines *i*, *i* show that the screw has a bearing and also has a collar butting against part of *A*; beyond the bearing the screw shows in full and apparently has a seat for some kind of an attachment which must cause the screw to turn, since a dowel-pin is shown in the seat. Inspecting the sectional view we find a screw similar to the one under discussion, with a ball handle and retaining nut on the end of it. As we find a note "*Two Handles Machinery Steel, Finish all over*," and as we cannot find any other place for the second handle, we naturally conclude that such a handle is to be placed on the end of the screw *C*. Now, from the fact that the screw is confined longitudinally by the collar and the ball handle, and that there is no thread on the part of the screw between them, we know that the nut must be to the right of the collar; since the part previously investigated is the only part we can find that directly surrounds the screw, we will now be justified in assuming that it is the nut we are looking for.

111. The fact that *A* and *B* are connected together by a screw provided with a handle for turning it will immediately suggest the idea that it is to be used for moving the part *B* along *A*, whence we conclude that *B* is a slide moving on *A*. Knowing this, the logical conclusion is that the piece *L* is a gib used for taking up the wear of the sliding surfaces, which view is proved to be correct when it is noticed, by reference to the sectional view, that a tightening of the setscrew will tend to draw the wearing surfaces together.

112. Looking now at the part *K*, at the left of *A*, in the front view, considering the part by itself, we cannot tell whether it is an integral part of *A* or a separate piece fastened to it. But as soon as we consider it in connection with the screw *C*, we see that the latter cannot be placed in position unless the part *K* is removable. From this we conclude that the part *K* is separate from *A*. The next question that suggests itself is: How is it fastened on? The note on the front view and the dotted screw heads in the sectional view show that screws with slotted heads used.

113. As far as the shape of *K* is concerned, the front view shows that it is a cone joining some presumably flat part. Referring now to the sectional view, we discover by measuring successively from the center line and center of the screw *C* that the dotted horizontal line *k'* represents the lower surface of *K*, and the absence of any other dotted lines in this part of the sectional view indicates that the profile of the flat part of *K* is the same as that of *A*.

114. On examining the sectional view, it is seen that some part of *D*, which from the section lining we know to be separate from *B*, projects downwards from the main body of *D* and is in contact with the upper surface *l'* of the part *B*. Referring now to the front view and looking along *l*, we find that the part under discussion is cylindrical; this is inferred from the dimension "*4" turned.*" The main body of *D*, and also the parts *G*, *H*, and *J*, may now be investigated in a manner similar to that in which the relation of *A*, *B*, *C*, and *K* was traced; it will then be found that *D* is a part similar to *A*. Furthermore, the investigation will show that *G* is a slide; this slide is movable by means of the screw *H*, which turns in the bearing *J*.

115. Referring again to the sectional view, we see that *B* and *D* are connected together by a pin *E*, whose purpose is unknown as yet. Examining this pin we notice that a hole is cut through its upper end and that a screw *F*, with a tapered shoulder to the right of its screw thread, passes through this hole. On close examination, we see that the hole in *E* is so placed that the tapered part of the screw *F* bears against the upper side of the hole. We further notice that the screw *F* is not used as a fastening device to hold any parts of *D* together; this conclusion is forced upon us by the fact that the sectional view shows *D* to be one piece. Now, we know from experience that a screw is used either as a fastening device or to transmit motion; as it obviously is not used for the purpose first mentioned, we conclude that it probably serves for the latter purpose. To make sure of this we trace out what will happen if the screw is rotated. We then noticed that if the screw is screwed inwards, it will raise the part *E*; but as *E* cannot move upwards by reason of being confined by the collar on it, it shows to us that screwing *F* inwards will force *D* down on *B*. The logical inference is that *E* and *F* form a clamping device intended to clamp *B* and *D* together.

Examining the pin *E* again, we do not find anything that would definitely tell whether it is round or square. Here judgment must be used. An experienced person would know upon the first glance that the clamping arrangement shown is an expensive one to make and one not likely to be adopted when it is only required to fasten two pieces rigidly together, in which case *E* might be either round or square. The next inference would be that it is used in order to allow *D* to be rotated around *E* and to be clamped in any position. This supposition requires the pin *E* to be round and is correct in this case.

116. Referring now to the ball handle *I*, of which only one view is shown, the question of whether it is circular or square is immediately settled by experience teaching us that a handle having the shape shown is not likely to be anything else but round, and in the absence of any note or indication to the contrary, we would be justified in assuming it to be round.

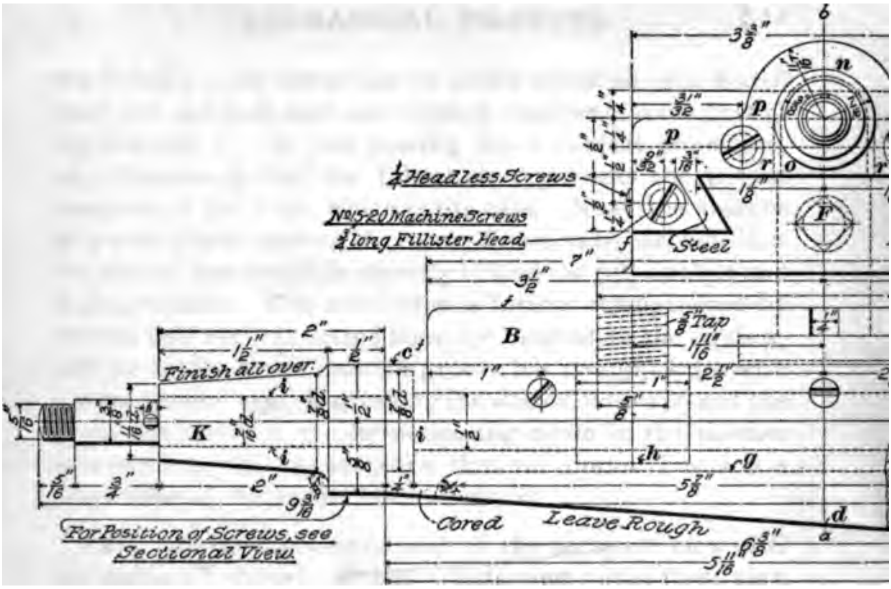
117. As far as the part *G* is concerned, the sectional view shows it to be cored out in order to pass over the nut in which the screw *H* works. The width and profile of the coring must be obtained from the front view, which it will be remembered is a view at a right angle to the sectional view. The natural assumption to make is that the lines giving the width and profile of the coring will be found directly in the vicinity of the screw *H* in the front view. Measuring from the center line of this screw in the sectional view upwards to the line showing the height of the coring, and then transferring this measurement to the front view, we find the full circle *n*. Now, as the coring is beyond the bearing *J*, we know that its profile would show in dotted lines and conclude that the circle *n* represents some part of the bearing *J*. As this bearing has a conical projection, the inference is that the full circle represents the largest diameter of the cone, which is the case. Now, the absence of a dotted line showing the coring forces us to conclude that the dotted line would be directly behind the full circle *n* and is thus hidden. This conclusion is further strengthened by finding two vertical dotted lines *r*, *r* tangent to the circle *n*, and we finally decide that the groove has straight sides with a semicircular top, as given by the dotted lines *r*, *r* and the upper semicircle of *n*. By measuring again in the manner previously explained, we decide that the dotted line *o* is a front view of the nut in which *H* works.

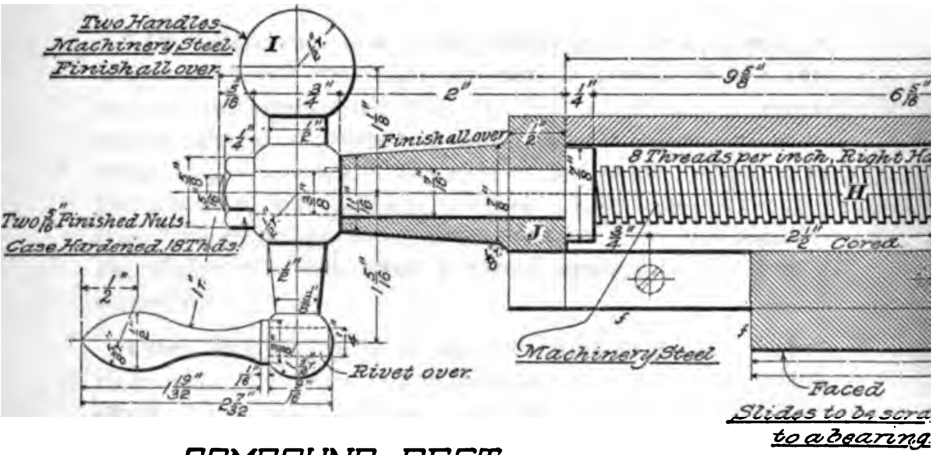
118. At the right-hand end of the sectional view of *G* we notice a **T**-shaped opening. Referring to the front view we can easily discover, by transferring measurements, that the dotted horizontal lines *p*, *p* show the length of the slot, which is seen to extend clear across *G*.

119. Referring now to the drawing of the tool post, it will be observed that only one view is given. While this does not definitely settle that the post is circular in cross-section, common practice would justify a person in assuming, in the absence of any note or any other indication to the contrary, that such was the case. This view is strengthened by the fact that some dimensions are marked *d*, signifying diameter, which term is rarely applied to any but a round object.

120. The two views of the collar give its shape. Referring to the front view, while there is no definite note to that effect, it would be inferred from the fact that a thread is shown that the lower part is separate, being, in fact, a circular nurled nut threaded to receive the upper part.

121. While, generally speaking, any one can learn to determine the shape of objects from a drawing, there are cases that arise in practice where this is very difficult





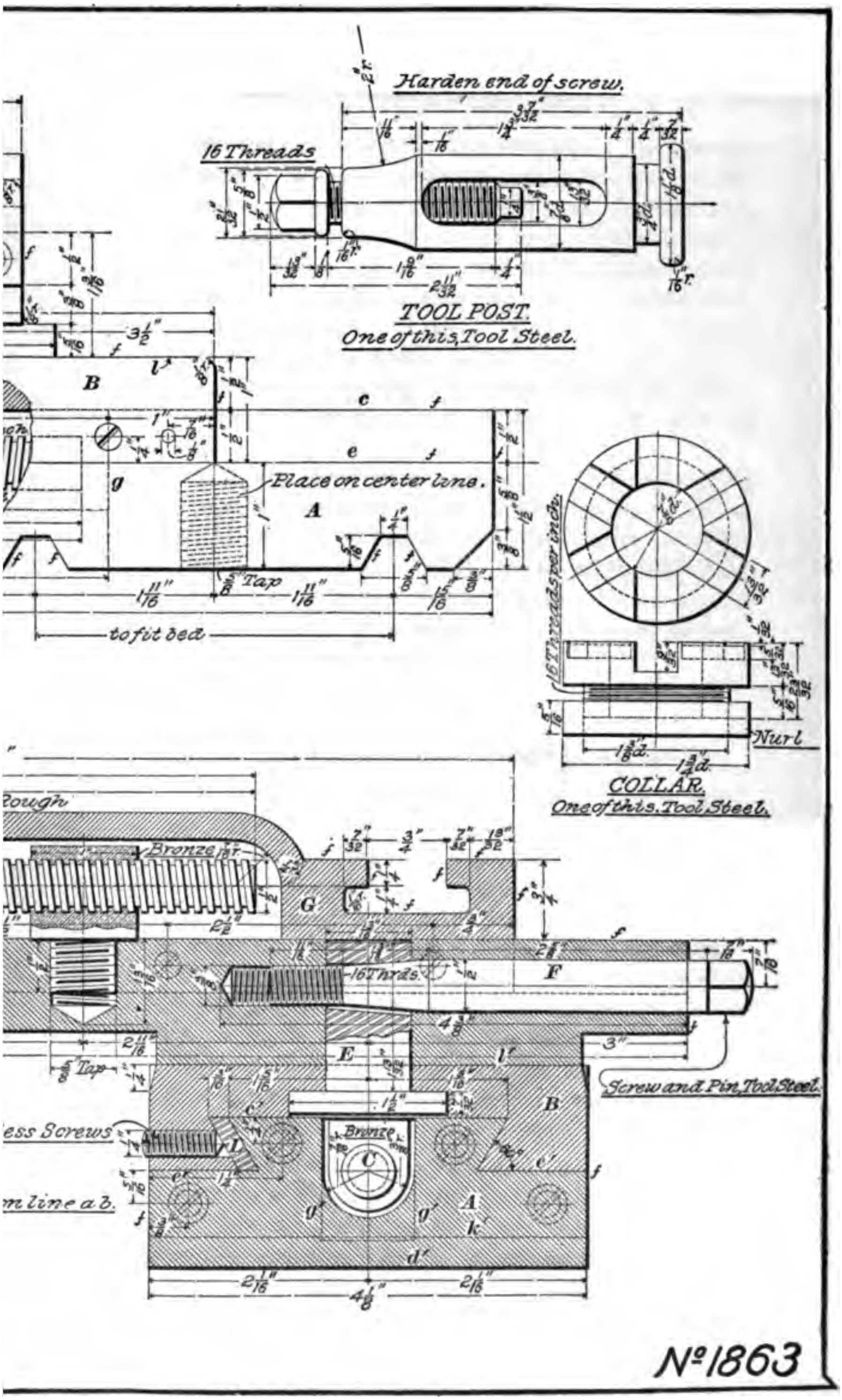
COMPOUND REST

FOR 10"X 36" SPEED LATHE.

CAST IRON, UNLESS OTHERWISE SPECIFIED,
ONE OF THIS. SCALE FULL SIZE.
THE SCRANTON TOOL WORKS,
SCRANTON, PA.
Drawn By J.A.G. Drawer 182.
Checked By C.P.T. Order No 97.
DATE OCT. 1-1900.

JUNE 25, 1893.

For notice of copyright, see pag



following the title page.

JOHN SMITH; CLASS No 4529.

without further verbal or written instructions. The cases in which this usually happens are where coring has various odd-shaped curved surfaces that curve in different directions, as occurs, for instance, with the steam ports and other passages of steam-engine cylinders and other similar work. Practical experience with a certain line of work, and, frequently, a knowledge of the object of the doubtful part, will often allow the reader to form a correct idea of what the draftsman is trying to convey; when this experience or knowledge is lacking, *consult somebody who is likely to know.*

Furthermore, the shape of an object does not necessarily in itself always reveal its purpose. Ability to determine at sight what an object is to be used for involves either a thorough knowledge of a particular line of work—in which case the purpose of objects coming within its range can usually be determined at sight—or a very wide general knowledge of engineering construction.

DRAWING PLATE, TITLE: COMPOUND REST

122. This drawing is at one and the same time a detail and an assembly drawing, as it not only gives all dimensions required for each and every part, but also shows how they are assembled. Drawings are frequently made in this manner in order to save some of the draftsman's time. The particular drawing shown exhibits a case where the draftsman, on account of lack of space, has been compelled to break the rules governing the arrangements of the views. Referring to the plate, the sectional view taken on the line *a b* should have been placed alongside and on the right of the front view, which obviously cannot be done without making the drawing to a scale smaller than full size.

123. To begin locate the center line of the screw *C* in the front view at a distance of 4¼ inches from the upper border line. The slide *B* being movable along the base *A*, it can be located anywhere within its range of movement;

but in order to get a convenient arrangement, it is recommended to locate the line *a b* passing through the center of the screw of the upper slide at a distance of $7\frac{1}{16}$ inches from the left-hand border line. Locate the center line of the hole in the base marked $\frac{5}{8}''$ tap $11\frac{3}{16}$ inches from the left-hand border line. The front view can now be drawn from the dimensions given.

124. Locate the center line of the screw *H* in the sectional view $8\frac{1}{4}$ inches from the upper border line and locate the center line of the handle *I* at a distance of $2\frac{5}{8}$ inches from the left-hand border line. The upper slide *G* being movable on its base *D*, the latter could be drawn anywhere within the range of movement; in order to have a good arrangement, however, it is recommended to locate the vertical center line passing through the part *E* at a distance of $12\frac{1}{16}$ inches from the left-hand border line. The sectional view can now be drawn without any special instructions.

125. The center line of the collar should be located at a distance of $1\frac{5}{8}$ inches from the right-hand border line and the center of the top view of the collar is to be located at a distance of $4\frac{7}{8}$ inches from the upper border line.

126. The center line of the tool post may be located at a distance of $1\frac{3}{16}$ inches from the upper border line and the right-hand end of the tool post at a distance of $1\frac{3}{8}$ inches from the right-hand border line. The tool-post screw being movable, it could be shown in any position within its range of movement; but in order to get the same arrangement as on the plate, the right-hand surface of its collar should be placed at a distance of $\frac{1}{8}$ inch from the left-hand end of the body of the tool post.

A number of reference letters printed in bold-face italics appear on this plate. The student should omit these letters on his drawing.

127. In order to emphasize the fact that different draftsmen prefer different styles of lettering and a different arrangement for titles, this plate has been given a title differing from those previously given. The first and second lines of lettering are easily made with the ruling pen, the inclination of the letters being 75°. The letters in the first line of lettering are $\frac{3}{16}$ inch high; in the second line all capitals and numerals are $\frac{1}{8}$ inch high, except the first letter of each word, which is $\frac{3}{16}$ inch high. The third to ninth line of lettering are written freehand. In the third and fourth lines, the lettering is $\frac{3}{32}$ inch high, except the first letter of each word, which is $\frac{1}{8}$ inch high. In the fifth line, all letters are $\frac{1}{8}$ inch high, except that the first letter of each word is $\frac{5}{32}$ inch high. The sixth line is the same as the third and fourth. In the remaining lines, the capitals are $\frac{3}{32}$ inch and the small letters $\frac{1}{16}$ inch high. The heading "Tool Post" is $\frac{1}{8}$ inch high.

TRACINGS

128. In actual practice in the drawing room, it is necessary to have more than one copy of a drawing. It would be very expensive to make a finished drawing every time an extra copy was wanted and to avoid this tracings and blueprints are made. Any number of blueprint copies can be made from the same tracing. A complete pencil drawing is made first; then, instead of inking in as heretofore, a piece of tracing paper or tracing cloth of the same size as the pencil drawing is fastened to the board over the original drawing. The tracing paper or cloth being almost transparent, the lines of the drawing can be readily seen through it, and the drawing is inked in on the tracing paper or cloth in the same manner as if inking in a finished drawing.

129. **Tracing paper** is but little used in this country. It is easily torn and cannot be preserved as well as tracing cloth. The two sides of the **tracing cloth** are known as the glazed side and the dull side; they are also known as the front and the back. The glazed side, or front, is covered with a preparation that gives it a very smooth polished surface; the back, or dull side, has very much the appearance of a piece of ordinary linen cloth. Either side may be used for drawing upon, but when the glazed side is used, care must be taken to remove all dirt and grease, otherwise the ink will not flow well from the pen. This can be done by taking a knife or a file and scraping or filing chalk upon the tracing cloth; then take a soft rag of some kind—cotton flannel or chamois skin—and rub it all over the tracing cloth, being sure to rub chalk over every spot. Finally, dust the rag and remove as much of the chalk from the cloth as can be gotten off by rubbing with the rag. The finer the chalk powder is the better. It is not usual to chalk the dull side, but it improves it to do so. The glazed side takes ink much better than the dull side, the finished drawing looks better and will not soil so easily, and it is also easier to erase a line that has been drawn on this side. Pencil lines can be more satisfactorily drawn on the dull side, and if it is desired to photograph the drawing, it is better to draw on this side. The draftsman uses either side, according to the work he is doing and to suit his individual taste, but if the glazed side is used, *it must be chalked.* The tracings are drawn in a manner similar to the finished drawings, the center lines, section lines, etc. being drawn exactly as previously described. In some offices it is customary to draw the center lines and dimension lines on a tracing in red ink, so that they may appear gray instead of white on the blueprint.

130. After having drawn the plate entitled Compound Rest, the student is required to make a tracing of the plate entitled Turret-Lathe Tools.

BLUEPRINTING

131. Blueprinting is the process of duplicating a tracing by means of the action of light upon a sensitized paper. The following solution is much used for sensitizing the paper : Dissolve 2 ounces of citrate of iron and ammonia in 8 ounces of water; also 1¼ ounces of red prussiate of potash in 8 ounces of water. Keep the solutions separate and in dark-colored bottles in a dark place where the light cannot reach them. Better results will be obtained if ½ an ounce of gum arabic is dissolved in each solution.

When ready to prepare the paper, mix equal portions of the two solutions, and be particularly careful not to allow any more light to strike the mixture than is absolutely necessary to see by. For this reason, it is necessary to have a dark room to work in. There must be in this room a tray or sink of some kind that will hold water; it should be larger than the blueprint and about 6 inches deep. There should also be a flat board large enough to cover the tray or sink. If the sink is lined with zinc or galvanized iron, so much the better. There must be an arrangement like a towel rack to hang the prints on while they are drying. For the want of a better name, this arrangement will be called a print rack. The paper used for blueprinting should be a good, smooth, white paper, and may be purchased of any dealer in drawing materials. Cut it into sheets a little larger than the tracing, so as to leave an edge around it when the tracing is placed upon it. Place eight or ten of these sheets upon the flat board before mentioned, taking care to spread flatly one above another, so that the edges do not overlap. Secure the sheets to the board by driving a brad or small wire nail through the two upper corners sufficiently far into the board to hold the weight of the papers when the board is placed in a vertical position. Lay the board on the edges of the sink, so that one edge is against the wall and the board is inclined so as to make an angle of about 60° with the horizontal. Darken the room as much as possible and obtain what light may be necessary from a lamp or gas jet, which should be turned down very low. With a wide camel's-hair brush or a fine sponge, spread the solution just prepared over the top sheet of paper. Be sure to cover every spot and do not get too much on the paper. Distribute it as evenly as possible over the paper, in much the same manner that the finishing coat of varnish would be put on by a painter. Remove the sheet by pulling on the lower edge, tearing it from the nail that holds it, and place it in a drawer where it can lie flat and be kept from the light. Treat the next sheet and each succeeding sheet in exactly the same manner, until the required number of sheets has been prepared.

Unless a large number of prints is constantly used, it is cheaper to buy the paper already prepared. It can be bought in rolls of 10 yards or more, of any width, or in sheets already cut and ready for use. There is very little, if anything, saved in preparing the paper, and better results are usually obtained from the commercial sensitized paper, since the manufacturers have machines for applying the solution and are able to distribute it very evenly.

132. In Figs. 15 and 16 are shown two views of a printing frame that is well adapted to sheets not over 17″ × 21″. The frame is placed face downwards and the back *A* is removed by unhooking the brass spring clips *B*, *B* and lifting it out. The tracing is laid upon the glass *C*, with the *inked* side touching the glass. A sheet of the prepared paper, perfectly dry, is laid upon the tracing with the yellow (sensitized)

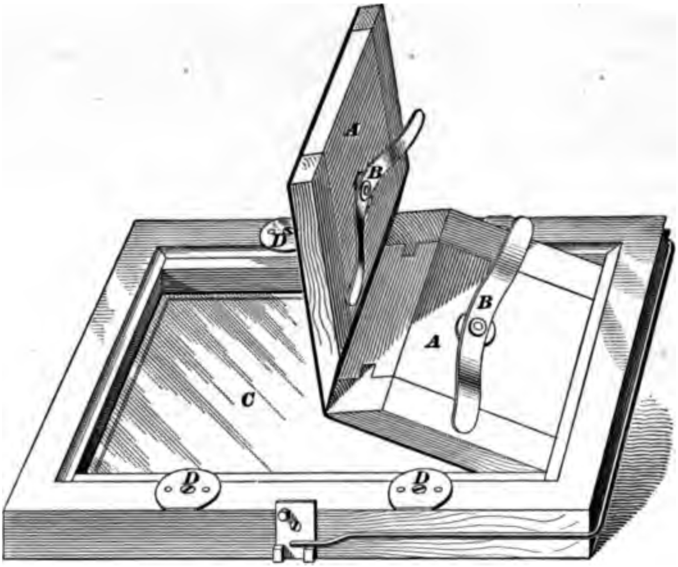


FIG. 15

side downwards. The paper and tracing are smoothed out so as to lie perfectly flat upon the glass, the cover *A* is replaced, and the brass spring clips *B*, *B* are sprung under the plates *D*, so that the back cannot fall out. While all this is being done, the paper should be kept from the light as much as possible. The frame is now placed where the sun can shine upon it and is adjusted, as shown in Fig. 16, so that the sun's rays will fall upon it as nearly at right angles as possible According to the conditions of the sky—whether clear or cloudy—and the time of the year, the print must be exposed from 3 to 15 minutes. The tray, or sink, already mentioned, should be filled to a depth of about 2 inches with clear water (rain water if possible). The print having been exposed the proper length of time, the frame is carried into a dark part of the room, the cover removed, and the print (prepared paper) taken out. Now place it on the water with the yellow side down and be sure that the water touches every part of it. Let it soak while putting the next print in the frame. Be sure that the hands are dry before

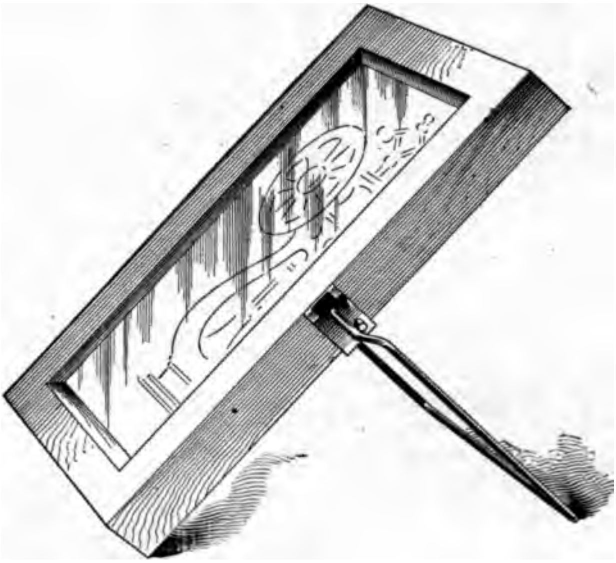


FIG. 16

touching the next print. The first print having soaked a short time (about 10 minutes) take hold of two of its opposite corners and lift it slowly out of the water. Dip it back again and pull out as before. Repeat this a number of times, until the paper appears to get no bluer; then hang it by two of its corners to dry on the print rack previously mentioned. If there are any dark-purple or bronze-colored spots on the prints, it indicates that the prints were not washed thoroughly on those spots. If these spots are well washed before the print is dried, they will disappear.

133. It is best to judge the proper time of exposure to the light by the color of the strip of print projecting beyond the edge of the tracing. To obtain the exact shade of the projecting edge, take a strip of paper about 12 or 14 inches long and 3 or 4 inches wide. Divide it into, say, 12 equal parts by lead-pencil marks, and with the lead pencil number each part 1, 2, 3, etc. Sensitize this side of the paper and, after it has been properly dried, place it in the print frame with the sensitized side and the marks and figures against the glass. Expose the whole strip to the light for one minute; then cover the part of the strip marked 1 with a thin board or anything that will prevent the light from striking the part covered. At the end of the second minute, cover parts 2 and 1; at the end of the third minute, parts 3, 2, and 1, etc. When twelve minutes are up, part 1 will have been exposed one minute; part 2, two minutes, etc., part 12 having been exposed twelve minutes. Remove the frame to a dark part of the room and tear the strip so as to divide it into two strips of the same length and about half the original width. Wash one of the strips as before described, and when it has dried, select a good rich shade of blue, neither too light nor too dark; notice the number of the part chosen, and it will indicate the length of time that the print was exposed. Examine carefully the corresponding part of the other strip, and the correct color of the edge of the print projecting beyond the tracing is determined. All prints should be exposed until this color is reached, no matter how long or how short the time may be; then they should be immediately taken out and washed.

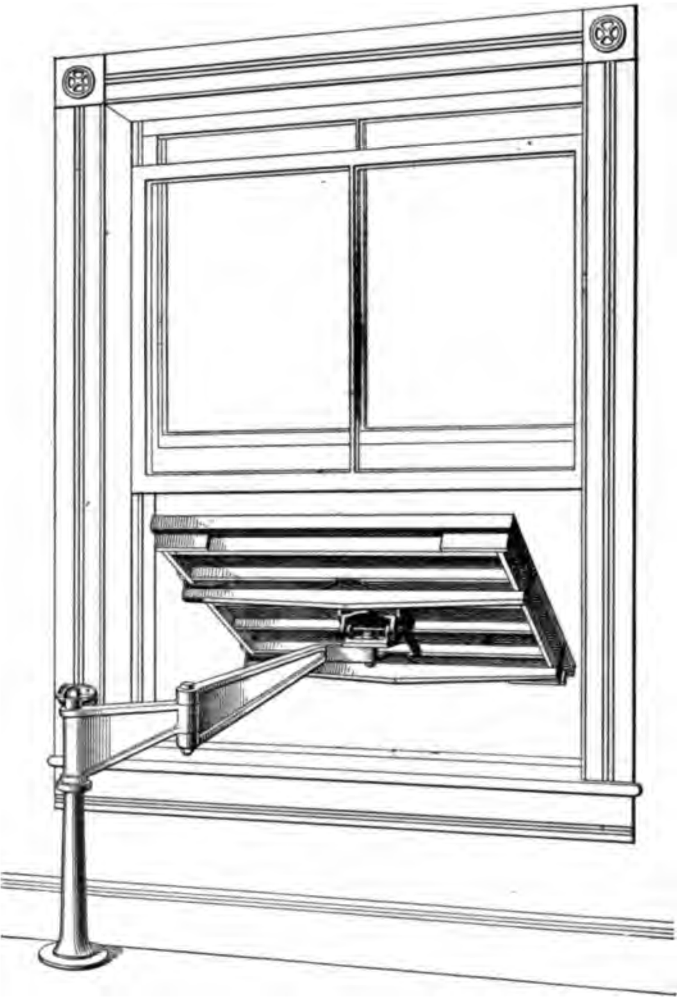


FIG. 17

134. In Fig. 17 is shown a patented frame which can be shoved out of the window and adjusted to any angle. When not in use, it can be folded up against the wall and occupies but little space. It is made in different sizes from 16″ × 24″ to 48″ × 72″. It is one of the best frames in the market, and is placed in such a position relatively to the window that the window can be lowered to the top of the main arm, when it is desired to keep out the cold during the winter.

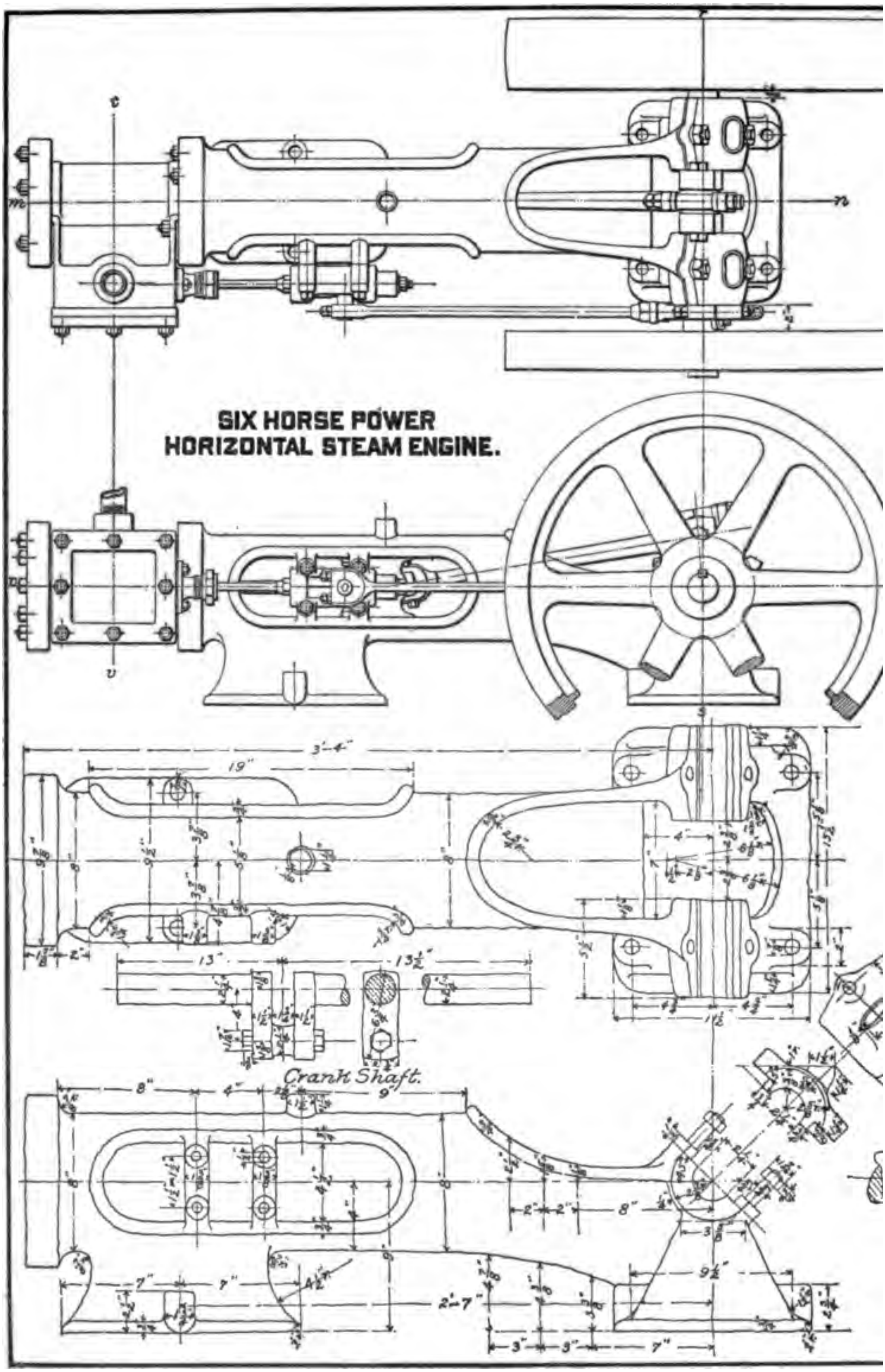
DRAWING PLATE, TITLE: SIX-HORSEPOWER HORIZONTAL STEAM ENGINE

135. Instead of making a finished drawing of this engine, as in the previous plates, from an exact copy, the student is given the rough sketches of the details of a **six-horsepower horizontal steam engine,** with full dimensions marked upon them; from these he is expected to make a general pencil drawing of the engine in two views—a plan and a side elevation. The details are not to be drawn.

The pencil drawing should then be traced according to the directions previously given. The details are not drawn to scale, but are fully dimensioned. In order to draw the engine to as large and as convenient a scale as possible, it is necessary to make this tracing a trifle larger than the plates that have preceded it. The size over all will be 14½″ × 18⅛″, with the usual border line ⅛ inch from each edge all around.

That the student may have a good idea of what he is expected to do, a greatly reduced cut of the general drawing is also given him. All dotted lines indicating parts not seen have been omitted in order to simplify the work. The scale to be used is 3 inches = 1 foot.

136. Draw the center lines *m n*, *p q*, *r s*, and *t v*. Draw the side elevation of the bedplate with the bearing caps in position, from the dimensions given on the detail sketches, taking care to make the parts that are likely to be hidden by the flywheel, eccentric rod, etc. light so that they may be easily erased before tracing. The drawing may be traced without removing the unnecessary construction lines, but it is better to do so, since it lessens the liability of inking in lines that will have to be erased from the tracing.



SIX HORSE POWER HORIZONTAL STEAM ENGINE.

JUNE 25, 1893.

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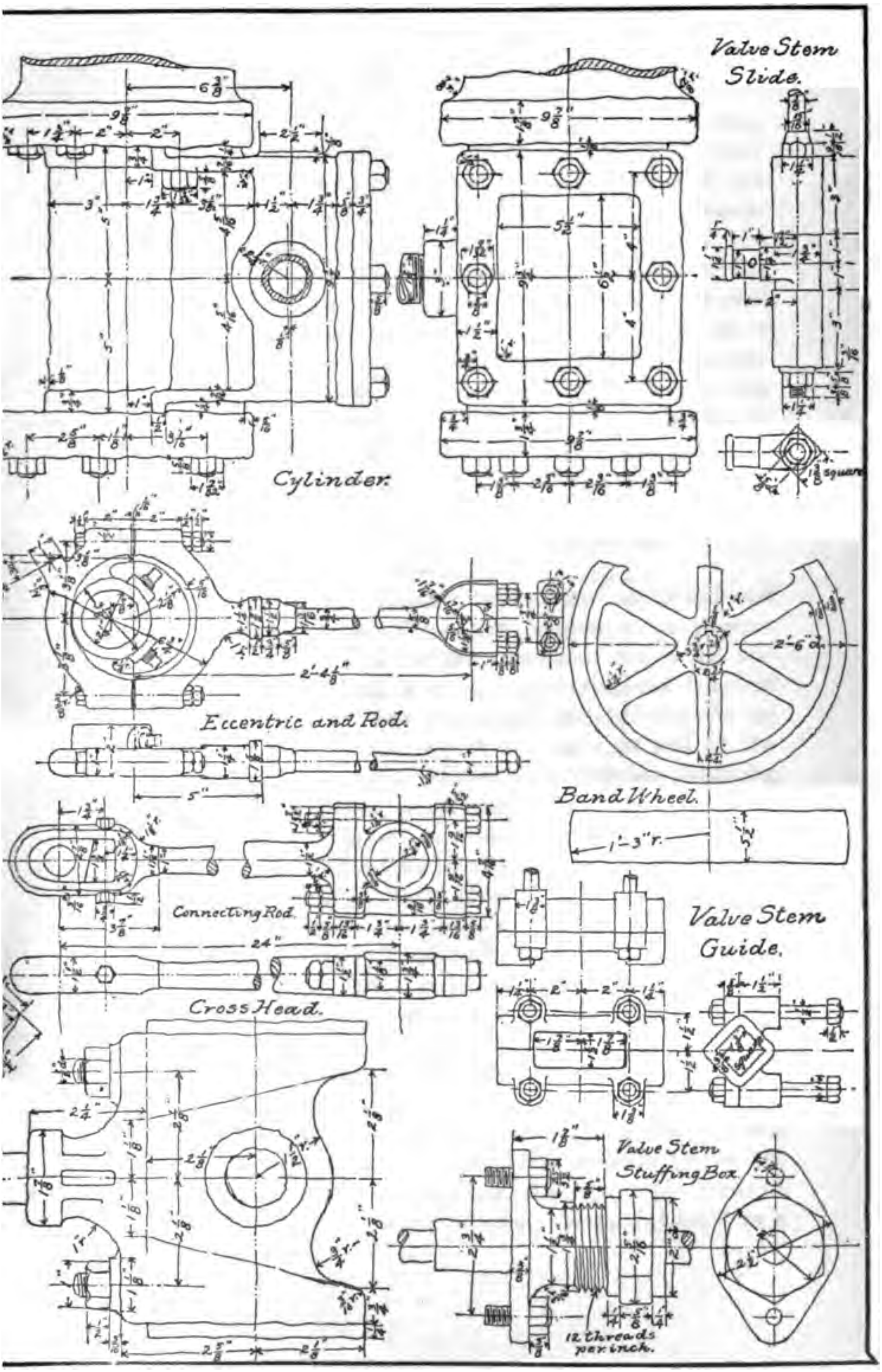

diately following the title page.

JOHN SMITH. CLASS Nº 4529.

Draw the plan of the bedplate with the bearing caps, studs and nuts, foundation-bolt holes, etc. shown in their proper places and positions. The different curves of the bearing caps in the plan should be constructed by projecting points from the view shown in the side elevation. In actual practice in the drawing room, three or four of the principal points (those that mark the limits) would be located and the curves sketched in freehand, they being inked in on the tracing by aid of an irregular curve. In such cases the draftsman has a good idea of the shape of the curves, owing to previous practice in drawing them. When drawing in the curves formed by the opening in the bedplate shown in this view, the student must exercise his own judgment regarding their shape, taking care not to get them too straight. The general drawing gives a good idea of their proper curvature.

Returning to the elevation, draw the crank and crank end of the connecting-rod in the position shown in the general drawing. With the center of the crankpin as a center and a radius equal to the length of the connecting-rod between its centers (obtained from the detail sketch), describe an arc cutting the center line $p q$ at a point that will be the center of the crosshead pin. Draw the crosshead, obtaining the dimensions from the detail sketch. Complete the connecting-rod in both views and draw the piston rod 1 inch in diameter. Draw both views of the cylinder with the nuts and the steam pipe in their proper position, getting all dimensions from the detail sketches.

Draw the center line of the valve stem in the plan view, and draw the stuffingbox, valve stem, valve-stem slide and its guide in both views. In order to determine the position of the valve-stem slide, it is necessary to locate the center of the eccentric. Referring to the general drawing, it is seen that the eccentric is on the dead center farthest from the cylinder. The offset of the eccentric is given as $\frac{7}{8}$ inch in the detail sketch; hence, when in this position, the center of the eccentric strap will be situated $\frac{7}{8}$ inch to the right of the crank-shaft center on the line $p q$. With this point as a

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Finally, draw in the bandwheel and flywheel (see general drawing for position). The flywheel will be of the same diameter as the bandwheel, but only 3 inches wide.

The pencil drawing is now completed. Before beginning to trace, erase the lines that are not to be inked in. This is not necessary, but it is better to do so, since it avoids confusion and lessens the liability of making mistakes. Some draftsmen prefer to redraw a portion of those parts that are to be inked in with a somewhat softer pencil and leave the light construction lines on the drawing rather than erase them; in some cases, this saves time.

The preliminary directions for tracing a drawing have been given previously. First, trace the side elevation, beginning with the flywheel, and then as much of the connecting-rod, eccentric, and eccentric rod as can be seen. Trace all those parts of the bedplate, cylinder, valve stem, stuffing-box, etc. that are seen. Then trace the plan view, letter the drawing, and draw the border lines. There will be no plate number for this tracing, but the student's name, class number, and the date of completion will be put on as before.

The student should exercise particular care to have every dimension scale exactly the size given in the detail sketches.

PRACTICAL PROJECTION

INTRODUCTION

1. Orthographic Projection.—When the mechanic is required to make any article whose form or dimensions are not previously known, it is evident that a description of the work in question should be furnished him. This description may be, and often is, given by verbal instruction; but in order to enable the worker to understand definitely what is wanted, the form of the object, its dimensions, and the quality of the material to be used should be stated. Instruction given in this way is, however, seldom satisfactory either to the workman or to his employer, since it is difficult in such cases to place the responsibility for any errors that may occur.

Written instruction, therefore, would seem to be preferable; but, since most objects would require an extended description, a shorter and more convenient method of conveying the desired information is to be sought. A **drawing** of the object is therefore made.

These drawings are generally made by a process termed *orthographic projection*, or, as it is usually called, **projection drawing.** Every detail of the object is correctly represented in this drawing, so that the workman knowing how to "read the drawing" may obtain his measurements therefrom for the construction of the object itself. He is also enabled by an examination of the drawing to understand exactly how the object will appear when completed. Hence, we have the following definition:

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Orthographic projection *is the process of making correct representations of objects by means of drawings.*

2. A Working Drawing Generally Necessary.—Before a pattern for any article can be made, a working drawing is needed. No pattern, however simple or plain, can be produced until we have something definite to work from.

The metal worker does not go to the trouble of preparing a drawing on paper for every piece of work he is called on to make, since many objects are so plain that a brief verbal or written description of their dimensions gives the mechanic all the information he needs to enable him to lay out his work.

If, for example, a tinsmith is called on to make a box out of IX tin, 4 inches long, 3 inches wide, and 1 inch deep, he immediately proceeds with the steel square to lay off the given sizes directly on the metal; but if the same mechanic is required to make a round pan having flaring sides or some article of a form not readily carried in the mind, there is one thing he must do before he can proceed with the work or even lay out the pattern—he must make a **working drawing** of the object.

3. What Constitutes a Working Drawing.—There are several ways in which this drawing may be made, depending altogether on how complicated the object is. In the case of the pan referred to, it may be desirable, by an application of certain principles, to omit the operation of making a drawing consisting of several views and proceed as with the box to "lay it out" directly on the metal. In this case, however, it will be found that the operation differs from that of making the box referred to, since it is first necessary to mark out the sizes and outlines as they will appear when the pan is completed. These sizes may or may not form a part of the pattern, but they are required as preliminary lines from which to "lay off," or "strike out," the pattern.

Marking out these sizes or dimensions of an object is really making a working drawing. This drawing may be *full size*—in which case it is referred to as a **detail drawing**—or it may be drawn to a scale, either larger or smaller than the object itself.

4. Where the Drawing Is Made.—In the case of plain articles, the necessary drawings may be made directly on the metal; in the majority of cases, however, the work is of a more or less complex nature, making it highly important that a full-sized properly made detail drawing be used. This is not always provided, and the mechanic frequently has to make his own detail from a small freehand sketch or possibly from a drawing made to a small scale. In the latter case, an enlarged drawing must generally be made before the work of laying out the pattern can proceed. This necessitates operations with drafting board and drawing implements—with which the student is already familiar. The proficiency that has been acquired may now be put into practical use in the operations to follow.

It is the purpose of this section to present methods by which working drawings may be *made* and *read*. These methods are presented in a practical way, that the principles laid down may be readily understood by the student.

GENERAL PRINCIPLES

5. Various Kinds of Drawings.—The most common representations of objects are those used for purposes of illustration merely and known as **perspective drawings.** They are of little value to the mechanic to serve as working drawings, since they are not drawn to a scale in the same way as a projection drawing. and to obtain measurements therefrom is an operation both complicated and indirect. A *photograph* is an ideal perspective picture, but no one would think of using a photograph as a working drawing. The photograph and the perspective drawing represent the object "as we see it," or as it appears to the eye of the observer, while the working drawing—the projection drawing—represents the object *as it actually is*, or will be when made.

A photograph, however, shows only such objects as really exist, while a projection drawing often shows objects that exist only in the imagination of the draftsman or a person capable of understanding, or "reading," the drawing. By means of such drawings the imagination is aided in picturing the object as already constructed—or, as we would say, it enables the mechanic to see the object "in his mind's eye."

6. What Is Shown in a Drawing.—The perspective drawing always shows more than one side of an object—generally three sides—while the working drawing seldom shows more than one side, that being the side towards the observer. The position of such other portions of the object as are not located on the side shown in the drawing may, however, be indicated in a projection drawing by dotted lines. No lines should be used in a working drawing that do not represent actual edges, or outlines, in the object itself. We sometimes find certain edges or outlines of an object represented in a drawing by heavy lines. These heavy lines are called **shade lines**; but since they are not essential features of the working drawing, no description of them is necessary in this section. There is also an elaborate system of representing the effect of light and shade on curved and receding surfaces by means of lines properly disposed over the surfaces shown in the drawing. Since these lines are for effect only, and their meaning is apparent to the observer, the principles governing their use are not made a part of the subject matter of this section.

7. Position of Observer. — Another point of difference between the perspective and the projection drawing is that, in taking the view of which the perspective drawing is a representation, the eye of the observer remains in a fixed position, and in the same relation to the drawing as the camera is to the photograph; while in that view of the object of which the projection drawing is a representation, the eye of the observer is always supposed to be directly over or opposite that point in the drawing which is being noted.

This may be illustrated by the student for himself in a very simple way. Place a sheet of paper on the drawing board and draw a horizontal line, say 6 inches long; now lay an ordinary 2-foot pocket rule against this line, in the manner shown in Fig. 1, and proceed to mark off the line to correspond with the divisions on the rule. He will find that he is obliged to get his eye directly over each mark on the rule, and to "sight" carefully down on to the rule before making each mark, very much as he would "sight" or look along a piece of work to see whether it is straight or not. It will be noticed also that he is making *one* eye do all the work in this "sighting," and, further, it will be observed that in making the markings he is moving his head as he progresses towards the end of the line. He is obliged to do this to keep his eye exactly over each point on the line as it is marked.

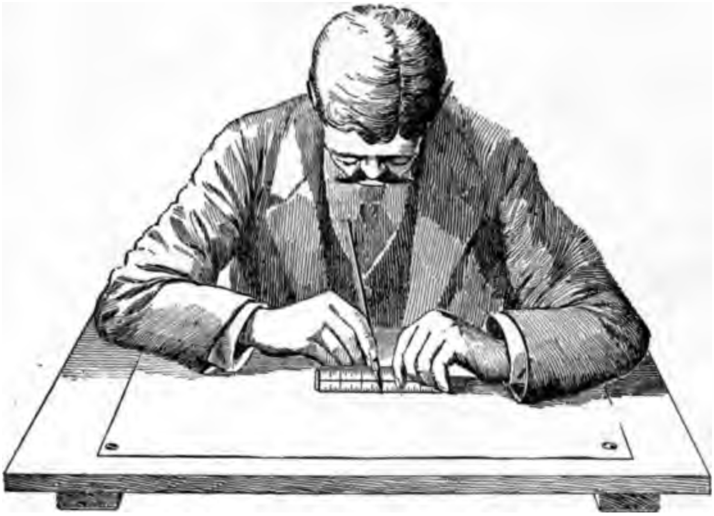


FIG. 1

8. Line of Sight.—The line first drawn on the paper is not the only one made use of in reproducing the markings on the rule. The student has unconsciously made use of another line, or, more properly, a set of lines, that are an important feature of projection drawing. These lines are those made use of in doing the "sighting" necessary for the marks; they are purely imaginary lines and are not represented in the illustration. They are very properly called **lines of sight.** The lines of sight in a projection drawing are *always perpendicular to the drawing*. They extend from a point in the eye of the observer to a point on the drawing that is directly opposite, as indicated by the point of the draftsman's pencil in Fig. 1.

These lines of sight—which, as stated, are only imaginary lines and are not represented in a working drawing—constitute one of the most important features of the projection drawing; for on these lines we are enabled to obtain the views from the object, and, by means of other lines, called **projectors,** bearing a certain relation to the lines of sight (as will be explained later), we can reproduce the views thus obtained on the drawing.

9. Lines of Sight Always Parallel.—When it is desired to make a drawing of any object, the lines of sight must be used in the same manner as in marking the divisions on the line in Fig. 1; that is, care must be taken to keep the lines of sight in any one view parallel to one another. We may take different views of the same object, or, to express it otherwise, we may take positions on different sides of the object, in order to obtain views therefrom; but in any view thus taken the above statement must be carefully observed and the lines of sight kept exactly parallel to one another.

10. Several Views Necessary.—We have already noted that a projection drawing seldom shows but one side of an object. Since there are no objects that present all their dimensions on any one side, it necessarily follows that, in order to convey a correct idea of the form of an object, it is necessary to make a drawing—or a *projection*—of as many sides as will enable the correct shape and dimensions to be shown. We may make these drawings from as many points of view as may be desired; but for certain reasons, to which attention will be called later, it is generally preferable to view all objects from six sides, which correspond to the six sides of a cube.

11. Before we proceed with the explanation of orthographic projection, it is important that the student should be informed about what are known as the *angles of projection*. That is to say, he must know something about the positions supposed to be assumed by the object when the different views are taken. Draftsmen generally recognize for projection drawing two principal angles in which the objects are placed, and the drawings made in these two angles are called, respectively, *first-angle projections* and *third-angle projections*.

These different positions of the object will be understood by the student from an examination of Fig. 2, in which four angles are formed by the intersection of the horizontal plane *A B C D* with the vertical plane *E F G H*. Then, of the four right angles included between the planes, the angle *B P G* is called the *first* angle, the angle *G P C* is known as the *second* angle, the angle *C P F* is the *third* angle, and the angle *F P B* is the *fourth* angle. In the view shown at (*a*) these two planes are assumed to be opaque and the object to be transparent. The lines of sight are projected from the eye of the observer to the plane of projection, *through the object*. In the view at (*b*) the planes are assumed to be transparent and the lines of sight pass from the object to the eye of the observer, *through the plane*. The effect of these different positions of the object is merely to change the relative positions of the different views when made on the drawing board.

The drawings made in this section are first-angle projections. Third-angle projections are made by processes similar to those here explained and the different positions of the views will be pointed out to the student at the proper time.

12. Plans and Elevations.—It being assumed that the object is in some fixed position, the various views take their names from the different positions of the observer in his view of the object. Thus, a view taken from above, or looking down on the object, is called a **plan**; so also is a

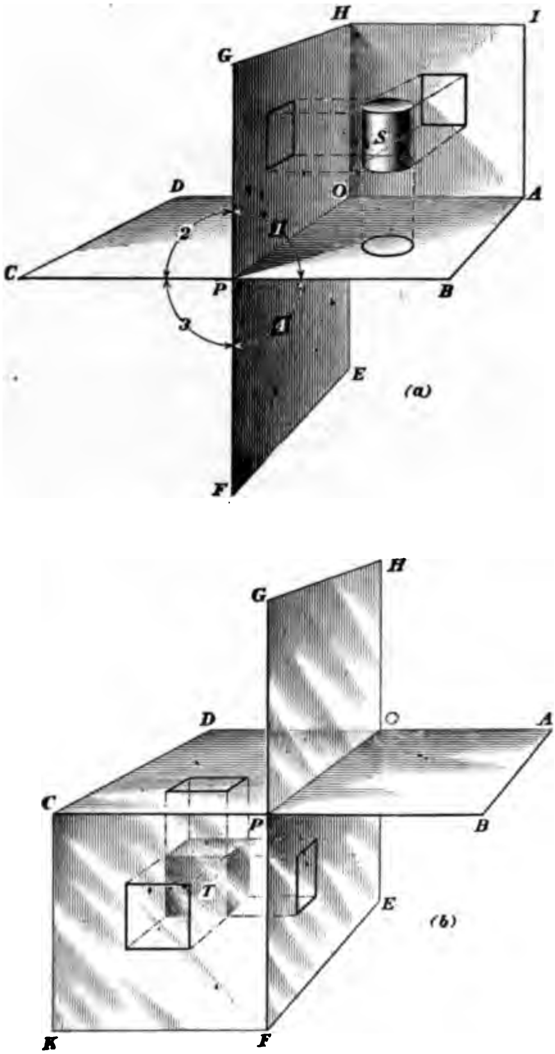


FIG. 2

view from beneath, or looking up at the object; thus we have the terms *top plan* and *bottom plan*. The two views thus obtained are frequently designated by terms that vary with the class of objects represented and not infrequently derive their names from some portion of the object itself. Thus, a top plan of a house is a view of the roof taken from above and is called a *roof plan;* while a *ceiling plan* is, as its name indicates, a view of another part of the house taken from the opposite direction. In the case of small objects generally, such views are termed *top plan* or *bottom plan*, as the case may be. These views, in certain cases, should be marked on the drawing, in order to guard against error. Here it should be noted that while the position of the object is not changed in making either the top or the bottom plan, yet the position of the observer is. When the two plans thus made are compared, it is found that the corresponding points of the drawings are changed in their relation to each other in the same manner as the hands of two persons that are standing exactly in front of and facing each other—the right hand of the one being opposite to the left hand of the other.

A view taken from the side of an object is called an **elevation.** That side of an object shown in any elevation gives its name to that drawing; thus, a view of the front of an object is called a *front elevation.* So, also, we have the terms *rear elevation* and *side elevation.* In some cases it may be more convenient to designate the elevations by the points of the compass; for example, the *north elevation* of a building is a projection of that part of the building which faces north, or, to state it as we have done before, that part of the building seen when looked at from the north.

13. Section Drawings.—Cases frequently occur in which the views or dimensions desired to be given on a working drawing cannot be shown in either plans or elevations. Under such circumstances, recourse is often had to a class of drawings termed **sections.** A section drawing is a projection of an object assumed to have been cut in two in a certain direction, usually at right angles to the lines of sight. Those parts of the object between the observer and the place where the cut is made are assumed to have been removed, so as to present an entirely new surface. This surface is not seen in the object itself, since the cutting is entirely imaginary—done simply for the purpose of showing some interior construction.

The cut just referred to may be made in a *horizontal*, a *vertical*, or an *oblique* direction, according to the way in which it is desired to show the section. Portions of surfaces that have been cut in this way are usually represented by certain conventional methods, indicating the character or composition of the material. A custom frequently adopted, and which will be followed in the drawings for this section, consists in designating surfaces exposed by the cut by a series of closely drawn parallel lines. Such lines are usually drawn at an oblique angle, as compared with the other portions of the drawing, and are called *cross-section* lines or *cross-hatching*.

14. A Set of Plans.—It is a common practice, when speaking of a set of drawings consisting of various plans, elevations, sections, etc.—as for a house or for some other object—to refer to them as a "set of plans." This is a collective phrase for the drawings and its use in this way is perfectly proper; when used in this sense, however, it is not understood as applying simply to a plan view as explained in Art. **12.** Drawings for large objects are frequently of a size such that the different views are more conveniently made on separate sheets. Architectural drawings are usually separated in this way, and it is often necessary for the one that is to read such drawings to arrange the sheets in a particular manner, in order that the relation between the views may be understood. This arrangement of the views will be considered later.

15. Foreshortened Views.—The lines in a projection drawing—or, as we shall term it hereafter, a *projection*—are either of the same length as the corresponding edges or outlines of the object itself, or they are *foreshortened*. No lines are represented longer than they actually exist on the object, except in cases where the drawing is made to an enlarged scale. All lines that are used to represent the edges or outlines of an object, and that are at right angles to the lines of sight in any view, are represented in that view by their true length. Lines that represent other edges or outlines, and are not at right angles to the lines of sight, are consequently represented shorter than they exist on the object, and are then said to be **foreshortened.**

16. Geometrical Forms.—The simplest geometrical form that we can imagine is a *point;* next we have a *line*, defined as the shortest distance between two points; then a *surface*, which is a flat, or plane, figure bounded by lines; and, finally, a *solid*, which in turn is bounded by different surfaces.

17. The Combination of Geometrical Forms.—Since the mechanic deals with objects of various forms that may be said to represent geometrical solids, we shall endeavor to convey an idea of the way in which the representation of these objects may be simplified by resolving them into their elements, or the parts that combine to produce these forms. We thus have to deal with points, lines, surfaces, and solids; of these four things, one only—the solid—can be represented by any actual thing or be such as to enable us to handle it, for there is no object that does not possess *length*, *breadth*, and *thickness* to a greater or less extent. It consequently follows that the other three forms are entirely imaginary. The student that can most readily conceive or imagine their existence in this way will most readily comprehend the principles involved in projection drawing and pattern drafting.

18. A Test of the Student's Imagination.—The following illustration will show how a solid may be resolved into the simplest of its elements and still retain its definite and characteristic form. We will suppose a sheet-metal box to be made in the form of a cube, each edge being 1 inch long. The six square pieces of sheet metal that make the sides of this box are to be lightly soldered together, "edge and edge." This box represents a geometrical solid, although it is by no means a solid considered in a physical, or practical, sense. It is, in fact, popularly spoken of as being "hollow"; but we could very readily convert it into a solid by filling it with molten solder. It represents, however, as it is, a geometrical solid, and as such we will consider the parts of which it is made up, without paying any attention in this elementary part of the subject to the material of which it is composed. We are now dealing with *forms* only, and until these principles are fixed in the student's mind no attention will be paid to other details.

19. We first look for the surfaces of this solid, of which we find six. The student may have some difficulty in understanding that the surfaces of this cube, which he can apparently distinguish by the sense of touch, exist only in an imaginary way. We refer him, therefore, to our definitions for the explanation of this seeming paradox. A surface, we have been told, has length and breadth, while a solid has these and also a third property—thickness. Now, as we have said before, we cannot consider the material of which the cube is composed, but if we handle it we will be obliged to refer to the metal of which the sides of the cube are made. These metal sides have thickness; it may be only a few thousandths of an inch, but still it *is* thickness, and consequently does not come within our definition of a surface. A surface may be compared to a shadow, which can be distinguished by its outlines or shape, but, as every one knows, is absolutely without thickness. It is in this sense that we refer to the sides of the cube as its *surfaces*. We find also that each of these six surfaces is bounded, or defined, by four edges or outlines, the lines in turn being terminated at the corners of the cube by points, of which there are eight. It will now be seen that these eight points, situated at the extremities of the lines, or edges, of the cube, perfectly define the shape and size of the geometrical solid.

If we could imagine the metal sides of the cube to disappear entirely, leaving only the points at the corners, we would still have as perfect a representation of the size and shape of the cube in our minds as if it had an actual existence and could be seen by the eye. For these imaginary points could then very easily be imagined as being connected by lines, and we could then "see" the surfaces, and finally the solid, existing in the same imaginary way. It will thus be comparatively easy for us to transfer a representation of this solid to our drawing, since we can use the points as the markings on the rule were used in connection with the lines of sight in a previous illustration.

20. The Imagination a Valuable Assistant to the Draftsman.—An object, or solid, of any conceivable shape may thus be resolved into its elementary parts or points. The drawing of the object, then, will consist simply of locating the positions of these points on the drawing. We may have drawings to make that will require the location of a hundred or more of these points, depending entirely on the form or shape of the object we are dealing with, but the principles are in all cases the same.

If the student, after resolving an object in this imaginary way, will carefully study or imagine the proper location of these points in their relation to the object itself, defining their positions on the drawing one at a time, much that may appear complicated at first sight will resolve itself into very simple and comparatively elementary work. Complicated work is usually nothing more or less than the aggregation of a number of simple operations that appear complicated only because they are combined. *There is no field of work to which the latter statement is more applicable than to that of the draftsman.*

It is in the "imaginary" way thus described that the student is directed to picture to himself each figure as presented to him for the making of the drawings on the plates. This part of the study is, as will be noticed, almost entirely the work of the imagination; but it should be practiced by the student for the sake of the assistance it will be to him later on.

The operations of projection drawing follow one another in a natural sequence, which we will proceed to trace out in a series of drawing plates. As the student follows these operations, keeping in mind the foregoing principles, he will have no difficulty in making or reading any drawing.

PLATES

21. Seven plates are to be drawn by the student in accordance with the directions given in this section. They are to be of the same size as those drawn for *Geometrical Drawing*, and the same general instructions regarding the preparation of the plates are to be observed; they must be drawn and sent to us for correction in the same manner.

The letter heading for each problem, which has heretofore been placed on the drawing, will be omitted, and the student is required only to designate each plate with the letter heading, or title, that is printed in heavy-faced type, both in this section and on the reduced copies of the plate. For this purpose the *block-letter* alphabet is used.

22. The dimension lines and figures shown in the first three problems of the drawing plate, title: Projections I, are to be especially noticed by the student. They are ordinarily used in all working drawings, and preference is invariably given to a dimension figure, rather than to the scale to which a drawing is made. Dimension figures are not to be placed on the plates, since the object in requiring the student to draw these projections is rather to enable him to gain an idea of their principles than to be able to make a finished working drawing.

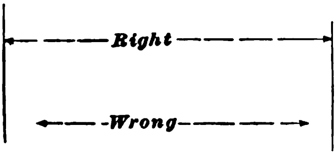
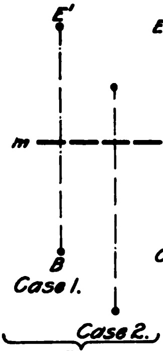


FIG. 3

Dimension and extension lines when used should be light broken lines. Care should be exercised to make the arrowheads as neatly as possible and of a uniform size—not too flaring. They are made with a steel writing pen and their points should touch the extension lines, as illustrated in Fig. 3.

PROJE

VP



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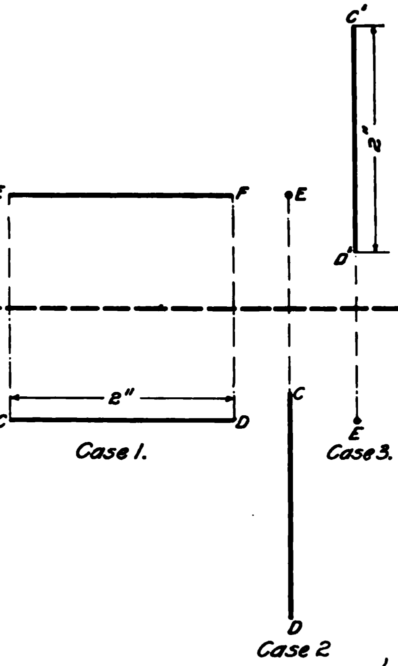

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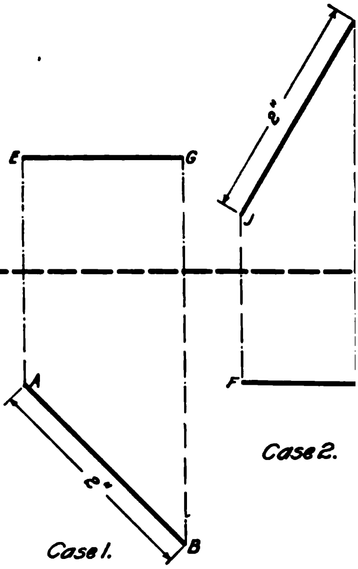
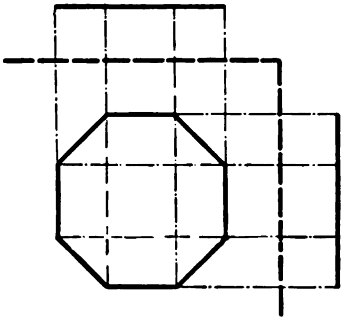
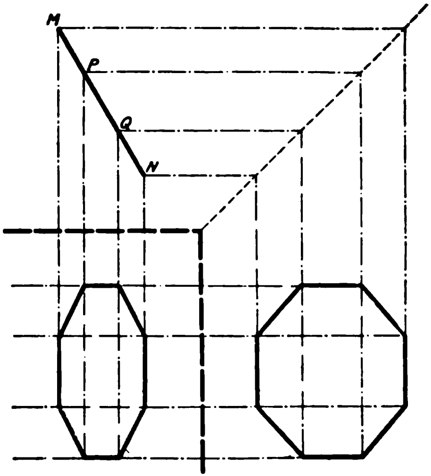


HP



PROBLEM 5.



PROBLEM 6.

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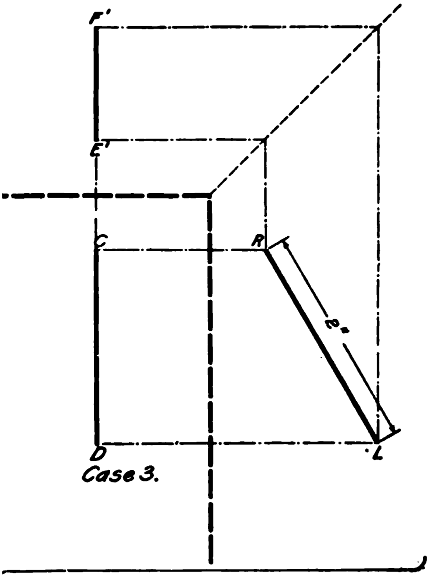
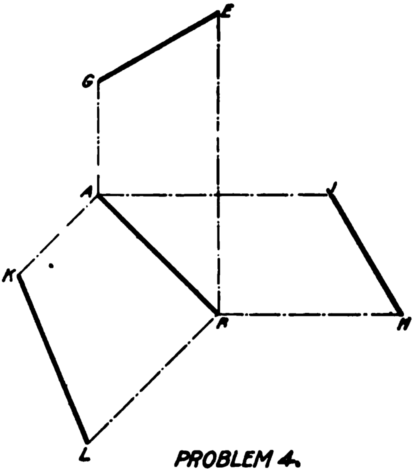


Case 3.

LEM 3.



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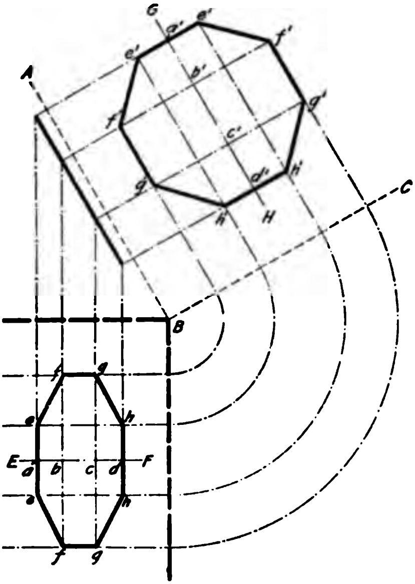

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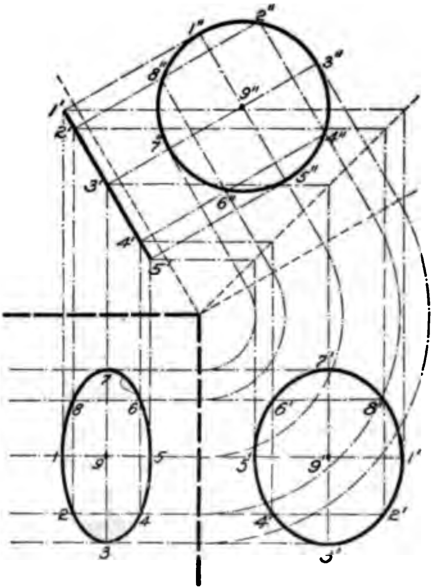

PROBLEM 8.

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JOHN SMITH, CLASS Nº 4529.

DRAWING PLATE, TITLE: PROJECTIONS I

23. General Instructions.—This plate is divided into four equal spaces, and each of these divisions, with the exception of the upper left-hand space, is again divided, by means of a central vertical line, into two equal parts. Use light pencil lines, as they are not to be inked in and are intended only to facilitate the location of the problems. These lines are not shown on the printed copies of the plates. Before attempting to draw any of the problems on the plate, the explanations accompanying each problem should first be carefully read and compared with the reduced copy of the plate and also with the illustrations in this section. The principles of projection drawing will thus be better understood by the student and their application readily made when more difficult drawings are undertaken.

The fundamental laws of projection are contained in the first four problems, and if these are thoroughly mastered by the student, the application of the laws to the remaining problems will be comparatively easy.

24. Why Different Views Are Drawn.—The names of the different views have already been noted; in this plate it is shown how they are distinguished from one another in a drawing. The relation of the different views to one another will also be explained.

Some objects in certain positions may have all their dimensions represented in two views—a plan and an elevation—but generally three views should be drawn. There are, indeed, many cases where views from each of the six sides, as well as sections and views taken from oblique positions, are necessary. It has already been observed that lines are represented in their true length only when at right angles to the lines of sight; consequently, since the position of the object in any set of views is not changed, it is necessary to change the position of the observer in such a manner as to bring the lines of sight where they will be at right angles to the lines in the object, thus enabling the latter to be shown in their true length on the drawing.

The student will readily perceive that in drafting-room work it is of the highest importance that the lines which combine to make up the surfaces of any object should be shown in their *true length*, or at least be presented in such positions that their true lengths may be easily found. Without these true lengths, no measurements can be obtained from which to lay out patterns, a pattern being merely a representation of the surfaces of some solid. It is necessary, therefore, to be prepared to take views of any object from any position; for there are many different forms, or shapes, of solids, and it is necessary to be able to show in its true length any line in the object that may be needed for a pattern.

25. The Base Line.—We shall first consider objects in positions that may be shown in two views. Draw a horizontal line through the central portion of the upper left-hand space on the drawing, as at *m–n* on the plate. In the portion of the space below this line are to be drawn the top plans of each of the two simplest forms, viz., the point and the line. The space above this line is to contain the elevations of the same forms. The line thus drawn is called a **base line** and defines the boundary of the surfaces on which we are to "sight," or, as we shall say hereafter, on which we are to *project* the lines of sight. It is necessary to call the imagination into use again and imagine the paper to be bent up at a right angle on this line.

26. Planes of Projection.—The drawing paper is imagined to be the surface that intercepts the lines of sight; and in the case of a plan, as seen from instruction already given, must be a horizontal surface, while in the case of an elevation, it is imagined to be a vertical surface. The different portions of the drawing on which the projections are made are called **planes of projection,** and are also distinguished by other names that designate the position they are supposed to occupy in intercepting the lines of sight. That portion of the drawing on which the elevation is drawn is called the **vertical plane** of projection; it is represented on this plate by the space above the base line; the portion below the base line is devoted to the plan and is called the **horizontal plane** of projection. We shall, for the sake of brevity, refer to these surfaces by the use of the letters **V P** and **H P**, respectively. Copy these letters into the upper and the lower left-hand portion of their respective spaces on the drawing, using for that purpose a block letter one-half the size of the title letter and leaving a distance of ¼ inch from the border lines of the spaces.

27. Foot of the Line of Sight.—Before proceeding with the drawing of this plate, it is desired to call the attention of the student to the distinction to be observed between the *imaginative* and the *practical* features of this subject. The imaginative feature is employed when a conception of an object is formed by the student in accordance with the instruction in previous articles, and also when the lines of sight are applied in the imaginary way, as in the "sighting" illustrated in Fig. 1. The application of the practical feature in this instance is made when the position of each division of the rule is indicated on the drawing by a pencil mark or dot. The practical part of the work is always accomplished by the aid of pencil and drawing instruments.

The two features are closely associated, since we cannot have a practical representation of any object without first having an idea, or an imaginative conception, either of the object itself or of the means of projection. The practical feature of the work was introduced in the illustration (Fig. 1) when a mark or dot was made on the paper, thereby indicating the position of the point at which the line of sight was intercepted by the plane of projection.

That point on any plane or drawing where a line of sight is intercepted is called the **foot** of the line of sight. When the foot of every line of sight that can be used on the elementary points of any object is thus represented on the drawing by dots, and connecting lines are drawn between such dots, the drawing is completed, and the object is said to be "projected."

28. Projectors.—If but one view of an object were required, the use of the lines of sight as previously explained (representing the imaginative feature) and the drawing of the dots and lines referred to in the previous article (representing the practical feature) would be all that is necessary for the student to understand before proceeding with the work on the drawing board.

Since it has been shown that several views are required, another important practical feature must necessarily be explained. This relates to the connection usually established between the different views of a drawing and the lines that are drawn in a certain manner between corresponding points in each view. These lines are usually not represented in a finished drawing, since they are in the nature of construction lines. They are essential, however, to the work of making the drawing, and it is very important that the student should thoroughly understand the principles by which they are employed. These lines are called **projectors** and may be defined as the *trace* of a line of sight, or the representation of the foot of a line of sight moving in a certain direction. Projectors are used in two ways, which are distinguished from each other for the present by the terms *primary* and *secondary.*

29. Primary Projectors.—This use of projectors is illustrated in Fig. 4, which shows the drawing bent up at a right angle along the base line *m–n.* The point *A* is projected to **H P** by the vertical line of sight *C B*; it is also projected to **V P** by the horizontal line of sight *D E*; *B* and *E* are dots at the foot of each line of sight.

It is assumed that the first position of the observer is at *C*; he then moves, in the direction of the arrow, along the dotted line to *D.* If, in so doing, he continues to sight through the point *A*, it is apparent that a line will be traced from *B* to *F* on **H P** and from *F* to *E* on **V P**. The upright portion of the drawing **V P** is now imagined to be bent backwards until laid flat on the drawing board, and it is evident that *E F B* is represented on the flat surface of the two planes of projection by the straight line *E' F B*. It may therefore be drawn as a straight line by the aid of the triangle and **T** square, the position of the point *A* in each view being determined by the points *B* and *E'* at the extremities of the line. These points (or dots, for points, being entirely imaginary, could not, of course, be actually represented)* *B* and *E'* are the projections of the point *A*—the line drawn between them (*B F E'*) is called a *projector*. When

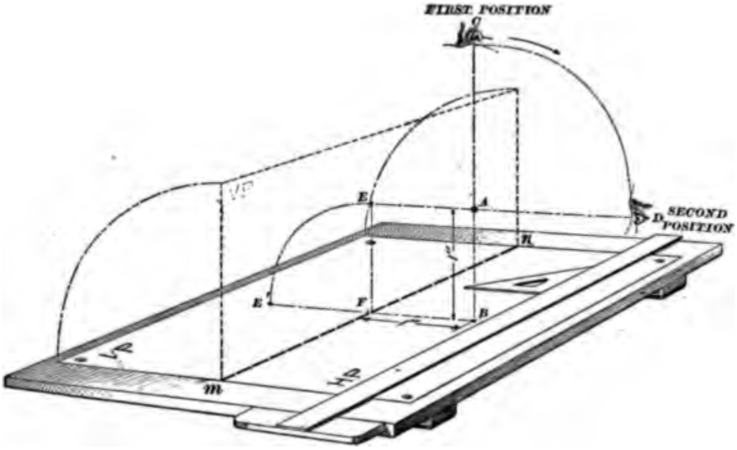

FIG. 4

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* Attention has been called to the fact that points, lines, and surfaces are entirely imaginary geometrical forms. This is true in the sense that the student must consider such forms in the imaginative study of this subject. When their representation on the drawing paper is considered, however, something that can actually be seen by the eye is required. Therefore, when a point is referred to in this section as pertaining to the drawing, it is to be represented by a neat dot in the proper place on the paper. In like manner, a line should be represented by a fine pencil mark drawn between two points marking its extremities. When the line is to serve a special purpose, as explained in this section, it is inked in in a particular manner characteristic of its use, in order that the drawing may be more easily read. A surface, therefore, would be represented by a portion of the drawing bounded by the proper lines and descriptive of the form of surface represented. Remember that accurate work cannot be done unless the pencil points are in good condition. The student should provide himself with a smooth file or piece of fine sandpaper and frequently sharpen the chisel point of the leads in both pencil and compasses, in order that fine sharp lines may be readily drawn.

projectors are used as in this illustration—that is, between two planes that may actually be bent up as shown in Fig. 4—they are said to be used in a *primary* manner.

The *secondary* use of the projector will be shown in connection with Problem 3, Case III.

The practical use of the projector is clearly shown in the following problems. It is a most important factor in the projection, and, as will be seen from instruction soon to follow, is often the first line to be used in a drawing.

PROBLEM 1

30. To project the plan and elevation of an imaginary point.

There are two cases of this problem, representing different positions of the point. Definite instructions are given for drawing the first projection and the student is expected to draw the second projection without further directions.

Case I.—*When the point is located 1 inch from each of the two surfaces* **V P** *and* **H P**.

This position is illustrated in Fig. 4, referred to in Art. **29.**

CONSTRUCTION.—Fix a point *B* (see plate) 1 inch below the base line on the drawing. This point should be $\frac{5}{8}$ inch from the left-hand side of the drawing and is the plan view of the point given in the problem. Bring the **T** square and triangle into position and draw the projector vertically upwards and across the base line. Fix a point at E' on the projector 1 inch above the base line. A projection drawing is thus made, showing two views—a plan and an elevation—of the required point, the position of which is thus definitely established.

Fig. 4 is an illustration of the imaginative feature and the projection drawing just made is a representation of the practical feature of the work—the part actually made by the draftsman. The intimate connection between the two features may be seen if the drawing just made is compared with the illustration in Fig. 4. Similar results are found to have been accomplished in both cases, the method last employed being the only one practicable for actual use. When inking in this drawing, make small round dots to represent the positions of the points, and always ink in projectors as light dot-and-dash lines, as shown on the plate. These dot-and-dash lines should be inked in in a uniform manner, as on the plate, the dashes being about ¼ inch in length and spaced about $\frac{1}{16}$ inch apart, with a light dot between each dash. Measure the distances by the eye and preserve a uniform shade for all projectors, thus giving the drawing a neat appearance. The base line should be represented by a heavy dotted line, as shown at *m–n* in the perspective illustration of Fig. 4.

When making the preliminary drawings, do not attempt to draw dotted lines with the pencil, since this is liable to affect the accuracy of the work. Keep the chisel point of the pencil sharp and draw as fine a line as can be distinctly seen. The contrast between the different lines on the drawing may then be clearly indicated when the work is inked in.

Case II.—*When the point is located 1½ inches from* **V P** *and ½ inch from* **H P**.

The student will fix the location of the point in the plan and elevation on the drawing without further instructions, bearing in mind the fact that distances from **V P** are measured on the plan and distances from **H P** are shown in the elevation. Reference to Fig. 4 explains this statement. Case II should be placed on the drawing about ½ inch to the right of the preceding figure.

PROBLEM 2

31. To project the plan and elevation of an imaginary line, the line being in a right position.

The term *right* position is used in connection with projection drawing as distinguished from the terms *inclined*, or *oblique*, position. The line, therefore, can be either in a horizontal position or in a vertical position and still be designated as in a *right* position. There are three figures for this problem, representing three cases where the line is in a right position and yet represented differently on the drawing. The different positions, the various distances, and the length of the lines for the three cases of this problem are clearly illustrated in the perspective drawings shown in Figs. 5, 6, and 7. Instructions are given for the drawing of Case I on the plate, but the student is expected to be able to make the drawings for Cases II and III without further directions than those contained in the illustrations. Be careful to preserve a distance of ½ inch between the drawings, so that the plate may present a neat appearance when completed.

Case I.—*When the line is parallel to both* **H P** *and* **V P.**

Explanation.—This position of the line is illustrated in Fig. 5, and, as apparent from that figure, the drawing is

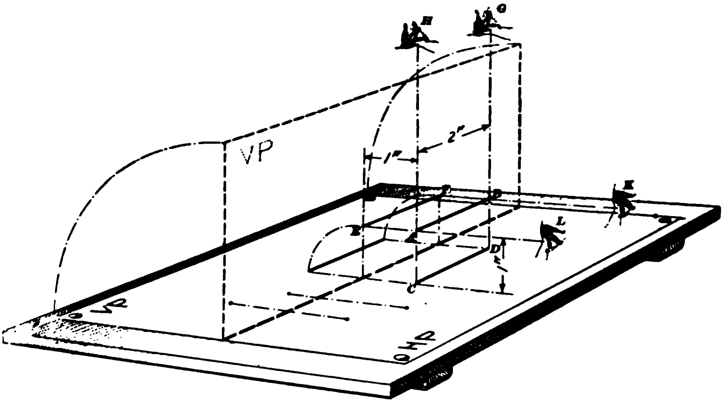

FIG. 5

merely an extension of Problem 1. Each end of the line is treated as a point, projected first to the plan and afterwards to the elevation in precisely the same manner as was the point in Problem 1, the only difference being that there are two points instead of one, for we cannot have a line without establishing at least two points.

This problem also illustrates another principle of projection already referred to, viz., all lines at right angles to the lines of sight in any view are shown in that view in their true length; or, in other words, the lines that are to be made on the drawing to represent the plan and elevation of *A B*, Fig. 5, will be of the same length as *A B* is indicated in the figure, viz., 2 inches. The angles *H A B* and *L A B* are right angles, although shown in perspective in the figure; and, since the lines of sight in any view are always parallel to each other, the angles *G B A* and *K B A* must also be right angles; consequently, as the line *A B* is at right angles to the lines of sight in both views, it must be shown in its true length in both the plan and the elevation on the drawing.

Construction.—To make this drawing on the plate, draw a horizontal line of the given length and the proper distance (i. e., 1 inch) below the base line. This will be the plan of the line *A B*. From each end of this line (*C D* in Fig. 5 and on the plate) draw projectors to the elevation; or, to use the term by which such operations are designated, project the ends of the line *C D* to the elevation. After measuring off the proper height above the base line, draw the horizontal line *E F*, which is the elevation of the line *A B*.

Note.—When a point is projected from one view to another, its projector (a straight line) is drawn from the first view to the view projected, and always at right angles to the base line.

Case II.—*Where the line is in a horizontal position and at right angles to* **V P**.

Explanation.—Fig. 6 illustrates this case. It will be noticed that the plan view of the line does not differ very much from the plan of the line given in Case I, merely that it is represented by a vertical line on the drawing in the plan in place of a horizontal line, as in Case I. The line *A B* is at right angles to the vertical lines of sight *G D* and *H C* in both cases. The line *A B* in this figure is in such a position that the horizontal line of sight *K E* passes through both points *B* and *A*. The projection of these points, therefore, on the elevation is the single point *E* at the foot of the line of sight *K E*. A single line of sight may pass through an unlimited number of points in any view, but the foot of such line of sight is always represented in that view by *one* point on the drawing. The student who is to read that drawing must picture to himself, or imagine, the position of these points as they are supposed to exist in the object of which any drawing is a representation. It is further to assist his imagination that other views are drawn, and by means of which the position of the different points may be definitely located. Thus, in this case, if we were to consider the elevation alone, without paying any attention to the plan, we would say that the point *E* represented merely some other point (imaginary, of course) that could be situated anywhere on the line of sight *K E*. A glance at the plan, however, shows not only where the location is, but how many points are represented. In this case there are two points represented by *E*, one directly in line with the other. The plan also shows how far apart these points are, and from the two views it can be further seen that the line is in a right position perpendicular to **V P**. If it were not, the two points would not be in the same line of sight, and would, consequently, require two positions on the elevation, whereas they are designated by the one position *E*, Fig. 6.

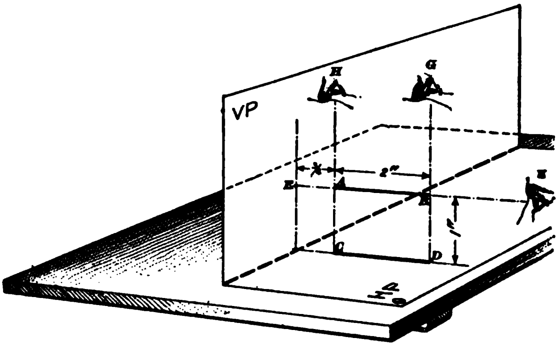

FIG. 6

Construction.—Since the drawings for this case have been shown in the preceding explanation to be similar to those of Case I, no definite instructions need be given. The two drawings differ in position only and are drawn as shown on the plate, the plan being first constructed.

Case III.—*Where the line is in a vertical position.*

This is shown in Fig. 7, each feature of which has already been explained in connection with Figs. 5 and 6. The drawing may, therefore, be made by the student in accordance with the dimensions given in the illustration.

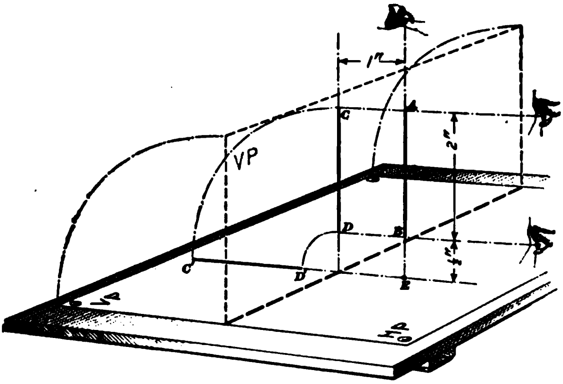

FIG. 7

32. Proof of a Projection Drawing.—The various cases of the foregoing problem represent lines in different positions, and, as in the case of the point in Problem 1, the student will see that these projections definitely represent the position of each line; further, that for each position indicated, but one line can be placed, or can occupy that position. It is recommended that the student prove this assertion as follows: Copy the projections of this problem on another piece of paper and bend the paper at right angles along the base line, as shown in the illustrations; take a piece of small wire of the given length, to represent the lines, and proceed to hold it, in turn, over the drawing for each case, at the same time "sighting," or using the lines of sight, as illustrated. It will be seen that, in order to make the foot of each separate line of sight come to the proper place on the drawing, the wire must be held in the position indicated in the statement of the case.

PROBLEM 8

33. To draw the projections of an imaginary line in a rightly inclined position.

There are three cases of this problem, in all of which the given line is *rightly inclined;* that is, the angle of inclination is such that the true length of the line may be shown in either a plan, a front elevation, or in some elevation that shall be at right angles to the front elevation. The different cases of this problem are presented in perspective views, from which the projection drawings are to be made by the student. They illustrate the principles of foreshortened views.

Case I.—*Where the line is horizontal, but inclined to* **V P** *at an angle of 45°.*

EXPLANATION.—This is shown in Fig. 8, which gives all the dimensions and distances necessary to enable the student to draw the projections on the plate. The student will note the position of each point and carefully observe the instructions

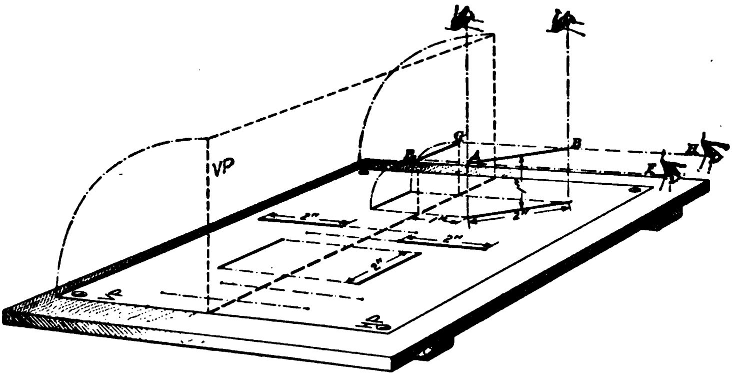

FIG. 8

for making the drawings. Bear in mind that, although the drawing is only that of a single line, careful study must be given to it, for the principles on which these simple projections are made are the same as for any other projection drawing. These principles are shown in a more comprehensive way in simple problems than if an object of complex form were presented, requiring a confusing number of points to define its outline. In Cases I and II of this problem, the plan is first to be drawn and the elevation projected therefrom.

CONSTRUCTION.—Since the line in Case I is in a horizontal position and therefore at right angles to the vertical lines of sight, it will be shown in its full length on the plan. The line is stated to be inclined to **V P** at an angle of 45°; draw the plan, therefore, at that angle to the base line and at such distance below the base line as indicated in Fig. 8 and shown at *A B* on the plate. Project the points *A* and *B* to the elevation, and at the given height above the base line draw a horizontal line between the projectors. This is the elevation of the line shown in the plan, and since its entire length is contained between the horizontal lines of sight *F E* and *H G*, Fig. 8, the line cannot be shown on the elevation as being any longer than the perpendicular distance between the projectors. This distance being less than the actual length of the line, the elevation is, in this case, called a *foreshortened* view of the line. It represents, however, the entire line, and reference to the plan is necessary in order to find its true length.

Case II.—*Where the line is parallel to* **V P** *but inclined to* **H P** *at an angle of 60°.*

CONSTRUCTION.—Fig. 9 shows that the plan is to be represented by the horizontal line *F H*. It also gives the length of the line in the plan, which is a foreshortened view. Therefore, draw *F H* 1 inch long and 1 inch below the base line. Draw the projectors and fix a point at *J* on the projector drawn from *F*, at the proper distance above the base line. This will be one end of the line in the elevation.

With the compasses set to 2 inches (the length of *A B*) and using the point *J*, fixed on the projector drawn from *F*, as a center, describe an arc intersecting the other projector. The point that represents the other end of the line is located at this intersection, and the line may then be drawn. Now bring the **T** square into position and prove by the triangle that the line *J K* is at an angle of 60° with the base line. It will be seen that it would have been possible, after fixing the position of a point at either end, to have drawn the line at once with the 60° triangle. Attention is called to both methods in order to show the student the connection between them.

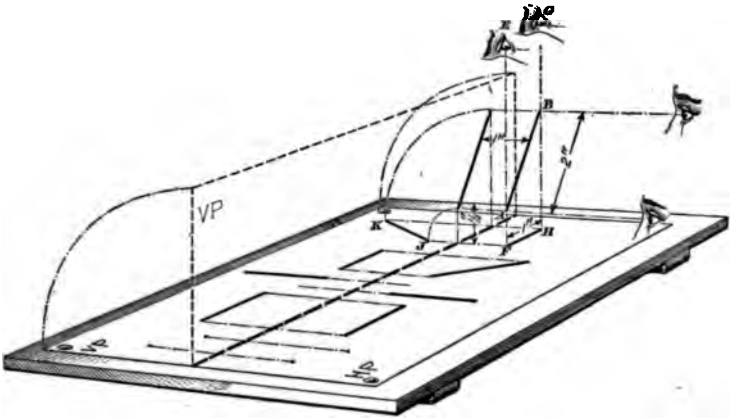

FIG. 9

Case III.—*Where the line is rightly inclined to* **V P** *at an angle of 60° and is also inclined to* **H P**.

EXPLANATION.—In Cases I and II, the line has been in such positions as to enable its full length to have been shown in one of the views drawn on the plate. In this case, a position is illustrated in which the line is shown foreshortened in both of these views. It will therefore require another view to be projected in order that the line may be shown in its true length. Since it is known that the line is rightly inclined, the additional view required will be at right angles to the base line. As another view is to be drawn, so another base line will be required. This base line, being merely the lower boundary of a surface supposed to be in an upright position to receive the lines of sight and at right angles to the surface of the elevation previously drawn, will, consequently, be at right angles with the base line in a drawing that shows only two views—such drawings as have thus far been made.

Fig. 10 contains the given dimensions, etc. for this position of the line. This perspective figure shows the interception

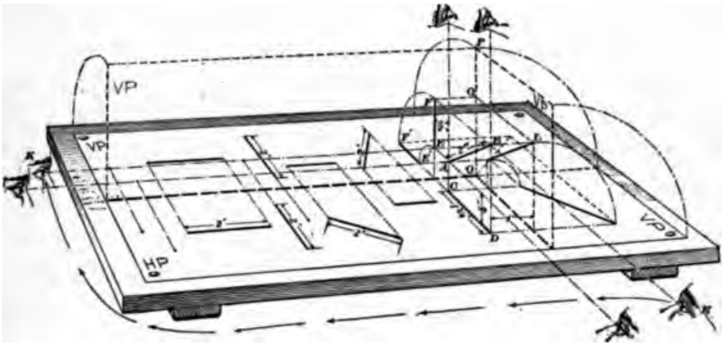

FIG. 10

of the lines of sight from still another direction than has been shown in the preceding illustrations. In the same manner a view may be obtained from any side of an object or at any angle other than a right angle. The method of accomplishing these results by the use of the **T** square and triangle on the flat surface of the drawing board will now be shown, and the illustration of the bent-up surfaces will not be continued beyond this problem. Such illustrations are, however, always implied in a projection drawing, for that part of the work is the imaginative feature previously mentioned, to which the attention of the student will be directed throughout this instruction.

Since the angle of inclination to the elevation is the same in this case as in the plan of Case II, the length shown on the elevation of this projection will be the same as in the plan of that case. In this drawing, the line is, however, in a different position as related to the plan, and from Fig. 10 it will be seen that it must be represented by a vertical line in that view on the drawing.

CONSTRUCTION.—Extend the base line on the plate to the center of the next space, from which point draw a vertical line downwards to the division line; these lines are to be inked in the same as the base line in the first space. In the smaller space thus enclosed on the drawing is to be drawn the plan for this problem, the front elevation occupying the same relative position as before, directly above the plan, while in the space at the right the side elevation will be projected. First, draw the elevation as at $E' F'$ on the plate, fixing the point E' at the specified distance above the base line. As the foreshortened length in the plan is given in Fig. 10 as 1¾ inches, draw the line $C D$ of that length, as shown on the plate, keeping the point C at its proper distance below the base line, as indicated in Fig. 10. The view to represent this line in its true length may next be drawn. It is known that this view must be one in which the line itself is represented at right angles to the lines of sight. There is a choice of two views for this projection—either to the right or to the left side. Having already utilized the space to the left on the plate, the side elevation is, in this case, projected to the right. The method employed in Case II might here be used to project the side elevation, but since it is customary, when a number of elevations are projected from the same plan, to facilitate the operation by drawing between such views lines that are termed *secondary projectors*, an explanation of their use is here presented.

34. Secondary Use of the Projector.—The term *secondary* is not applied in the case of projectors as indicating an unimportant or infrequent application of these lines. The name is used rather to distinguish operations in which similar principles as applied to the imaginative features are differently represented on the drawing in the application of the practical features of the work. In fact, both uses of these lines are required in most drawings. It is therefore essential that the student should become familiar with the various means employed in producing them on the drawing.

35. It is evident, from an inspection of Fig. 10, that the eye of the observer at *E* in moving around along the broken line in the direction of the arrows to take position at *K* would trace a line from *F*, through *O*, to *L*. The part of this line (*F O*) that shows on the front elevation is parallel to the base line of that surface; and, also, that part of the line (*O L*) shown on the side elevation is parallel to the base line of the side elevation. It is seen that the definition of the projector, as previously given, applies equally to the lines *F O* and *O L*. If the two planes of projection represented by the two upright surfaces could be bent in the same relation to each other as were the plan and the elevation in Fig. 4—i. e., on the line *P Q*, Fig. 10—the use of the projectors in these views would be no different from that already described. It is customary, however, in first-angle projection, to assume that such upright surfaces are always bent downwards and away from the plan; to accomplish this result, the secondary use of the projector is employed. Suppose, now, that the upright surfaces, represented in Fig. 10, were bent backwards until laid flat on the drawing board. Evidently, there would be an appearance presented similar to that shown in Fig. 11, and an open space would be shown on the drawing board included between the angle *P Q P'*. The paper on which the drawing is made is not of an irregular shape, thus to be bent up at will; further, the operations

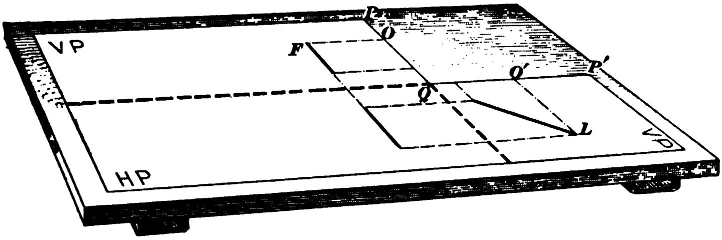

FIG. 11

performed in projection drawing are such that they can be accomplished only on the flat surface of the drawing board.

36. It is found that similar results may be obtained in two ways, both being easily affected by the aid of the drawing instruments. The first is known as the *angular* method, and is thus accomplished: If the projectors *F O* and *O' L*, Fig. 11, or any other corresponding set of projectors, parallel to their respective base lines, are extended until they intersect each other, it is found that *all the intersections are on a diagonal line terminating exactly at the intersection of the base lines.* It is also found that *this diagonal line exactly bisects the outer angle formed by the base lines.* Applying these principles, therefore, to the drawing, bisect the outer angle formed by the base lines on the plate and produce the bisector indefinitely towards the right-hand side of the space. The outer angle formed by the base lines in this case being an angle of 270°, the bisector may be drawn with the 45° triangle, since a line thus drawn will be at an angle of 135° with both base lines.

37. Fig. 12 is a reproduction of the projection drawing from the plate, showing the bisector drawn as previously directed. Draw *F' x* and *E' y* parallel to the base line in the front elevation; from their intersections with the bisector at *x* and *y*, draw *x L* and *y R* parallel to the base line in the side elevation. Project the points *C* and *D* from the plan to the side elevation by the use of primary projectors, as previously described. The side elevation of the line *A B* of Fig. 10, then, is a line drawn between points of intersection of the primary with the secondary projectors, as shown by *R L*, Fig. 12. The lines *F' x*, *x L*, and *E' y*, *y R* are called *secondary* projectors, and are used,

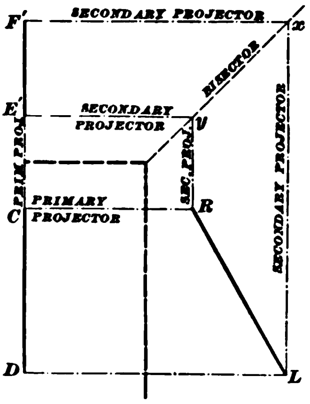

FIG. 12

as in this case, when projecting points between views that are related to one another in the manner shown. Projectors are used in a similar way when, for reasons that will be shown later, the base lines are at an angle other than a right angle. In all cases, the outer angle is bisected as shown in Fig. 12, and the secondary projectors are drawn parallel to their respective base lines.

Note that the front elevation, shown in Fig. 12, is a foreshortened view, and corresponds in length with the perpendicular distance between the secondary projectors in the side elevation.

38. Reading a Drawing.—The ability to read a drawing consists of the intelligent comparison of the different views and is well illustrated in the projections just drawn. The different views—or the different *projections*, as they are called—must never be considered as drawings apart from one another. Each projection is shown to be necessary in order to enable the position of some point or element of the object to be established in the reader's imagination.

PROBLEM 4

39. To draw the projections of an imaginary line in an obliquely inclined position.

The projections of this problem are to be drawn by the student in the next space on the plate, following the instructions here given.

CONSTRUCTION.—Draw the base lines as in the last space used for Problem 3, but place the lines ¾ inch higher on the plate and extend them ½ inch farther to the right in the space. These base lines will be used in the construction of the projections as before, but will not be inked in on this drawing; they are construction lines only—to be erased from the plate after the drawing is completed. It has been shown that base lines are necessary for determining the position of the different points on a drawing and are essential in establishing the first few points in any projection; but as the drawing progresses and other lines are produced, any right line in a view—i. e., a line at right angles to the lines of sight—may be used as a base from which to establish the position of points in a drawing.

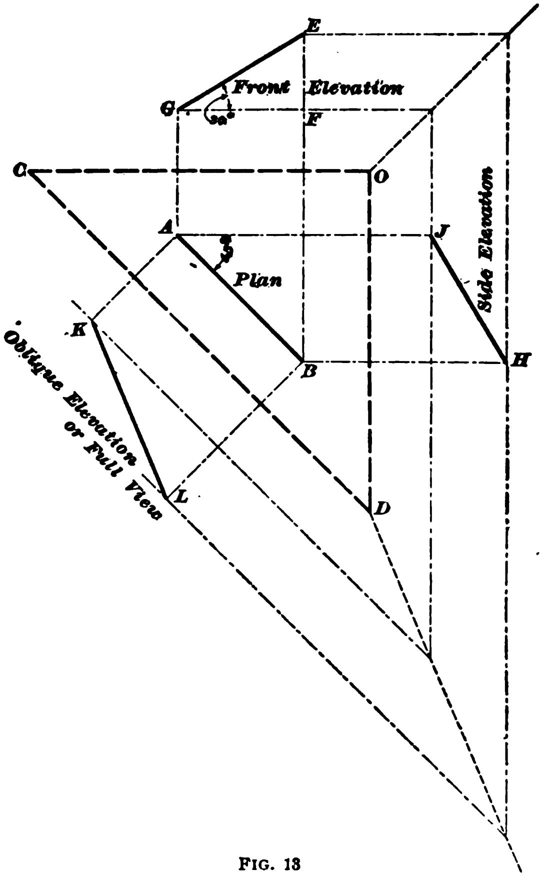

FIG. 13

Represent a foreshortened view of this line in the plan by a line 1½ inches long, drawn at an angle of 45° with the base line of the front elevation, as shown at *A B*, Fig. 13; draw the front elevation (also a foreshortened view) at an angle of 30° with the base line. Draw the line *A B* in the plan and *G E* in the front elevation in such positions that the end of either line nearest to the base line shall be ½ inch from that line. Next draw the side elevation as explained in Problem 3, Case III, and it will be seen, when the side elevation is completed, that *H J* is also a foreshortened view, not representing the true length of the line.

An elevation will now be projected in which the line may be shown in its true length. This will be an elevation whose surface is parallel to the line. Draw the base line of this surface ½ inch from the line *A B* on the plan and parallel to that line, as at *C D*, Fig. 13. This figure is an illustration of the projection drawing, showing all the lines used in its construction, certain of which, as already explained, are not to appear in the completed drawing on the plate. Note that the oblique elevation *K L* is projected in the same manner as the side elevation was drawn, the only difference being that the outer angle *O D C*, formed by the base lines, is greater than a right angle, but is treated in the same way. This completes the problem, and in finishing the figure on the plate, the student will ink in only the different views and the primary projectors, erasing all other construction lines.

40. Finding True Lengths by Triangles. — It is possible to find the true lengths of lines from a plan and any elevation showing such lines obliquely inclined by a shorter method than that given in Problem 4. This is accomplished by the use of the right-angled triangle. If such a triangle is constructed, with its base equal to the length of the line shown in the plan and its altitude equal to the vertical height shown in the elevation, *the hypotenuse will be equal to the true length of the line.* This is shown in Fig. 14, in which *A B* is made the same length as *A B*, Fig. 13, and *B C*, Fig. 14, is equal to the vertical height shown in the elevation, i. e., *E F*, Fig. 13. Since *A B C* is a right angle, the hypotenuse *A C*, Fig. 14, is equal to the true length of the line. This statement is of

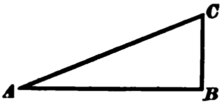

FIG. 14

the greatest importance to the draftsman and should be proved by the student. Construct a triangle on a separate piece of paper and set off the lengths from the drawing with the dividers; afterwards compare the length of the hypotenuse with the length of the line shown in the oblique elevation, or full view. This is an illustration of a principle of much use in later problems, and one on which certain important principles of patterncutting depend.

41. All Projections Depend on Similar Principles.—There is no conceivable position of a line that may not be shown or its true length not be ascertained by the application of the principles contained in the foregoing simple problems. Lines have been used to illustrate these problems drawn at such angles as were conveniently made with the T square and the 45° or 60° triangles, but any angle or any position could as well have been represented, since the principles are in any and all cases the same. We will now proceed with the representation of flat, or plane, surfaces.

42. Planes, or Plane Surfaces.—All drawings made to represent surfaces are composed of lines that bound, or limit, their borders, or sides. These drawings, therefore, will differ from those of the foregoing problems only in the fact that they are the representation of lines shown in their relation to one another. There are, however, certain principles relating to flat, or plane, surfaces that must be borne in mind, since they influence this relation of the different lines in a drawing.

43. That the student may have a thorough knowledge of the principles employed in the representation of surfaces, it is essential that he first have a clear conception of what a plane is. A plane surface, as has been stated, has only an imaginary existence, being bounded, or enclosed, by imaginary lines; this surface may be in any conceivable position, but is always a flat surface. If viewed from a certain direction—viz., as if "on edge"—it would be represented by a single straight line. If the student can imagine a plane surface indefinitely extended in every direction beyond the boundary lines of the figure, he will have a very good conception of a plane; any number of points or lines, the positions of which are anywhere on this surface thus extended, are said to be "in the same plane" in relation to one another.

44. To illustrate: Suppose two flat-top tables of the same height are on the floor of a room perfectly level and of indefinite extent. Here is a practical representation of two planes, both of them in a horizontal position; one plane is represented by the floor, while the other plane is parallel to the first and "passes through" the tops of the tables. The surfaces represented by the tops of the tables are said to be "in the same plane." The tables may be placed some distance apart, yet the straight edge of a ruler laid across their tops would exactly coincide with the upper surfaces of both tables and would remain in contact at all points for every position of the ruler.

The plane surface represented by the top of one table is said to be "in the same plane" as the corresponding surface of the other table. The same could be said with reference to any other surfaces answering the same test. Any number of flat surfaces are said to be in, or to "lie in," the same plane with one another, and the same is true of any lines or points used to define any surface or position in that plane.

The planes in the foregoing illustration of the floor and tables are horizontal planes, but may be imagined in any position, vertical or inclined, needed for the projections of a drawing.

45. How the Position of a Plane Is Determined. Since any two points determine the position of a line, so any three points not in the same straight line determine the position of a plane. To illustrate: Take a square piece of cardboard, thick enough to remain flat, and push pins of equal length through each of the four corners so that they will resemble the legs of a chair. The object will stand firmly when placed on a level surface with the points of the pins down, for the reason that all the points represented by the ends of the pins are in the same plane. If one of the pins is withdrawn and a shorter one inserted in its place, the cardboard will not be stable when placed as before, and can be "rocked," for the point at the extremity of the short pin is not in the same plane with the other three. Two planes are thus defined—one determined by points at the extremities of the three long pins and the other by points at the ends of the short and the two adjacent pins. Both of these planes may be imagined as extended indefinitely, one plane being inclined to and intersecting the other.

Again, a flat sheet of metal may be supposed to represent a plane surface. All points that may be located on this sheet are in the same plane; but if a sheet that is "buckled" is chosen, it is possible to locate some points on the surface of that sheet higher or lower than others, and the points would then be in different planes. The connection between the plane and the plane surface, then, is such that, to be defined as a plane surface, every point on that surface must be in the same plane.

46. In drawing different views for the illustration of the plane surface, we shall first use the octagon, requiring the projection of eight points and the intermediate lines. The use of the word "imaginary" in connection with the statement of the problem will henceforth be discontinued, since it has been clearly shown that all surfaces, as well as other geometrical elements, depend for their existence on the imaginative feature referred to in previous articles. It will be understood, therefore, when any geometrical element is mentioned, that the practical feature is to be employed—the imaginative, of course, being implied.

PROBLEM 5

47. To project three views of an octagonal surface, representing it in a horizontal position.

The three views consist of a plan, front, and side elevation. A perspective view of the surface in the required position is shown in Fig. 15.

Explanation.—All lines used to define this surface in the plan are at right angles to the vertical lines of sight; and since the lines will thus be drawn in their full length in that view, the surface will there be shown in its full dimensions.

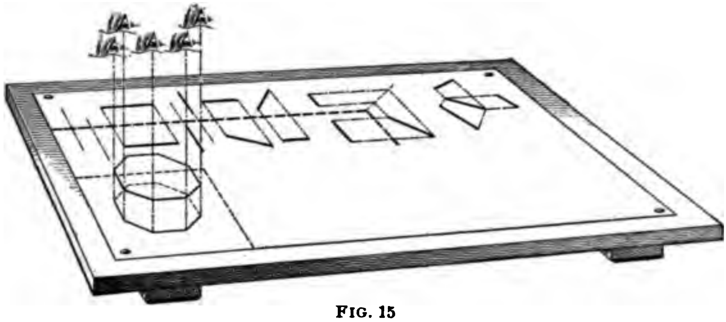

Fig. 15

This principle also applies to any view of a plane surface in which all its lines are at right angles to the lines of sight. The plan of the surface, then, will be a true octagon, and may be drawn on the plate with lines tangent to a circle 1½ inches in diameter, using the **T** square and 45° triangle for that purpose.

Construction.—Draw the base line for the front elevation 3 inches above the lower border of the drawing, and draw the vertical base line (for the side elevation) 2¾ inches from the left-hand border. Describe the circle previously mentioned in such a position that the nearest edges of the octagon will be ½ inch from each base line; the figure may then be completed in the plan. In this and the remaining problems to be drawn on this plate, the right views are to be drawn ½ inch from the base line in all cases. In both elevations, the lines of sight in crossing the surface pass also

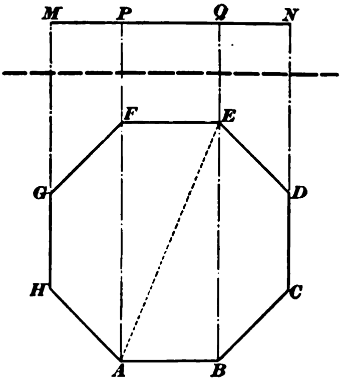

Fig. 16.

through the points in that portion of the surface farthest from the observer; and as the eye of the observer travels from points opposite to *M* and *N*, Fig. 16, in tracing the front elevation, the foot of every line of sight would be projected on a single line on **V P**. The elevation of the surface, therefore, is represented on the drawing by the single straight line *M N*, Fig. 16. Project the front and the side elevations in their proper places, completing the problem.

NOTE.—The single line that constitutes each elevation of this problem represents the eight lines of the octagonal surface shown on the plan. This is shown in Fig. 16, which is a copy of the plan and front elevation on the plate, lettered for convenience of reference. Two of the lines in the plan, *A B* and *F E*, Fig. 16, are shown in their full length by that portion of the line *M N* included between the points *P* and *Q*; since *F E* is directly in line with *A B* in the elevation, as already explained, it is shown by the same line *P Q* used to define *A B*. The line *H A* is shown foreshortened at *M P*; and as *G F* is directly behind *H A*, *M P* represents *G F* also; *Q N* bears the same relation to *B C* and *E D*. The line *G H* is represented in the elevation by the point *M*; *N*, in like manner, represents *D C*. Thus, the line *M N* represents a certain view of the eight lines *A B*, *B C*, etc. to *H A*, and also a view of the surface defined by those lines.

The above is very important and should be carefully read, as it shows the application of principles to Problem 5.

PROBLEM 6

48. To project the views of an octagonal surface that is in a rightly inclined position.

NOTE.—No perspective figure is shown for this problem, and the projections will be made on the plate from the following directions. The same figure is used for this problem as for Problem 5. These drawings are really a continuation of that problem; and since a full view of the surface is shown in the plan of Problem 5, projectors will be drawn from that view, in order to define the plan of this problem.

CONSTRUCTION.—Draw the horizontal base line in the next space on the drawing and at the same distance from the lower edge as the corresponding base line was drawn in the space for Problem 5; draw the vertical base line 1¾ inches from the left side of the space.

The front elevation of this problem will first be drawn. Draw a line inclined to the horizontal base line at an angle of 60° and equal in length to the line shown in the front elevation of Problem 5; the lower end of this line should be ½ inch above the base line, as previously explained. It should be drawn in such a position on the plate that vertical projectors from the ends of the line will pass through the central portion of the horizontal base line. Mark the position of the points indicated by the projectors, as at *M*, *P*, *Q*, and *N*, Fig. 16, and from these points draw four vertical projectors to the plan. Intersect these with horizontal projectors drawn from the plan in Problem 5; draw the connecting lines between corresponding points thus projected; this produces a figure that is the plan of the octagonal surface in the rightly inclined position indicated by the front elevation.

Project the side elevation by the use of secondary projectors, as previously explained. Reference to the copy of this plate will be of assistance to the student during the projection of these views. The completed drawings are there shown and the method of projecting between different views is indicated by projectors partially extended towards the left of the plan of this problem.

49. Basis of Projection.—The plan and side elevation of this surface are foreshortened views. There are, however, two lines in each view shown in their true length; this may be proved by a comparison of the figures with the plan of Problem 5; the other lines in each case are foreshortened.

It does not necessarily follow, however, that any of the lines in a rightly inclined view are shown in their true length. Had the angle of inclination been along the line *A E*, Fig. 16, every line would have been foreshortened; again, in the case of surfaces having curved or irregular outlines, the least angle of inclination in any direction would preclude the possibility of representing, in a foreshortened view, any portion of the outline in its true length.

It will thus be seen that the point is the only geometrical element not subject to change, or variation, in any view. It may therefore be relied on as a basis of projection. The outline of any surface in the different views is determined by *first fixing the location of points at the extremities of the boundary lines of such surface*, afterwards drawing the connecting lines, as in this problem.

PROBLEM 7

50. To project a full view of a surface from a given plan and elevation showing that surface in a rightly inclined position.

A full view of any surface may be projected by assuming a view to be taken at right angles to a line in which the entire surface is represented, for in such a view the lines of sight are at right angles to the outlines of the surface.

CONSTRUCTION.—In the next adjoining space to the right on the plate, copy the plan and the front elevation of Problem 6, placing the projections so that they will occupy the same relative position in the space. To obtain a full view of the surface in this problem, a view must now be assumed at right angles to the line in the elevation; in other words, the elevation must be considered as a plan and a new front elevation projected therefrom. The plan copied from Problem 6 is used as a base plan, secondary projectors being drawn from thence in the manner shown on the plate and described in the following article. The projectors in this case are drawn by the arc method, sometimes more conveniently employed than the angular method previously described.

51. Arc Method of Drawing Secondary Projectors.—Draw a base line, for the projection of the full view, from the intersection of the base lines previously drawn and parallel to the line that represents the surface of the octagon, as *A B* (see plate). At the point of intersection of the base lines (*B*) erect a perpendicular to the oblique base line *A B*, as *B C*, producing it indefinitely towards the right. The positions of all points in the plan are now to be located on this line in the same relative position as they would occupy if projected horizontally to the vertical base line in the drawing. The points are accordingly projected horizontally to the vertical base line *B D*; thence, by using the compasses and describing arcs from a center *B*, located at the intersection of the base lines, they are projected to the line *B C*. The projectors are then continued beyond *B C*, but parallel to *A B*; they are there intersected by primary projectors drawn from corresponding points in the elevation, as shown. Locate the various positions of the corresponding points at the intersections of these projectors and produce the full view of the octagon by drawing the connecting lines.

It will thus be seen that the drawing of secondary projectors by this method involves *first*, projection to the nearest base line; *second*, the describing of arcs from the center shown; and *third*, the continuation of the projectors parallel to the base line of the desired view. If the drawing has been carefully made, it will be found that the surface thus defined is an exact counterpart of the plan in Problem 5, and that the full view projected in this problem is in the same relation to the elevation as the plan in Problem 5 is to the elevation of that problem.

52. Views Necessary for the Projection of the Full View.—When it is desired to project a full view of any surface that is represented in a drawing in an inclined position, it is necessary to have one view that will show all the points of that surface as contained in one line. A projection must also be drawn at right angles to that view, in order that such dimensions of the surface as are at right angles to those in the first view may be shown in their true length.

53. Full Views Sometimes Obtained Without Projection Methods.—A comparison of the views in the projections of the last problem will prove that the vertical primary projectors included within the surface of the octagon shown in the plan are of the same length as the secondary projectors in the view last projected. This knowledge may be used to some advantage in producing a full view without using all the projectors employed in this problem.

Thus, draw a horizontal center line through the plan, as *E F* (see drawing on the plate for Problem 7) and draw primary projectors from the elevation to the full view in the regular way; at a convenient distance draw a line at right angles to, and crossing, these projectors, as *G H*. This line will be the center line for the full view, the points of which may then be located with the dividers in the following manner: Set the dividers to the length *a e* in the plan and set off a corresponding distance at *a' e'* in the full view; in like manner make *b' f'* equal to *b f*, etc., as shown on the plate. Complete the outline of the full view, then, by drawing connecting lines as heretofore.

This method is generally followed in pattern drafting, since it requires less time than to draw full projections as in the construction of the problem, and there is less liability of error.

54. If the student that does not clearly understand the principles by which these projections are made will cut a piece of cardboard to the same size and shape as the plan of Problem 5 and hold it in such positions that the foot of the lines of sight falls on the points designated on the drawings for the different views, he will at once see the correct position of the surface as represented in each view.

55. Surfaces Bounded by Curved Lines.—Surfaces that are defined by curved lines do not present any points from which to make projections. In making such projections, the same principles are employed, however; but it is first necessary to establish a number of points at various positions on the curved lines. The points thus established are then projected in the same way as in the foregoing problems. When, for purposes of projection, points are located on the outline of a curved surface in any view, it should be observed that they are so placed that, when the points thus located are projected to a line that represents an edge view of that surface, each end of that line is defined by the projection of a point.

PROBLEM 8

56. To project views of a plane surface defined by a curved line, the surface being in a rightly inclined position; also, to project a point located on that surface.

NOTE.—Before making the projections of this problem, the lines to remain on the drawing for Problem 5 should be inked in, and, to avoid confusion, all other lines not to be inked in on that figure should be erased; the circle drawn for Problem 5 may then be redrawn for this problem.

CONSTRUCTION.—The surface for the projections of this problem is that of the circle to which the sides of the octagon in Problem 5 are tangent. After describing the circle, the next step is to locate points on its circumference. Do this by first drawing a vertical and a horizontal diameter, and then drawing, with the 45° triangle, two other diameters at right angles to each other, thus locating eight points at equal distances on the circumference. Those points indicated by the horizontal diameter will, when projected to the elevation, define the ends of the line in that view.

Also, locate a point at the center of the circle. The plan and elevation of the surface thus projected is shown in Fig. 17, the points being denoted by numerals. Project these points to the elevation of Problem 5, using lines easily erased, since they are not to appear in that problem when the plate is finished. In the last space on the plate draw horizontal and vertical base lines in the same corresponding position as in the space for Problem 6. The line that represents the front elevation of the circular plane surface is then drawn in the same position as in the elevation in Problem 6; with the dividers, locate points thereon in the same position as the points projected from the circle to the front elevation in Problem 5, as shown in Fig. 17. Project these points vertically to the plan and intersect these projectors with horizontal projectors drawn from the circle in Problem 5.

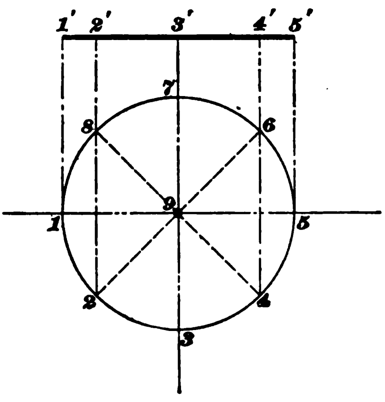

FIG. 17

When projecting points across a drawing—as from the space occupied by Problem 5 to the drawing for this problem — it is not necessary to draw lines the entire distance. By carefully placing the edge of the **T** square on each point in turn, corresponding lines may be drawn across the plan in this problem. This saves erasing unnecessary lines, but care must be taken, when making projections in this way, to observe that points thus located are at the intersections of projectors drawn from points in corresponding positions in each of the views. Find the location of each point thus projected and through these points trace the curve that represents the foreshortened view of the circle, using the irregular curve for this purpose. Thus a plan and a front elevation of the circular plane surface is drawn, in which the surface is represented in a rightly inclined position.

Project the side elevation by the angular method of secondary projectors, as previously explained, and designate the point in both views by a small dot at the center of the surface. Finally project the full view by the arc method, as in Problem 7. In this problem a good test of accuracy is afforded if, after the nine points have been projected to the full view, a circle with a radius of ¾ inch, described from the central point, passes through the other eight points.

57. Importance of Accuracy.—Next to a knowledge of the principles of projection, neatness and accuracy are the prime requisites in a drawing. The student should

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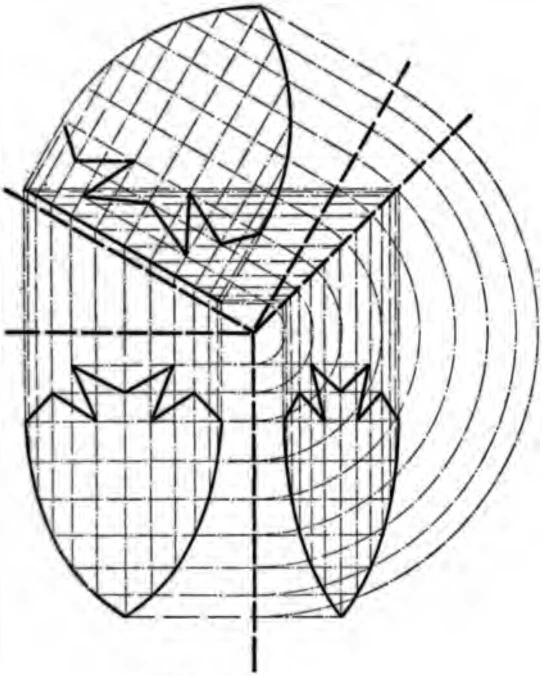

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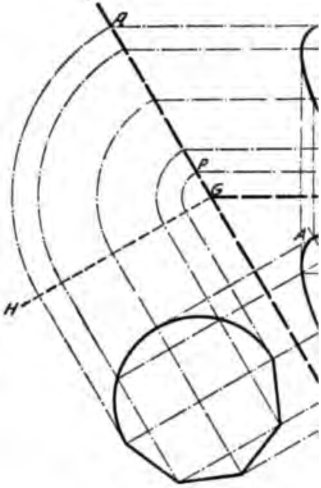
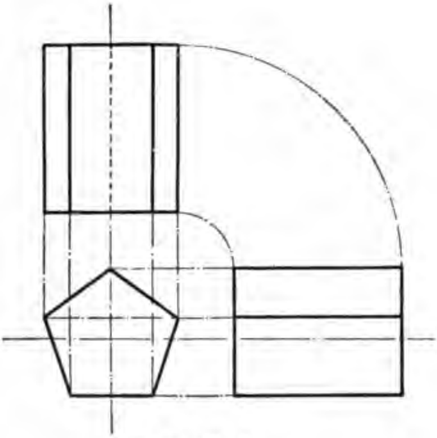
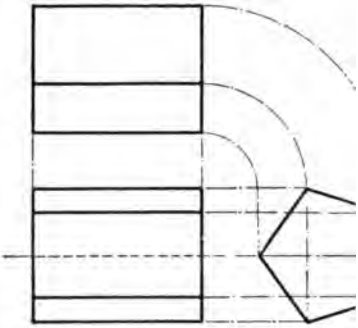




PROBLEM 12.
Case 1.



PROBLEM 12.
Case 2.

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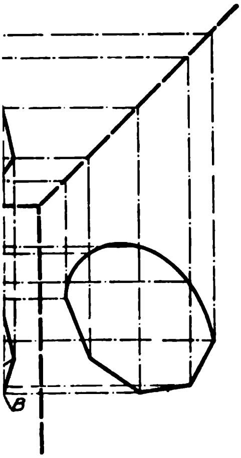

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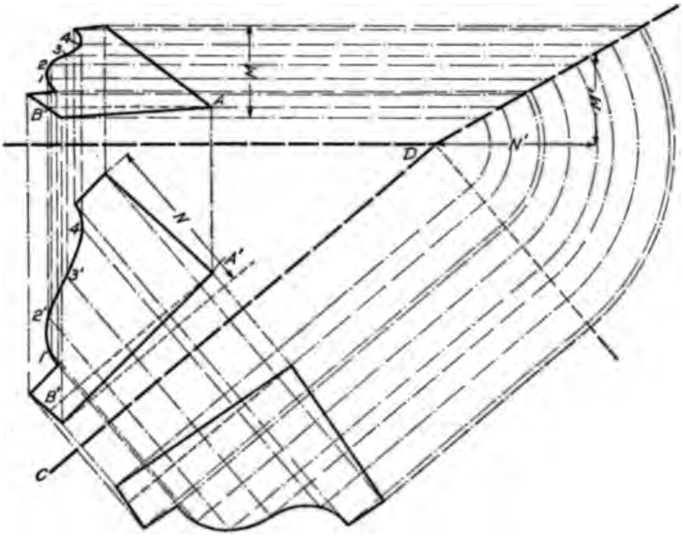
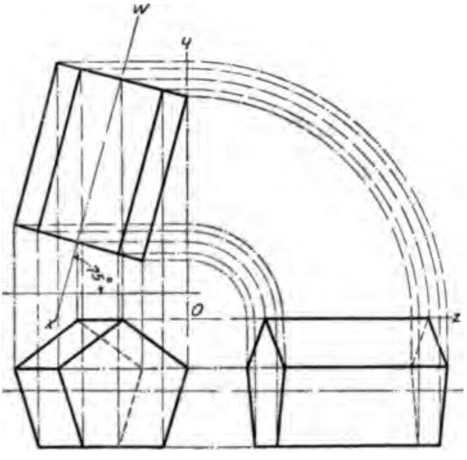
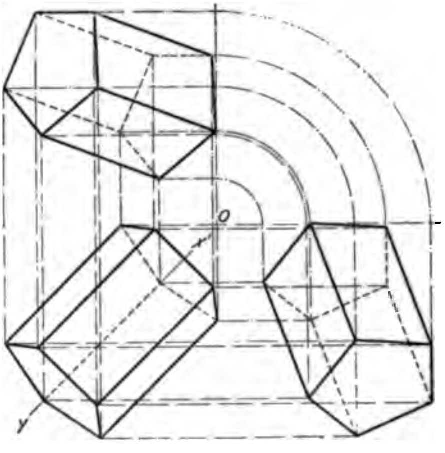


PROBLEM 11.



PROBLEM 12.
Case 3.



PROBLEM 12.
Case 4.

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served.

carefully observe that, when the points determined by the intersection of lines are used as centers for arcs or circles, the needle point of the compasses should be placed exactly on that position; again, drawing three or more lines that shall intersect at the same point is very commonly required in projection drawing and in pattern drafting; this is not an easy thing to do accurately unless carefully practiced by the student. It is needless to state that unless the work is accurately done it is of no value.

When putting in the figures for the dimensions on drawings, care should be observed that they are placed on those views in which the lines and surfaces are shown in their true length. Do not designate a foreshortened view of a line or surface by a dimension figure, when another view is given in which the true length is shown. Again, do not repeat the same dimension on different views of the same drawing; thus, in Problem 2, Case I, the length of the line is given as 2 inches in the plan, and it is obviously unnecessary to give the same dimension in the front elevation.

The student may ink in all the problems on the plates, but the letters used to describe the different positions and lines are not placed on the drawing. The date, name, and class letter and number are inscribed as in the plates of *Geometrical Drawing*.

DRAWING PLATE, TITLE: PROJECTIONS II

58. The problems for this and the succeeding plates should be practiced on other paper and then copied on the drawing that is to be sent in for correction. The student can thus judge better as to the relative position the figures should occupy and the completed plates will present a neat appearance. In making the projections on this and the succeeding plates, the views may be assumed to be ¼ inch from their respective base lines, as this will enable the projections to be kept in closer proximity. The base lines are not to be inked in on this or the following plates. Divide this plate by a central horizontal line; the part of the drawing above this line is divided into three, and the part below the line into four, equal spaces.

PROBLEM 9

59. To project a side elevation and a full view of a rightly inclined plane surface defined by an irregular outline.

This is a problem in which the student has an opportunity to use, in a practical way, the knowledge of projection thus far gained. The surface to be projected is shown in a rightly inclined position in Fig. 18, which is a foreshortened view of the surface. This figure is to be copied, in the size indicated by the dimension figures, into such a position in the upper left-hand space on the drawing that the projections when completed will occupy about the center of the space.

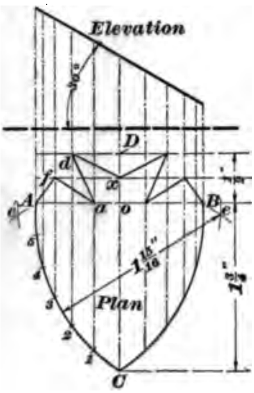


FIG. 18

CONSTRUCTION.—First draw the horizontal line $A B$ 1¾ inches long and bisect it at o by the vertical line $C D$. Make $o D$ ½ inch long and bisect it at x, as shown, making $o C$ 1¾ inches long. Set the compasses at a radius of $1\frac{15}{16}$ inches, and with A and C, respectively, as centers, describe arcs intersecting at e; with the same radius and with B and C as centers, describe arcs similarly intersecting at e'. From these centers (e and e') describe the arcs $A C$ and $B C$, thus producing the curved outline of the lower portion of the plan. Next, divide these arcs, by spacing, into six equal parts, thus locating the points *1*, *2*, *3*, *4*, and *5*; from these points draw vertical lines, as shown in Fig. 18. Complete the upper outline of the surface as represented in the figure; thus, locate a at the intersection of the vertical from *1* with a horizontal from A; d, at the intersection of the vertical from *2* with a horizontal from *D*; *f*, in like manner, at the intersection of a vertical from *3* with a horizontal from *x*.

The points at the extremities of these lines and those located on the curved outline are now to be treated as in former problems and the projections made in the usual way.

CAUTION.—When making the projections for this problem, the student must observe the precautions given in regard to the taking of the same corresponding points in each view. Project the side elevation first; it may be desirable for the student to ink in that figure, in order to avoid the confusion arising from a number of lines crossing one another on the drawing. Use the angular method for the secondary projectors in projecting the side elevation and the arc method for the full view. Since it is often necessary, when developing patterns, to draw several views over one another in this way, the student should accustom himself to drawings that have a complicated appearance from this cause, and should learn to follow each set of projectors as readily as though they were in separate drawings. During the construction of this projection, it will be noticed that the base line for the full view, in order to be drawn from the intersection of the other base lines, will fall below the front elevation of the surface. This is unimportant, however, since its purpose is the same, and the result is merely that of a slight appearance of crowding on the drawing.

PROBLEM 10

60. To project views of a plane surface in an obliquely inclined position.

EXPLANATION.—A full view of the surface to be projected in this problem is shown in Fig. 19, the dimension figures giving the size in which it is to be drawn by the student. The upper portion of this surface is defined by a semicircle, the lower by one-half of an octagon. The purpose in selecting a surface of this outline is to give the student some practice in the projection of both straight and curved outline surfaces.

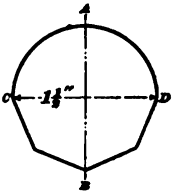
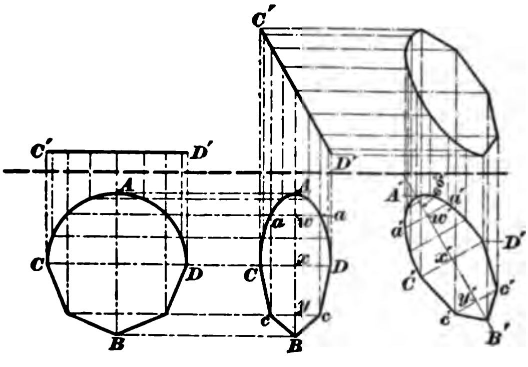


FIG. 19

It has been shown that, before an inclined view of a surface was projected, a right view—i. e., a right plan and elevation, as in Problem 5—has first been drawn. These views alone are projected in drawings of simple or plain objects, it being obviously unnecessary to show any object in a working drawing in a position not commonly occupied. But owing to the different shapes of objects, variously outlined surfaces are presented in a diversity of positions, and it is essential that the student should be capable of projecting any surface into any conceivable position and of drawing a full view from such a projection.

CONSTRUCTION.—The method of drawing oblique views of surfaces is shown in detail at (*a*), (*b*), and (*c*), Fig. 20, the



(*a*) (*b*) (*c*)

FIG. 20

projections at (*c*) being the ones required for the plan and elevation of this problem. Lay a separate piece of paper over the drawing of Problem 9 on the plate, and reproduce thereon the projections shown at (*a*) and (*b*), Fig. 20, in accordance with principles already explained. Next, draw the plan and elevation at (*c*) in their proper places on the plate. The drawing shown at (*a*) may be seen to be similar to that of Problem 5 of the preceding plate; (*b*) is projected directly from (*a*), in the same manner as Problem 6, the angle of inclination being 60°. The plan of this surface in (*b*) is then copied at (*c*) in such a position that the center line $A B$ makes an angle of 60° with the base line of **V P**. This is accomplished by first drawing the center line $A' B'$ at the given angle in (*c*), noting thereon the position of the points w', x', and y'; draw perpendiculars through these points, and make $w' a'$ in (*c*) equal to $w a$ in (*b*), $x' D'$ equal to $x D$, etc. The outline of the plan at (*c*), therefore, is exactly the same as it is shown in (*b*), the only difference between the two views being the fact that the line $A' B'$ in (*c*) is inclined to **V P**, while in (*b*) it is perpendicular to that plane.

Let us consider what changes have here been represented. Cut a piece of cardboard to the outline and size shown in Fig. 19 and compare it with the different positions in the drawings just made. It will be seen that the cardboard must be held in a horizontal position to coincide with the drawing at (*a*); to represent the drawing at (*b*), the point C must be raised until the line $C D$ is at the angle of 60° with **H P**. The plan at (*b*) is, therefore, a foreshortened view of the surface, although its elevation may still be represented by the single line $C' D'$. Now turn the cardboard to the position indicated in (*c*), that is, so that the line $A' B'$ makes an angle of 60° with **V P**.

It will be seen that the line $C D$ in its relation to **H P** is not affected by this change, its angle with **H P** remaining as before; *therefore, the vertical distances to be shown in the elevation of* (*c*) *will be the same as in the elevation of* (*b*), and may be projected directly to (*c*) from (*b*), as shown in Fig. 20. Draw horizontal projectors from the elevation at (*b*) to the elevation in (*c*), intersecting them, in the manner shown, by primary projectors drawn vertically upwards from the points in the plan at (*c*). Trace the outline of the surface thus indicated through the intersections of projectors drawn from corresponding points in each view. The projections shown at (*c*) being completed on the plate, the paper on which (*a*) and (*b*) were drawn may now be removed and the side elevation required for the problem projected by the angular method previously described. Three views are thus shown, in all of which the surface is represented as inclined at an oblique angle to the lines of sight; all these views, therefore, are foreshortened. Oblique views may always be drawn in this manner; that is, a right view is first drawn; next, a rightly inclined view is projected, the desired angle being represented in the elevation. The plan thus produced is then redrawn for the oblique view and its elevation projected as in this problem.

61. Position of Full Views: How Determined. To project a full view of this surface it is first necessary to determine whether any of the lines or distances in any of the views are shown in their true length, but without having recourse to the projections made on the separate paper, since projection methods are to be used. This may be done by comparing the relative position of any two points in the outline of the surface, as located in the plan and elevation. If it is found that a line drawn between any two of these points in the elevation will be parallel to the base line (and therefore at right angles to the vertical lines of sight), that line will be shown, of course, in its true length on the plan. Any other lines parallel to it will also be shown in their true length. It is found on examination that points in the elevation corresponding to the positions represented by *A* and *B*, Fig. 19, are located on the same horizontal projector; therefore, a line drawn between these points as they are located on the plan will be represented in its true length in that view; and a view projected from these points in the plan by primary projectors drawn at right angles to this line, intersected by secondary projectors from the front elevation (by a modification of the method used in Problem 7), will be a full view of the surface.

Draw the oblique base line in the proper position, i. e., parallel to that line shown in full length in the plan (*A B* on the plate), as above explained, and at such distance away from the plan as directed in the instructions for drawing this plate, producing the line indefinitely towards the upper portion of the drawing, as shown on the plate at *E F*. In this case, the line thus drawn defines the inclination of the surface, since the angle is the same in both plan and elevation, viz., 60°. The full view is projected as follows: Draw the line *G H* at right angles to the base line *E F* and from the intersection of *E F* with the horizontal base line. By the arc method, draw secondary projectors from the elevation, as shown; intersect these projectors by primary projectors drawn from the plan at right angles to *E F*, thus producing the full view of the surface. It will be found that this full view is an exact counterpart of the surface shown in Fig. 19, and should correspond to the preliminary drawing in the plan at (*a*) on the separate paper. Note that the portion of *E F* included between *p* and *q* corresponds in length to that of the rightly inclined front elevation at (*b*), Fig. 20. This may be seen by comparing that portion of the line with the view on the preliminary drawing.*

The student should now be able to recognize any view of a surface in any position; that is, he should be able to tell whether a view represents such a surface in a right position, a rightly inclined position, or an obliquely inclined position; and by the application of the principles illustrated in the foregoing problems and the exercise of a little judgment, he should be able to project any surface into any desired position. Or, being given a surface in a position indicated by a properly projected plan and elevation, he should be able to produce the full view and designate the angle of inclination.

The following problem will serve as a test of his progress. The principles involved have already been presented and the method of application will be readily understood. The angle of inclination in the plan is not the same as is shown in the elevation; both angles are to be determined by projection methods.

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* It should be noted that this is the case only when the angle of inclination is the same in both views.

PROBLEM 11

62. To project the full view of an irregularly outlined surface obliquely inclined.

The projections of this problem are to occupy the upper right-hand space of the drawing plate. The plan and front elevation are shown in Fig. 21 and are reproduced full size on the sheet opposite this page.* The outline represented is frequently used as a "stay," or profile, to which moldings are formed in cornice work. Since the view shown is known to be obliquely inclined, its dimensions are foreshortened, and their true lengths are to be found by projection methods, as follows:

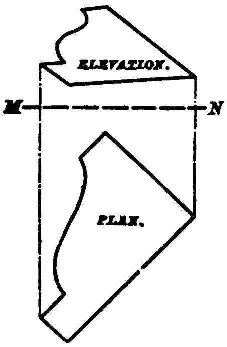

FIG. 21

CONSTRUCTION.—The student should detach the sheet opposite this page and paste the plan and elevation in such a position in the third space on the plate that the base line *M N*, Fig. 21, will be 2½ inches below the top border line and exactly horizontal. Locate a number of points on the curved outline in the elevation, by equally spacing that portion of the figure with the dividers, as shown at *1*, *2*, *3*, and *4* in the drawing for this problem on the plate, and project these points to the plan. To ascertain the angle of inclination in the plan, draw a horizontal line through the widest portion of the figure in the elevation, locating, if possible, one end of the line at an angle of the surface—as the line *A B*. Project this line to the plan at *A' B'*, as explained in Art. **61**; parallel to this line erect the oblique base line *C D*.

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* This sheet is not inserted in the bound volume containing this Paper.

The angle formed by these lines (*A′ B′* and *C D*) with the horizontal base line is the angle of inclination of the surface to **V P**, or that angle shown in the plan. The angle of inclination to **H P**, or that shown in the elevation, is most easily found by constructing a right-angled triangle whose base and altitude are equal to certain distances found in the plan and elevation; that is, the base is equal to the extreme width of the figure in the plan, taken at right angles to the line of inclination in that view (shown by the dimension *N*). The altitude is the vertical height shown in the elevation at *M*. Construct this right-angled triangle on the horizontal base line extended, as shown at *N′ M′*, and locate one end of the base at *D*, the intersection of the base lines. Extend the hypotenuse indefinitely towards the right of the drawing. Next, intersect primary projectors drawn from the plan to the oblique view by secondary projectors drawn from the elevation by the arc method, as shown on the plate. The full view of the surface is then traced through the intersections of these projectors, completing the problem.

63. Projection of Solids.—We now come to the projection of solids, which, as before noted, are merely various combinations of surfaces. Projections of surfaces in a variety of positions having already been made, we shall encounter no new principles in the projection of solids, the surfaces of which are projected in the same manner as has been shown in preceding problems.

Since lines intersect in a point, so surfaces intersect in a line, and in drawing projections of solids it is necessary only to find the true projections in any view, or set of views, of those lines that represent the correct intersections of the adjacent surfaces; this is a comparatively easy thing for the student to do, if he will use proper care and diligence in the application of the principles of the preceding problems.

64. Projection of the Cube Illustrated.—Every solid consists of a number of surfaces, each of which is differently shown when the solid is projected to the various views.

This is due to the fact that the observer is assumed to occupy a different position in each view of the solid thus projected. In certain positions some solids show one or more of their surfaces directly behind another surface of the same size and shape. This would be the case if the projections of the cube, referred to in Art. **18**, were drawn as shown in Fig. 22. When the cube is in a right position—that is, with two surfaces horizontal, and that surface nearest the observer in a side view in such position that the lines defining that surface will be at right angles to the lines of sight, as indicated in Fig. 22—it is evident that the surface parallel to and behind

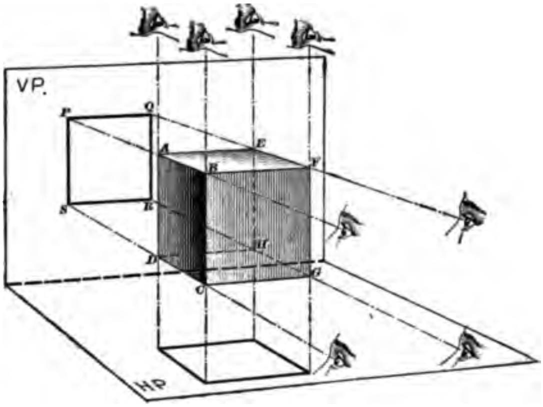


FIG. 22

the front surface will be projected by the same lines of sight as the front surface. Therefore, in such a case, a projection of the front surface of the cube is equivalent to a projection of the entire cube. Each projection of the cube in the plan, front, and side elevations is a square, the sides of which are 1 inch long, while the views are arranged, as shown in Fig. 23, in such a way as to appear related to one another.

65. In "reading" the projections shown in Fig. 23, we merely compare the surfaces of the cube as they are shown in the different projections. Thus, the surface *A B C D*, Fig. 22, is represented in the plan of Fig. 23

by the line *A B*, and in the front elevation by the line *B C*, while a full view of that surface is shown in the side elevation; so, in like manner, the position of each surface of the cube may be determined. Note that, in each full view, two surfaces are represented; thus, in the front elevation of Fig. 22, the surfaces *B F G C* and *A E H D* are projected on **V P** as one surface at *P Q R S*. It is thus shown that surfaces in their relation to one another, when combined in one view, as in the projection of a solid, partake of the same principle that has been shown in its application to points and lines, viz., *surfaces whose outlines are contained in the same lines of sight in any view are projected in that view as one surface.*

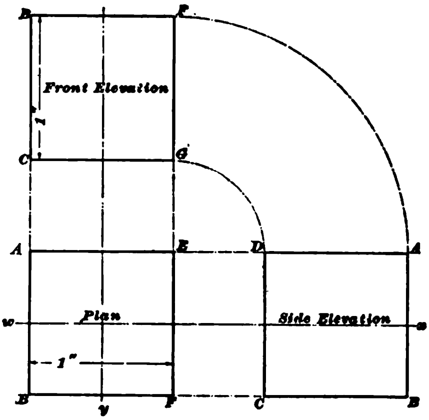


FIG. 23

The difference in position of the several views due to the use of first- and third-angle projection may easily be illustrated in connection with Fig. 23. Here the cube is represented as having been projected in the first angle. If the student will now turn the book "upside down," he will see the three views of the cube in the relative positions they would occupy were the drawing made by the third-angle method. It will be seen that the effect is merely that of rendering the plan uppermost on the drawing. The front elevation then is seen below the plan, while the side elevation, as in first-angle projection, is at the side of the plan. In the case of a simple solid like the cube, the use of different angles of projection has but slight effect, but when a more complicated solid is represented on the drawing, it becomes necessary for the one that would "read" the drawing to know which angle has been used by the draftsman.

Note that in Fig. 23 the secondary projectors are described from *E* as a center, the lines of the cube being used as base lines, as mentioned in Art. **39.** When this short method is adopted in the case of secondary projectors, the center from which they are described must be located at that intersection of the primary projectors nearest the two views between which secondary projectors are drawn. A further illustration of this will be given in connection with a later drawing.

66. Hidden Surfaces: How Indicated.—When the form of a solid is such that in any view a smaller surface is hidden by a larger, the smaller surface is not shown in that view, although frequently its outline may be defined by dotted lines on the drawing. This applies also to projections in which two or more solids are shown in positions such that some of their surfaces are completely or partially hidden by other surfaces nearer the eye of the observer. Only such surfaces as receive the lines of sight directly from the eye of the observer are shown in a view by full lines, although, as mentioned above, the outline of such other surfaces as it may be desirable to show in a drawing may be indicated by dotted lines.

67. Facility in Reading Drawings Acquired Only by Practice.—The reading of working drawings is, therefore, a comparatively easy matter, if the student will resolve each portion of the object represented into its respective surfaces and look for the various outlines as they are shown in the different projections. If this is found a difficult task, the surfaces may be further resolved into lines and points, whose respective positions may then be located in each view shown. It is not to be expected that the position of every surface in a complicated drawing will be seen by the beginner at a single glance—an expert seldom acquires such proficiency—but as "practice makes perfect," the student may easily accustom himself, by careful study of the various positions of the surfaces composing the solids that are projected in the following problems, to the more or less complicated projections found in the various mechanical and architectural journals, in shop drawings, or in such other projection drawings as are within his reach.

68. The Center Line.—It has been found convenient, when making projections of objects, to make use of a line that is imagined to pass through the central portion of the solid as it is shown in any plan and elevation. Such a line is called a *center line*, and in many projections it is inked in when the drawing is finished, since it frequently affords a convenient means of indicating certain positions of the figure, besides assisting in the location of the several surfaces of the solid in the different views. This line, however, is central only in its relation to the object of which the drawing is a representation, and not in relation to the planes of projection. This may be better understood by considering the center line as the projection of an imaginary surface (or plane) that passes through the central portion of the figure. It is generally represented in those views only in which that imaginary surface can be shown in one line, or, as we have said before, as if "on edge." Thus, in the right view projected in Fig. 23, the lines $w\,x$ and $y\,z$ are center lines, represented on the drawing by the broken-and-double-dotted lines shown in Fig. 23. The practical use of the center line will be illustrated in the succeeding problems by the projection of solids into various positions.

PROBLEM 12

69. To draw the projection of a given solid, several positions being indicated.

The solid for the projections of this problem is shown in perspective in Fig. 24. It is, as the figure shows, a pentagonal prism; that is, a solid whose ends are pentagons and parallel to each other and whose sides are parallelograms. The dimensions are given in Fig. 24; three projections are to be drawn, each showing the solid in a different position; each projection will be complete in a set of views consisting of a plan and front and side elevations. Each set of projections is to occupy one of the remaining spaces on the plate, the prism being shown in the positions indicated by the following cases:

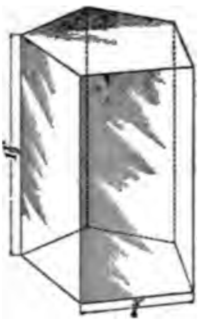


FIG. 24

Case I.—*In an upright position, the side nearest the observer being parallel to* **V P**.

The projections showing this position of the prism may be readily drawn by the student from the explanations and instructions already given. The plan, which is a pentagon with ¾-inch sides, should be drawn first, according to instructions given in *Geometrical Drawing*. Draw the center lines as in Fig. 23 and ink them in, completing the drawing as shown on the plate. All lines that represent the intersection of surfaces—i. e., the edges of the prism—and that intercept the lines of sight directly from the eye of the observer are to be represented by full lines on the drawing. All hidden edges are to be shown by dotted lines.

Case II.—*In a horizontal position, the upper side being parallel to* **H P**, *and the ends of the prism parallel to* **V P** *in the side elevation.*

In this case, which differs from Case I only in the position of the prism, the side elevation should be drawn first. It may here be mentioned that, in drawing different views of objects in right positions, the question as to which view is drawn first is merely a matter of convenience, depending on the form of the object represented. Thus, in Case I the plan is drawn first; in this instance it is, however, more convenient to draw the side elevation first. In this drawing it is only in the plan and the side elevation that the center line can be shown, since an edge view of the plane represented would not be given in the front elevation.

Case III.—*In a rightly inclined position, the angle of inclination of the center line (and consequently of the prism) to* **H P** *being 75°.*

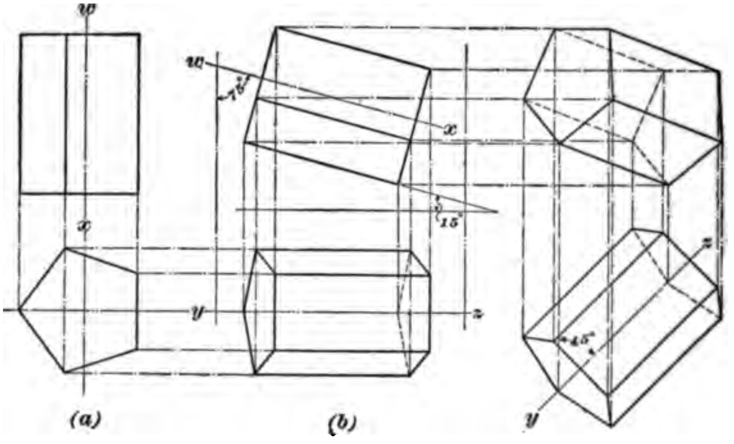
Explanation.—The sides of the prism are in the same relative position to **V P** in the front elevation as in Case I. (See the plate.) The projection of solids to inclined positions is accomplished in the same manner as the projection of surfaces to similar positions in preceding problems. Practical use may be made of the center line in this drawing; this line may be shown in the front elevation, but cannot be continued to the plan from that view, since it is clear that its true position may not be shown there in one line.

Construction.—First draw the line wx in that part of the space devoted to the front elevation and at an angle of 75° to the base line. The line wx is the center line of the front elevation. Next, copy the front elevation of Case I in the same relative position on this line, thus producing the rightly inclined front elevation. Draw primary projectors vertically downwards from all points of this front elevation, and intersect them with horizontal primary projectors drawn from the plan of Case I, in the same manner as the plan of Problem 6 on the preceding plate was produced. Next, draw the outline of the plan by connecting the intersections of projectors that have been drawn from corresponding points in each of the two views. Project the side elevation by means of secondary projectors described from the intersection of the upper and right-hand primary projectors, that is, at O on the plate (see Art. **65**). When drawing secondary projectors for solids whose surfaces do not extend to the outer primary projectors in the adjacent views, note that the primary projector must be produced as at Oy and Oz.

Case IV.—*In an obliquely inclined position, with the center line at an angle of 45° to* **V P** *and 15° to* **H P**, *the upper side being in such a position that a full view will be shown of its upper and lower edges.*

Explanation.—As in Problem 10, where a surface was projected to an obliquely inclined position, so in this problem some preliminary work must be done on another piece of paper. These preliminary projections are shown in Fig. 25, of which (*a*) is the right plan and elevation and (*b*) is the rightly inclined drawing.

Construction.—On a separate piece of paper construct the projections shown at (*a*), Fig. 25; copy the elevation produced at (*a*) in such a position at (*b*) that the angle of the center line *w x* is 15° to **H P**, as required by the conditions of the problem. Next, project the plan in (*b*) from the plan of (*a*), in connection with the elevation of (*b*), as indicated by the primary projectors drawn from these views. Redraw the plan of (*b*) at (*c*) and give its center line *y z* the required angle of 45° to **V P**. Next, produce the elevation in (*c*) by drawing primary projectors from the elevation in (*b*) and intersecting them by primary projectors drawn from corresponding points in the plan in (*c*). These operations are precisely similar to those used in producing the obliquely inclined views of the surface in Problem 10; and if the extra piece of paper is laid over the drawing in such a manner as to leave the lower right-hand space exposed, the



(a) (b) (c)

Fig. 25

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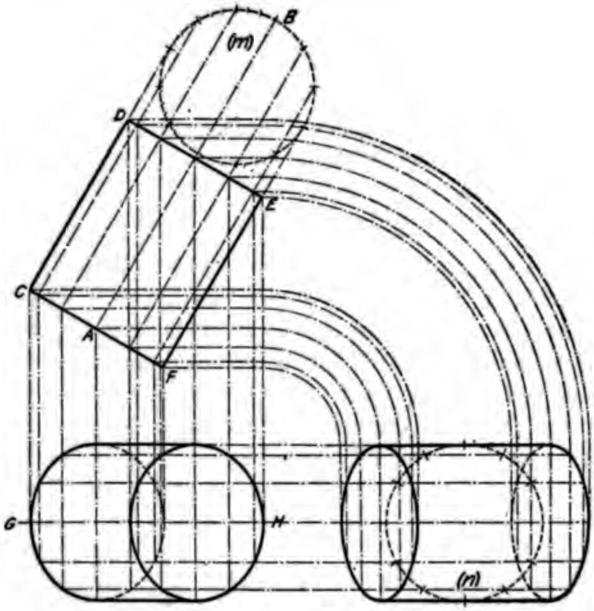
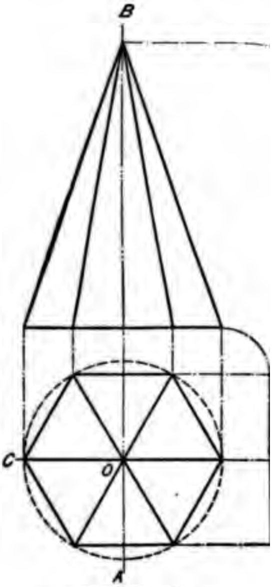
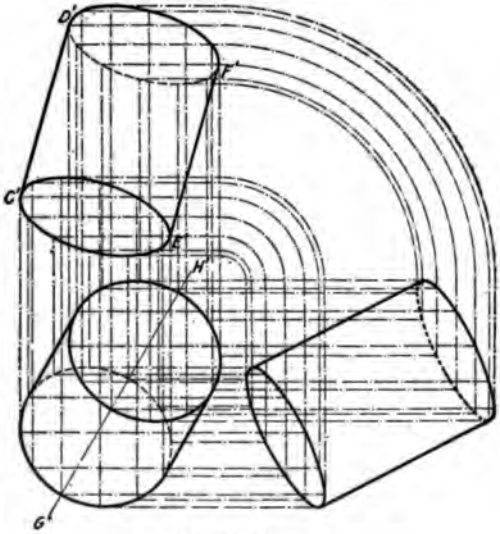


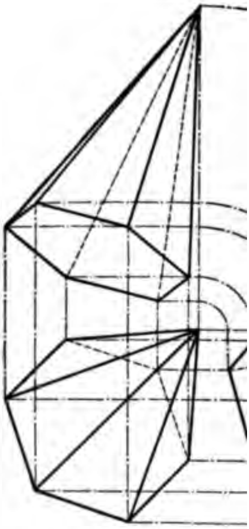
PROBLEM 13.
Case 1.



PROBLEM 14.
Case 1.



PROBLEM 13.
Case 2.



PROBLEM
Case 2.

JUNE 25, 1893

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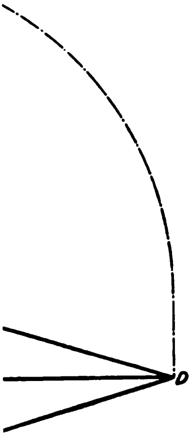
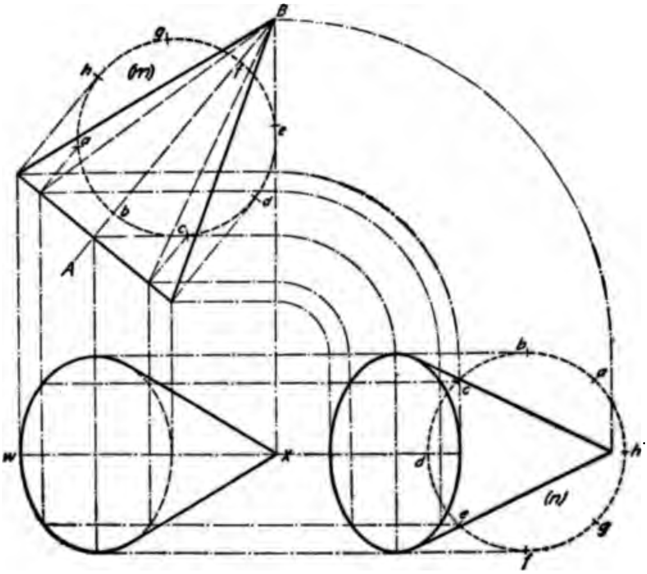
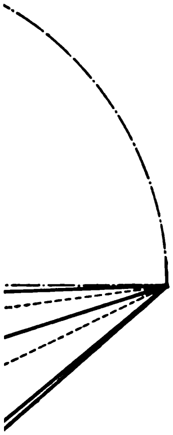
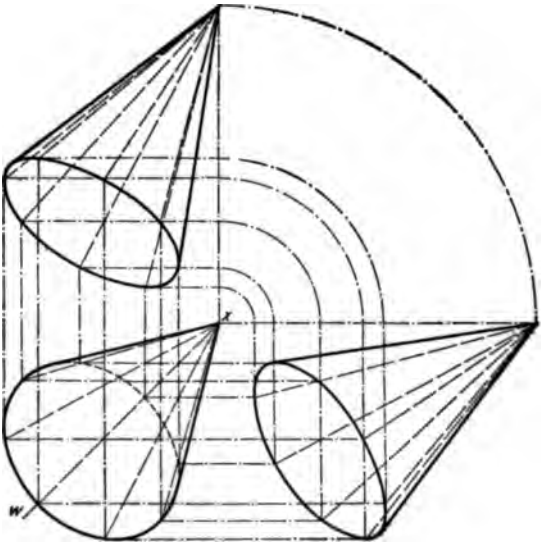




PROBLEM 15.
Case 1.





PROBLEM 15.
Case 2.

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drawings shown in Fig. 25 at (*c*) may be projected directly to their proper position, thus producing the plan and front elevation required. The side elevation is then projected by means of secondary projectors described from the center *O*, as shown on the plate. It is thus shown that oblique views of solids are projected by the same methods as those used in the case of surfaces.

NOTE.—The appearance of this plate will be improved if the base line of the front elevation of this case is placed ⅝ inch higher than in the preceding cases.

70. The Axial Line.—In views such as are projected in this drawing—i. e., obliquely inclined views—the center line, as the representation of the central plane of the figure, can be shown only in the plan; that is, in the position of the solid shown in this case. The position of the solid might be such that the center line would be shown in one of the elevations, or possibly not in any right view. However, it is sometimes represented in drawings merely as a central line, and not as the representation of a central plane; it may in such cases be projected to the other views by means of points located at convenient distances on the line. It is then called an *axial line*, or *line of axis*, and represents the position of the axis of the solid. It is not projected in the cases of the preceding problem, since the figure is not of such a form as to demand the location of an axial line. The axial line is similar in its use to the center line and is represented on drawings by the same kind of a broken-and-dotted line.

In order to avoid confusion, the center and axial lines are usually indicated by small lettering placed conveniently near one end of the lines; thus, "center line" or "axial line."

DRAWING PLATE, TITLE: PROJECTIONS III

71. Three problems are to be drawn on this plate, each of which will require two sets of projections. Like the projections of Problem 12, they consist of a plan and front and side elevations. Each set of projections occupies one space on the drawing, and the plate is divided into six equal spaces by a single horizontal and two vertical lines. The different cases of each problem are drawn in the same vertical division; thus, Case I of Problem 13 occupies the upper left hand space and Case II of the same problem is directly under it. Cases I and II of the other two problems on the plate are to be placed in the same way in the remaining spaces.

PROBLEM 13

72. To draw the projections of a cylinder.

The method of projection used in the case of surfaces having curved outlines has been shown in a preceding problem. Two such surfaces are presented in the ends of the cylinder shown in perspective in Fig. 26, and the figure also gives the dimensions of the solid as it is to be drawn on the plate. Since, with the exception of those at the intersection of the ends, there are no edges formed by the curved sides of the cylinder, there will be no full lines on the drawing except those required to show the ends in their different positions and the outline of the sides. A front elevation of the cylinder in the right position indicated by Fig. 26 is therefore a parallelogram, the length of whose horizontal sides is equal to the diameter of the circular surfaces at the ends of the cylinder—the vertical sides equal in length to the height of the cylinder. Its entire dimensions and its form are indicated in a plan and front elevation, and there is no need of making any further drawings to enable the mechanic to understand the shape of the solid thus represented.

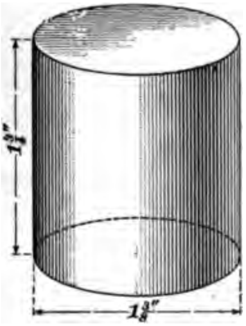

FIG. 26

Case I.—*When the cylinder is rightly inclined.*

EXPLANATION.—In this drawing the cylinder is inclined at an angle of 60° to **H P**. Rightly inclined views of solids are often drawn by a method somewhat shorter than that shown in preceding problems. By this method, temporary views are drawn in convenient positions on the paper and rightly inclined views are projected, as shown in this case. Less space is required for the drawing and a saving of time is effected; the principles involved, as will be seen from the following construction, are identical with those of the preceding problem.

CONSTRUCTION.—First draw the center line in the elevation at the given angle—as *A B* in the drawing on the plate. Describe the circle shown at (*m*), which represents a full view of the end of the cylinder; next, draw the front elevation *C D E F* on the center line *A B* according to the given dimensions. Describe a circle similar to (*m*) at (*n*); this is a temporary view of the end of the cylinder and corresponds to the plan of the prism at (*a*), Fig. 25. Locate a convenient number of points at equal distances on the outline of each full view thus drawn at (*m*) and (*n*). Project the points of (*m*) to the elevation *C D E F*, and thence draw primary projectors vertically downwards; intersect these primary projectors by other primary projectors drawn horizontally from similar points located on the outline of the full view at (*n*). Trace the outline of the plan thus produced through points of intersection corresponding to those on the full views. The temporary full views (*m*) and (*n*) may then be erased from the plate. Project the side elevation by means of secondary projectors described by the arc method, thus completing the drawing.

Case II.—*When the cylinder is obliquely inclined.*

EXPLANATION.—The method of projecting the drawings required for this case is similar to that already given for oblique views of surfaces and solids, and has been fully explained in Art. **60**, and also in connection with Case IV of Problem 12. A right view is first drawn [as the elevation and full view (*m*) of the preceding case]; next, a rightly inclined view. The rightly inclined plan thus drawn is then recopied at the given angle, thus producing the plan of the oblique view; from this plan, in connection with the rightly inclined elevation, the obliquely inclined elevation is projected. The rightly inclined plan and elevation having been drawn in Case I of this problem, the plan there shown may be redrawn for the plan of this case.

CONSTRUCTION.—On a separate piece of paper reproduce the plan and front elevation of Case I and fasten this paper by thumbtacks to the drawing board, towards the left of the space used for this case. Next, redraw the plan of Case I in its proper place on the plate for this case, and in such a position that the line *G H* of Case I forms an angle of 60° with the base line of the front elevation, as shown at *G' H'* on the plate. Then produce the front elevation by drawing primary projectors upwards from the plan and intersecting them by similar projectors drawn horizontally from the rightly inclined elevation on the attached sheet, which may then be removed. Trace curves through the points thus projected and draw the tangential lines, as previously described and as shown at *C' D' F' E'* on the plate. Project the side elevation as in preceding problems, taking special care to project from similar points in each view.

PROBLEM 14

73. To draw the projections of a hexagonal pyramid.

A *pyramid* is a solid whose base is a polygon and whose sides are triangles uniting at a common point called the *vertex*. The pyramid for the projections of this problem is shown in Fig. 27, where its dimensions are clearly indicated. Since these drawings are very easy and are constructed in a manner similar to those of preceding problems, definite instructions are omitted, and the student is expected to be able to complete the drawings by the aid of the brief explanations that follow.

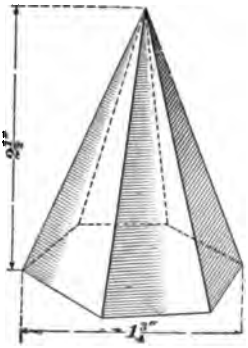

FIG. 27

Case I.—*When the pyramid is in a right position.*

Explanation.—The plan of this projection is most conveniently drawn first, a circle 1¾ inches in diameter being described from *O* as a center, as shown on the plate. The edges of the pyramid are then drawn: a horizontal diameter and two diameters at angles of 60° with the first represent the upright edges; chords of the arcs thus designated are then drawn, and the plan of the pyramid is complete. Next draw the center lines *A B* and *C D*; set off the height of the pyramid on the line *A B* and complete the front elevation by the aid of primary projectors, as shown on the plate, the side elevation being projected as in former problems.

Case II.—*When the pyramid is in an obliquely inclined position.*

Explanation.—The angles of inclination in the projections of this case are 60° to **H P** and 45° to **V P**. Preliminary

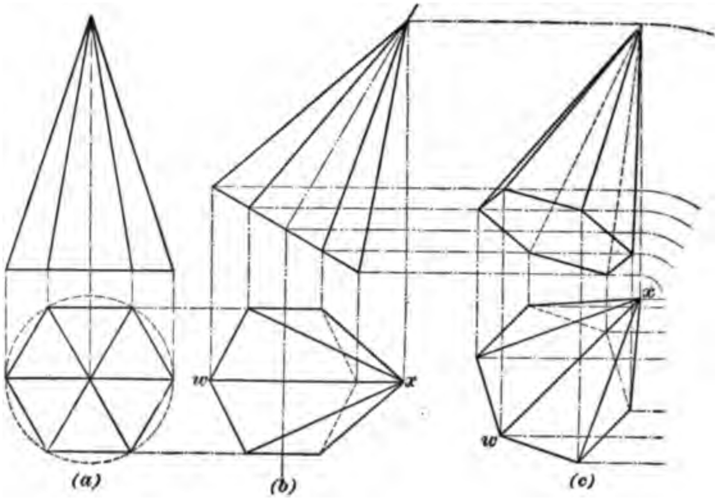

Fig. 28

drawings are required on separate paper, as shown in Fig. 28, first, as at (*a*), showing a right view of the pyramid; and second, as at (*b*), showing a rightly inclined view, the angle of inclination (of the center line) being 60° to **H P**. The plan produced at (*b*) is then copied on the plate in such a position that its axial line will make the required angle, viz., 45° to **V P**, as shown by the line *w x* at (*c*), Fig. 28. The front elevation is then projected as in Case II of the preceding problem, that is, by vertical primary projectors drawn from the plan in (*c*), intersected by horizontal primary projectors drawn from the elevation in (*b*). The projection of the side elevation by the arc method of secondary projectors is also similar to the preceding projections, as will be seen from an inspection of the plate.

PROBLEM 15

74. To draw the projections of a cone.

The *cone* is a solid that may be produced by the revolution of a right-angled triangle around one of its sides as an axis. Its base, therefore, is a circle, and its curved surface tapers uniformly towards a point at the top called the vertex, or *apex*. Like the cylinder, its entire form and dimensions are presented in a plan and a single elevation showing a right view of the cone. The cone for the projections of this problem is shown in perspective in Fig. 29, which gives the dimensions that the cone is to present on the plate. The methods used are precisely similar to those used in the case of the hexagonal pyramid in the preceding problem.

Case I.—*In a rightly inclined position.*

Explanation.—In order to produce the rightly inclined front elevation, a construction similar to that used in Case I of Problem 13 is here used. The drawing differs from that projection only in the form of the solid. The angle of inclination in this case is 50° to **H P**.

Construction.—Draw the center line *A B* (see the plate) at the given angle, that is, 50° to the horizontal. Next,

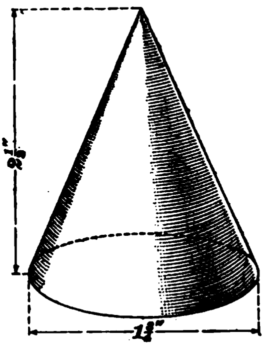

FIG. 29

construct the triangle representing the elevation of the cone and describe the circle at (*m*)—a temporary full view of the base. Describe a similar circle at (*n*), also a full view of the base, and locate a convenient number of points on the outline of each full view—in this case eight—as shown at *a*, *b*, *c*, etc., on the plate. Project the rightly inclined plan in a manner precisely similar to that used in the view of the cylinder in Case I of Problem 13. Erase the temporary views (*m*) and (*n*), and project the side elevation as in preceding projections.

Case II.—*In an obliquely inclined position.*

EXPLANATION.—The angle of inclination to **H P** is the same as in the drawing last made, and the plan of that projection may be recopied for the plan of this case, but it is to be drawn on the plate in such a position that the center line *w x* will make an angle of 45° to **V P**. The plan and front elevation of the preceding case must be redrawn on separate paper and temporarily fastened over the drawing towards the left of the space required for this case, in order that the projection of the front elevation may be drawn. As this process is similar to that used in preceding constructions, no further explanation will be given. Complete the projections in the plan, front, and side elevations as shown on the plate.

NOTE.—When inclined views are drawn of solids having curved surfaces (as the cylinder and the cone), the circular ends should first be projected. The outline of the curved surface is then represented as tangent to the base, or bases, of the solid, and without regard to the intersection of such outline with any given point on the base outline.

75. Self-Reliance.—The student that has intelligently completed the projections of the foregoing problems and has made frequent use of the imaginative feature of this subject, as previously explained and directed, should now possess a very complete knowledge of the methods of projection used in representing plain solids in various positions.

The projection of irregularly outlined figures has not been presented, since the methods are identical with those already shown. The student should acquire a degree of self-reliance in this work; for if he is to depend on having the projection of every conceivable form described for him, the principles governing those projections will become a secondary matter, whereas the practical draftsman requires, above all else, the faculty of recognizing the principles by which to define and project the various forms occurring in the course of his work.

NOTE.—The student should understand that the percentage of marking adopted for these plates is based on the degree of accuracy in which the projections are drawn to the angles of inclination, as well as on the quality of neatness attained in the finish of the drawings.

DRAWING PLATE, TITLE: SECTIONS I

76. Use of Section Drawings.—A section drawing, as previously explained, is a projection of a portion of a solid, in a view where the solid is intersected by a plane. This plane—sometimes called a *cutting plane*—may pass through the solid in any direction; that portion of the solid between the plane and the observer is assumed to have been removed. Section drawings are useful in many ways, for by such means the construction of interior parts of objects may be shown. It is desirable in many cases to show some particular form that a solid of peculiar shape possesses at a place where it cannot be presented in an exterior view, and in such cases sections are projected. The section is considered simply as a surface that would appear on the cutting plane in any view of the object projected. The outline of this surface, therefore, will depend on the form of the object, the number of surfaces intersected by the plane, and the angle of inclination of the cutting plane.

To project the views of a solid in which a new surface is thus presented, it is necessary to consider the various surfaces that originally composed the solid. The first view drawn is always that in which the cutting plane is represented as *on edge*. The intersections of the original surfaces

of the solid with the cutting plane are thus shown; and by using methods of projection already presented, any view of the section may then be drawn. Those portions of the solid assumed to have been removed are not shown in a section drawing, although their position is sometimes indicated by dotted lines.

77. Sections of the Sphere.—The solid that presents the most simple illustrations of sections is the *sphere*, or *globe*. This solid, also called a *ball*, is such as would be generated by the revolution of a circle around its diameter as an axis.

The cylinder, cone, and sphere are sometimes called the "three round bodies," or the *solids of revolution*. It will be shown later that their imaginary formation by such revolution may be taken advantage of by practical short methods of projection, the principles of which are based on this knowledge.

A full view of any section of a sphere is a circle. If the cutting plane passes through the center of the sphere, the full view of the section is a circle whose diameter is the same as the diameter of the sphere—or the *great circle* of the sphere, as it is called. If the cutting plane intersects the sphere in any other way, the section is still a circle, but of smaller diameter, and is measured from a view in which the cutting plane is shown on edge.

78. Sections of the Cube.—Fig. 30 is a projection drawing of the cube referred to in former illustrations and shows a vertical section. It will be noticed that the view of the section in the elevation is the same as the view in the front elevation in Fig. 23. This is always the case in sections

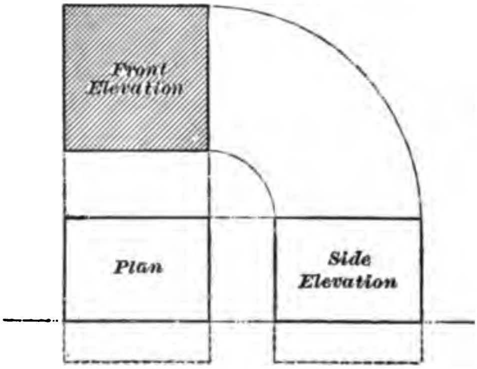

FIG. 30

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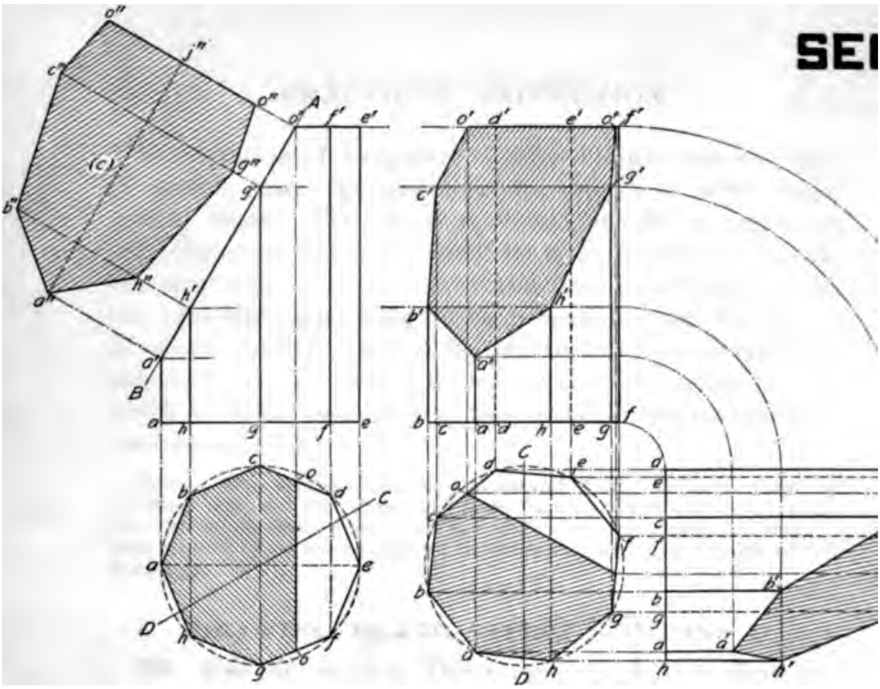
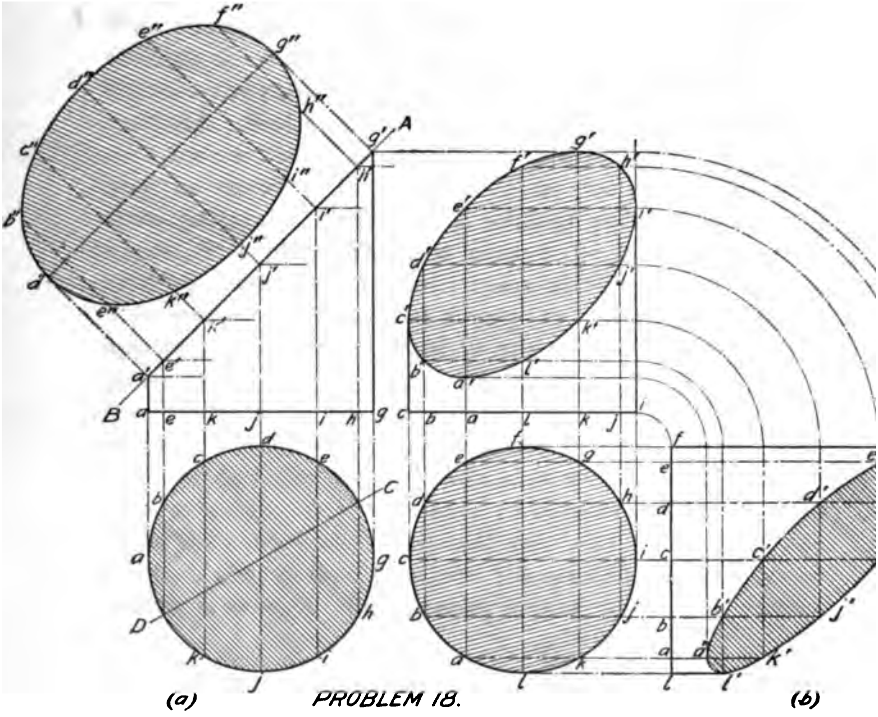


(a) (b)

PROBLEM 16.



(a) PROBLEM 18. (b)

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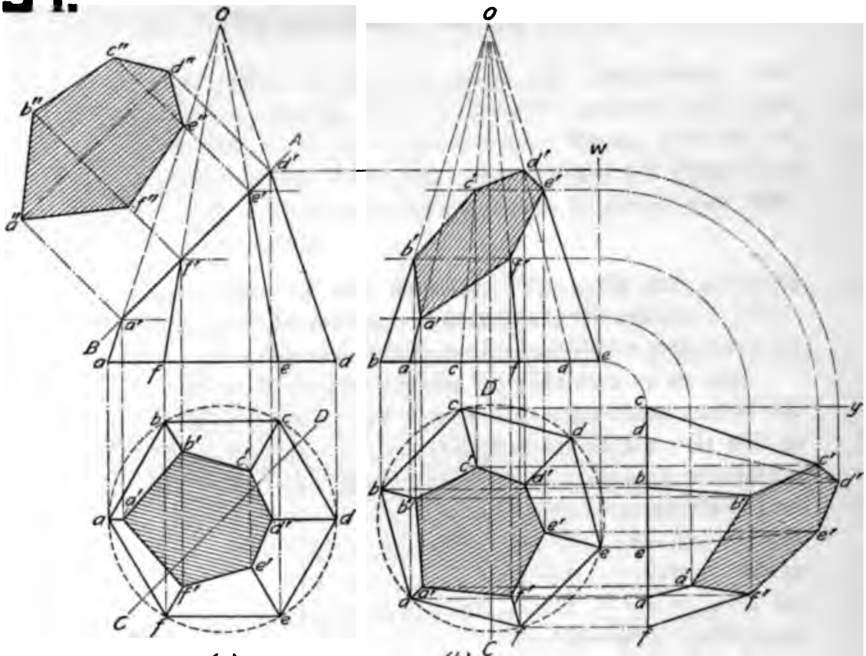

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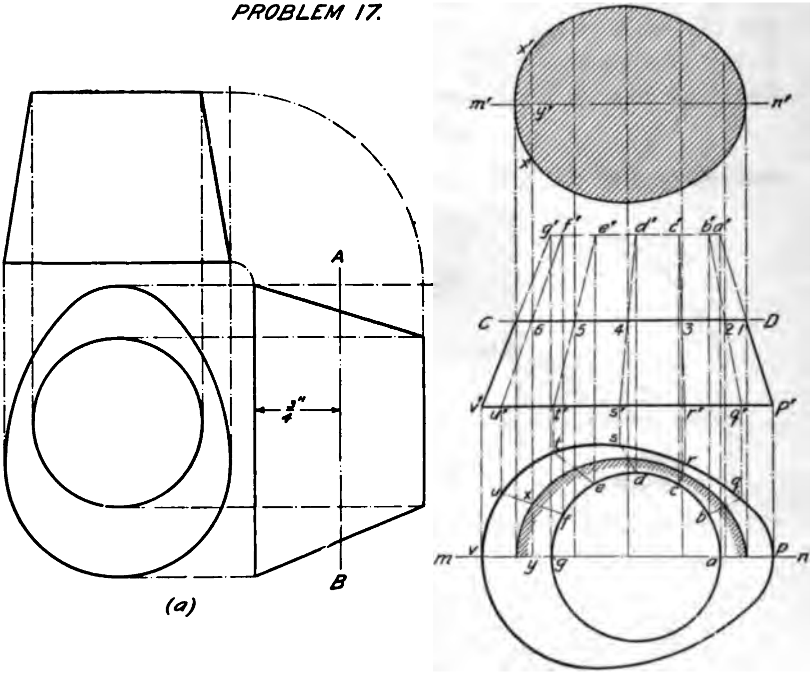

(a)

(b)

PROBLEM 19.

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of the solid with the cutting plane are thus shown; and by using methods of projection already presented, any view of the section may then be drawn. Those portions of the solid assumed to have been removed are not shown in a section drawing, although their position is sometimes indicated by dotted lines.

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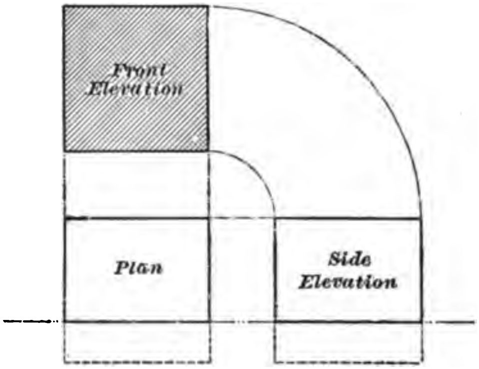


FIG. 30

of regular prisms where the cutting plane is parallel to the ends of the prism. Fig. 31 represents a diagonal section of the cube, the measurements of which will be apparent to the student from an inspection of the drawing. Fig. 32 is an oblique section, in which but three sides of the cube are intersected by the cutting plane; in this figure, the full view of the sectional surface is projected. Fig. 33 represents a section taken at a still different angle and position of the cube. *When a cutting plane passes through a solid having parallel sides, in any direction that causes it to intersect both of those parallel sides, those sides are shown in any view by parallel lines.* Note that in the sectional views of Fig. 33 the opposite edges of the surfaces are defined by parallel lines; thus, since *A B* and *C D* are parallel to each other in the plan of the cube in Fig. 33, so *A' B'* and *C' D'* are in the same relation to each other in the side elevation; also, *A'' B''* and *C'' D''* in the full view of the section. The same is also true of *A C* and *B D*, as may be seen from a comparison of the views.

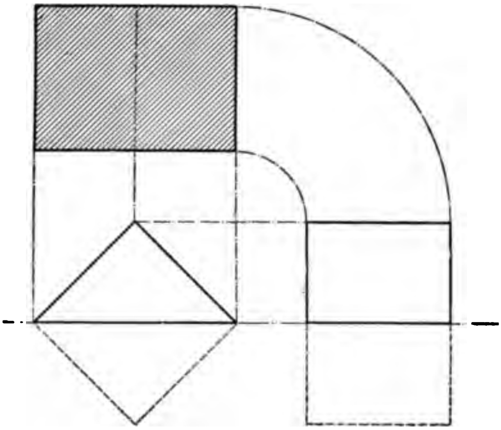


FIG. 31

79. How the Cutting Plane Is Represented.—When the cutting plane is shown on edge in a view, it is usually indicated by the same kind of a broken-and-double-dotted line used for the center line and the axial line. The use of this line for these three purposes is somewhat puzzling to the beginner; but as the student is now able to read projection drawings, he can readily determine which purpose the line is intended to serve. It is customary, however, as already mentioned, when center and axial lines are used, to mark them as such by neat lettering. A section line in a complicated drawing is usually designated by a letter placed at each end of the line, to which reference is made in the following manner: If, in the view where the cutting plane appears as a line, it is lettered A–B, the full view of the section is designated as a "section on the line A–B."

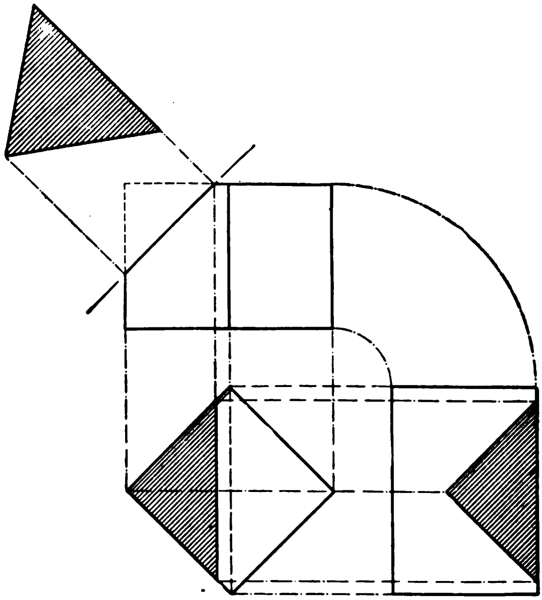


FIG. 32

80. The problems for this plate, of which there are four, consist of the projection of the section drawings indicated in the accompanying illustrations. They are to be reproduced on the plate by the student to the dimensions given on each figure. The cutting plane is indicated by the line *A–B*, and the views shown may be understood by a careful study of the figures on the plate illustrating each problem. The direction of a certain line in each plan is changed in these views, and the cross-section is accordingly represented as foreshortened in the front and side elevations. A view is also to be projected in which a full view of the sectional surface will be seen. The plate is to be divided into four equal spaces by horizontal and vertical lines. Problem 16 is to occupy the upper left-hand space.

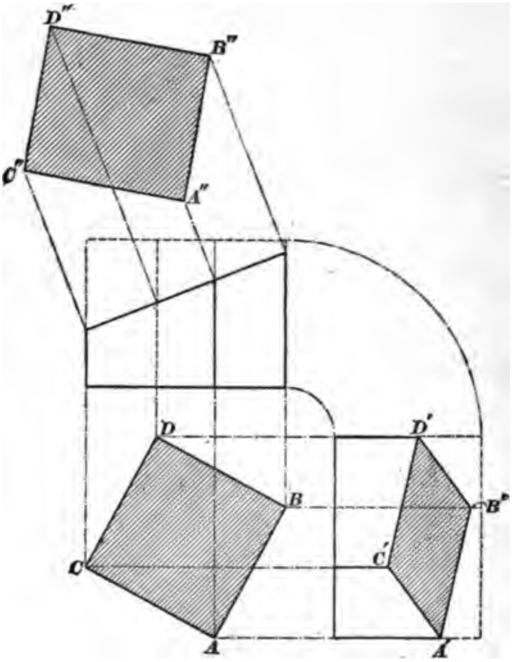


FIG. 33

The student is recommended not to refer to the reduced copy of the plate more frequently than is necessary to enable him to fix the location of the views on the drawing; he should learn to depend on his own knowledge of projection.

PROBLEM 16

81. To project sectional views of an octagonal prism, the cutting plane crossing the solid at an oblique angle and leaving a portion of the upper surface intact.

Explanation.—The position of the prism and the angle of the cutting plane is shown in Fig. 34. This figure is to be drawn, as at (*a*) in the left-hand portion of the space, to the size required by the dimension figures. The plan is then copied to the right, as at (*b*), but in a relatively different position, as will be seen by the arrangement of the letters and the direction of the line *C D*. Thus, the edge *a*, which in the plan of (*a*) is on the extreme left of the figure, occupies a position nearer the lower part of the drawing in the plan at (*b*). This shifting of position of the plan may be effected by describing circles circumscribing the octagons and drawing the diameter *C D* at an angle of 30° to **V P**, as shown in the plan at (*a*). This diameter is then drawn in a vertical position in the second plan (*b*), after which the arc *D a*, as measured on the first, may be set off with the dividers on the second, plan. The projections of the front and side elevations are then made in the regular way. The projection of the full view is accomplished by drawing projectors at right angles to the cutting plane *A B*. At a convenient distance, as shown at (*c*), draw the center line *a'' j''*, and from this line set off with the dividers the distances from the line *a e* as found in the first plan. As similar positions have corresponding letters in the different views on the plate, the student will have no difficulty in recognizing the method of transfer, it being the same as that mentioned in Art. **53.** Observe that the sides of the section *b' c'* and

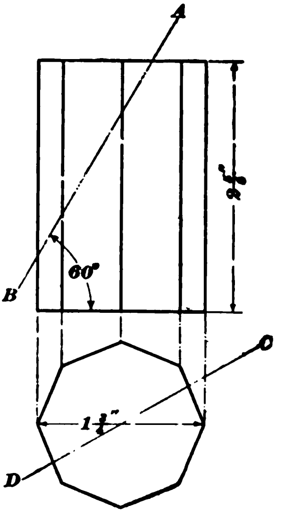


Fig. 34

g' o', and *b'' c''* and *g'' o''*, are parallel in every view shown, since the sides *b c* and *g o* (*f*) are parallel, as shown in the plan of the solid. The same is true of *h g* and *c o* (*d*), as seen at *h'' g''* and *c'' o''* in (*c*) (see Art. **78**).

The student will finish the drawing on the plate in as complete a manner as in the drawing of the cube in Fig. 30, but omitting all reference letters. The surface of all sections in each view is to be cross-hatched, as in Figs. 30–33.

PROBLEM 17

82. To project sectional views of a hexagonal pyramid.

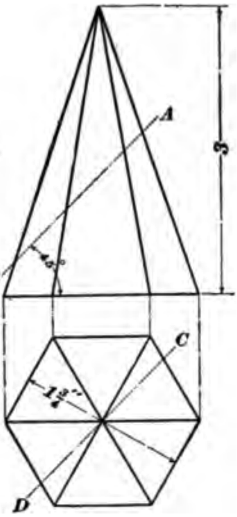


FIG. 35

The dimensions and position of the pyramid and the angle of the cutting plane are indicated in Fig. 35. Any section of a pyramid taken at right angles to the axis of the solid is a polygon having an outline similar to the base of the pyramid. The polygon representing the section of any pyramid thus intersected varies in size as the cutting plane passes through the pyramid at a point on the axis nearer the base or the vertex of the pyramid. If the cutting plane passes through the vertex and the base, or through two sides and the base of the pyramid, the section is a triangle. Any other section is a polygon having sides unequal in length, but equal in number to the sides of the polygon forming the base of the pyramid.

EXPLANATION.—The projections of this solid and its sections do not differ materially from those of the preceding problem. The change of position in the two plans is accomplished by means of the circumscribing circle, the diameter in the first plan being drawn at an angle of 45° and the arc *C a* being measured in a similar manner. This drawing may be more accurately made if the edges of the pyramid are continued by dotted lines to the vertex *O* in the front elevations, as shown on the plate. A comparison of the reference letters in the different views on the plate will assist the student in the work of drawing these projections, as each line shown is similarly lettered in all views.

PROBLEM 18

83. To project sectional views of a cylinder.

The cylinder possesses some points of similarity to the regular prism with respect to its section, viz., a section parallel to its ends has the same outline as the ends, while a section parallel to its axis is a parallelogram. An oblique section of the cylinder is, however, as will be observed during the projection of this problem, an ellipse; and it will further be noted that a certain view of an ellipse is a circle.

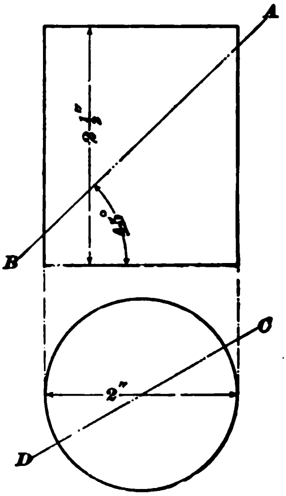


FIG. 36

EXPLANATION.—The position of the cylinder and the angle of the cutting plane is shown in Fig. 36. Make the edge *a–a'* of the elevation at (*a*) $\frac{5}{16}$ inch long. The plan of the cylinder is first divided into an equal number of spaces (in this case 12), the points being lettered *a*, *b*, *c*, etc., as shown on the plate. These points are then assumed to represent edges, as in the case of the octagonal prism, and are projected to the elevation, their respective positions there being indicated by similar letters; thus, the edge *a–a'* in the elevation represents the edge *a* as shown on the plan. The change of position in the second plan is effected by drawing the vertical diameter *l f*, which is shown as *C D* in the first plan at an angle of 30° to **V P**. The position of all points is then noted in a manner similar to the two preceding problems and the projections are completed as heretofore. After tracing the curve of the ellipses through the points located in the different views, the sections are indicated by cross-section lines, as before directed.

PROBLEM 19

84. To project views of an irregularly formed solid; also, to project a sectional view from a given cutting plane.

EXPLANATION.—This solid, shown in perspective in Fig. 37, may be described as a transition piece, that is, a form used to connect openings or outlines unlike in shape. Such solids are frequently used in the sheet-metal trades, particularly in boiler and pipework. The dimensions of this solid are better shown in the projection at Fig. 38. Its upper base is a circle 1½ inches in diameter, while the lower base is an oval, which may be drawn by the method shown in *Geometrical Drawing*. The end circle of the lower base is described with a radius of 1 inch from a center located at *B*, Fig. 38, the distance between the centers of the upper and lower bases, as measured on the plan between *A* and *B*, Fig. 38, being $\frac{5}{8}$ inch. The vertical height of the solid is 1½ inches.

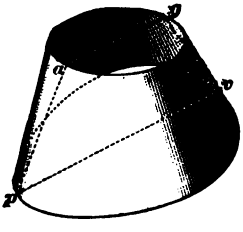


FIG. 37

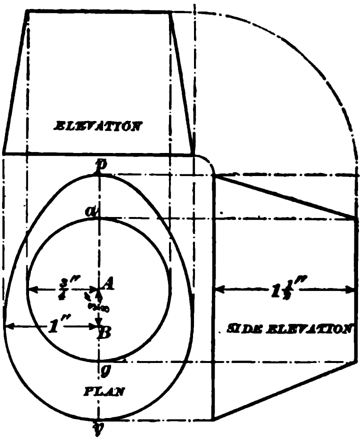


FIG. 38

The plan and front and side elevations are drawn as shown on the plate, and since the methods of projection are no different from preceding problems representing regular solids, definite instructions for this work are omitted. Particular attention is directed to the method of finding the outline of the section, as its application to various processes of patterncutting is of great importance. The direction of the cutting plane is shown in the side elevation at (*a*) by the line *A B* on the plate. The plan and elevation is to be redrawn on the plate in the position shown thereon at (*b*), and the full view of the section is projected towards the upper part of the drawing. As it is desired to show a section through a certain portion of this solid, it is first necessary to locate a number of points on the line that represents the cutting plane. Since the solid presents no edges that intersect this plane, a number of lines are to be assumed as drawn on the curved surface of the solid.

CONSTRUCTION.—First draw a horizontal line through the central portion of the plan, as *m n* in the drawing on the plate; this divides the figure into symmetrical halves. Next, by spacing, divide the outline of each base into the same number of equal parts, as at *a*, *b*, *c*, etc. on the upper base and *p*, *q*, *r*, etc. on the lower base. Project the points thus located to the elevation and draw connecting lines in both views, as shown. Across the elevation draw the line *C D*, representing the cutting plane, and project the intersections of this line at *1*, *2*, *3*, etc. to the plan, producing each line thus drawn until it meets the horizontal line *m n*. The full view of the sectional surface may now be drawn as follows: Draw projectors vertically upwards from the points *1*, *2*, *3*, etc. in the elevation, and at a convenient height from that view draw the horizontal line *m' n'*. The width of the surface, as measured on each line, may now be set off with the dividers, as directed in Art. **53**; thus, take the space *x y* from the plan and transfer it to corresponding positions at *x' y' x'* on the same line in the full view. Transfer, in like manner, the length of each vertical dotted line shown in the plan to the full view and trace an irregular curve through the points thus located, completing the projection. This curve may also be traced through the plan and there indicated by short cross-hatching, while the full view of the sectional surface is designated, as heretofore, by cross-section lines in the manner shown on the plate.

85. Practical Method of Representing Certain Sections.—In working drawings, particularly those made for the execution of sheet-metal work, a section drawing is often made directly over another view, and the lines indicating the section are distinguished by short cross-hatching, as shown on the plate in the plan of the preceding problem. Short portions of sectional views are also frequently shown in this way on working drawings, in order to indicate the form, or profile, of moldings, the different planes of adjacent surfaces, or the different levels at which certain notations

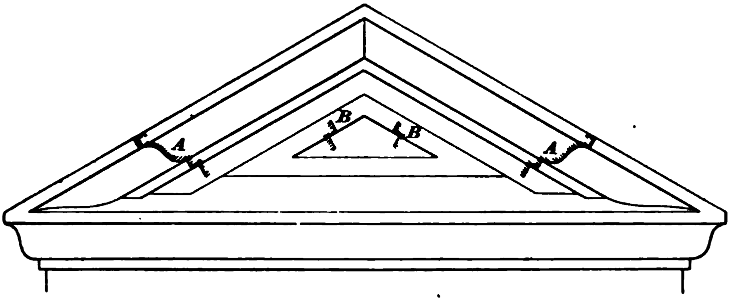


FIG. 39

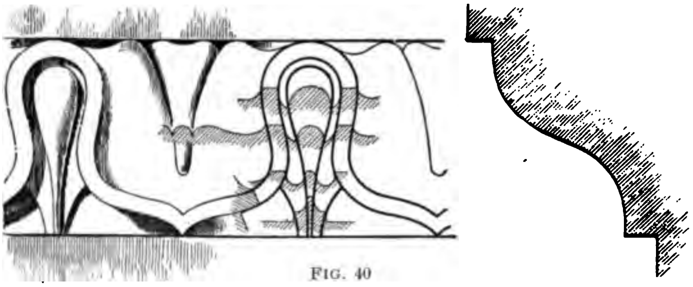
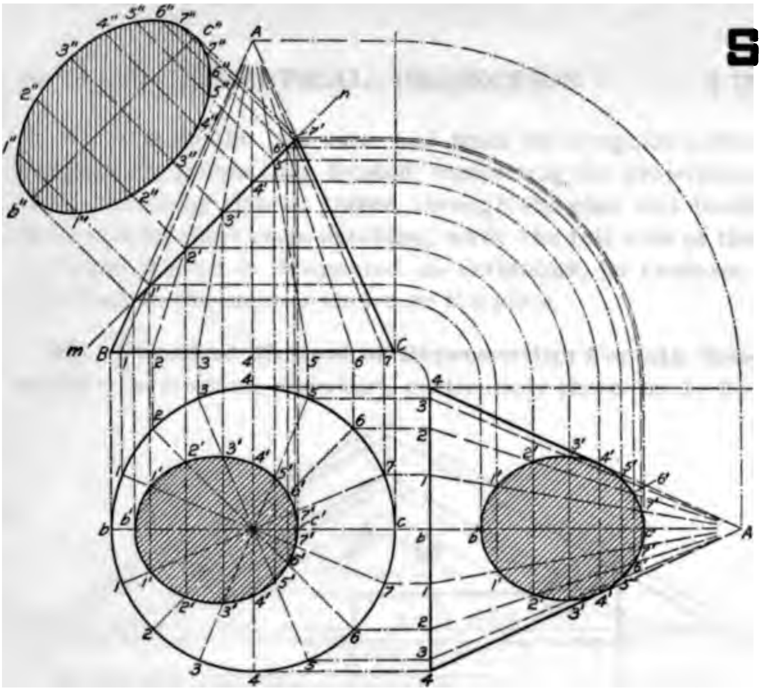
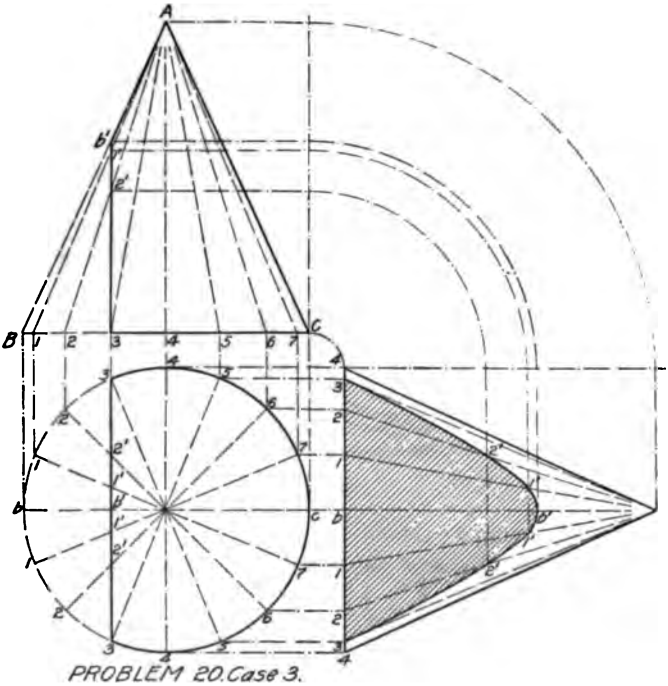


FIG. 40



PROBLEM 20.
Case 1.



PROBLEM 20. Case 3.

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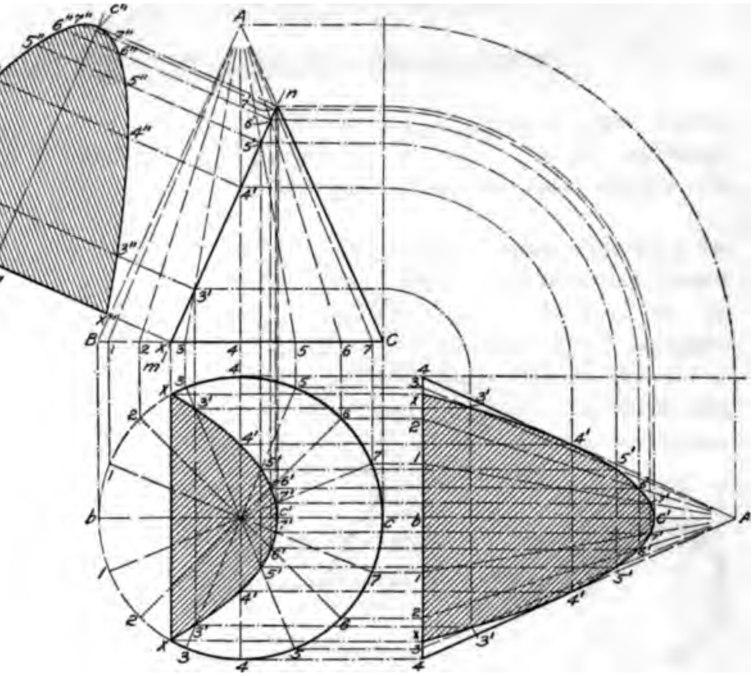
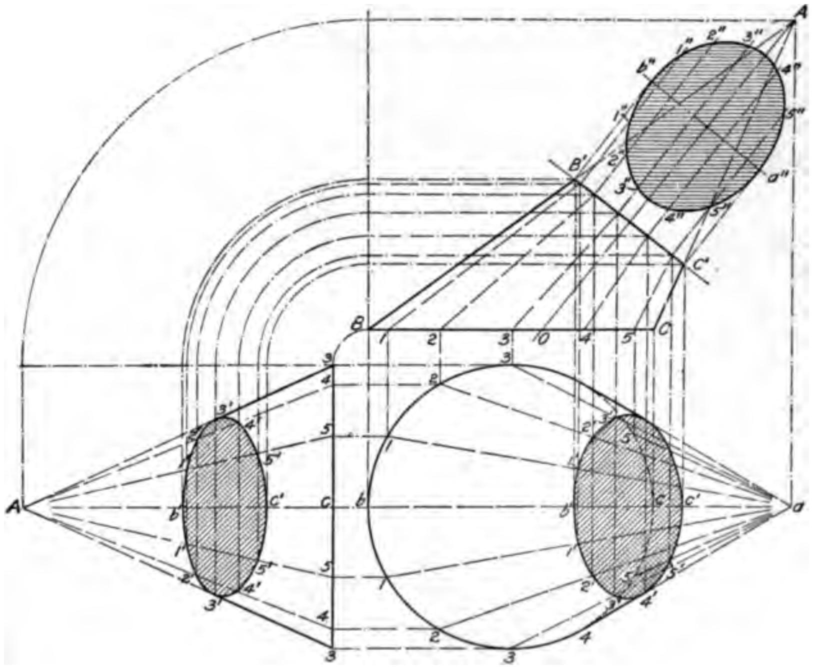


PROBLEM 20.
Case 2.



PROBLEM 21.

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are made. Thus, in Fig. 39, which shows a gable finish, the section at *A* is the profile, or "stay," of the molding, while the section at *B* indicates that the panel shown is a *sunk* panel.

In detail drawings (mere projections drawn full size) for decorative sheet-metal work, sections are frequently shown at different portions of the drawing, as in Fig. 40, an inspection of which will enable the student fully to comprehend the character of the various parts without the aid of another view. Sections, therefore, properly understood and used, may be employed by the draftsman in many ways, and are frequently a means of shortening the labor of drawing intricate projections.

The lines representing the sides of the octagonal shaft in Fig. 41 are broken in the lower part of the figure, this being a means of indicating that the full length is not shown in the drawing. Reference to the dimension figures gives the reason, it being obviously unnecessary to make a drawing extending the full length of a simple form.

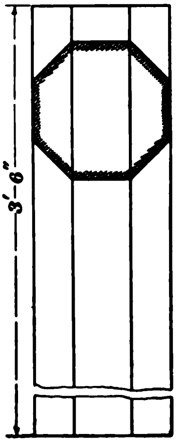


FIG. 41

DRAWING PLATE, TITLE: SECTIONS II

86. Conic Sections.—The divisions on this plate are the same as on the preceding plate. The two problems to be constructed are those relating to sections of the cone. Besides their use in many calculations of the arts and higher sciences, conic sections are of great value to the architectural and mechanical draftsman, for the curves thus developed possess great beauty and symmetry, and when used in moldings present pleasing architectural effects of light and shade.

The full view of the section of a regular cone made by a cutting plane parallel to its base is a *circle;* if the cutting plane passes through the vertex and the base of the cone, the section is a *triangle;* if the cutting plane is at an oblique angle to the base, which angle is less than the angle made by the elements of the cone with the base, the section is an *ellipse;* if the cutting plane is parallel to any line drawn on the convex surface of the cone from the base to the vertex—that is, parallel to any element of the cone—the section is a *parabola;* if the cutting plane is at any angle (but not passing through the vertex of the cone) greater than the angle which the elements of the cone make with the base, the section is an *hyperbola.*

87. Elements of the Cone.—In the construction of Problem 19, where it was desired to project a section of a solid whose curved surface presented no edges, it was found necessary to assume lines on that surface in order to establish points of intersection with the cutting plane. This process must be followed with all solids whose surfaces are curved, and in the case of the cone the lines thus assumed have a further use, as will appear later. The manner in which these lines are located on the surface of the regular cone is of special importance and is as follows: The outline of the base in the plan is first divided by spacing it into a number of equal parts (16 in the plan of Fig. 42). The points thus located are then projected to a view in which the cone is represented as in the elevation of Fig. 42—that is, a right view—and lines are then drawn from these points to the vertex. They are also shown on the plan in the same relation, each line being represented as a radius of the circle at the base of the cone. These lines are called the *elements* of the cone.

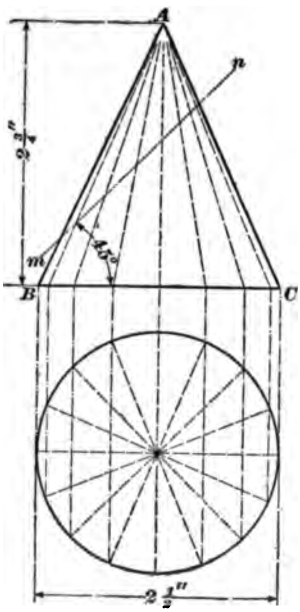


FIG. 42

In such a view of a cone as presented in the elevation of Fig. 42, only two of these elements (viz., *A B* and *A C*) can be shown in their true length—all the others are foreshortened. This must be borne in mind by the student, for it is evident that only on such lines as are shown in their true length can measurements be obtained for any points of intersection. Those points that are located on foreshortened elements are determined in a particular way, as will appear during the construction of the following problem.

The plan and front and side elevations and the full view of the section for each problem and case are to be drawn on this plate in the same corresponding relation as on the preceding plate.

PROBLEM 20

88. To project sectional views of a cone.

The three cases of this problem should be carefully studied by the student, for on the application of the principles here shown depend nearly all operations of pattern drafting that relate to so-called flaring work. A clear conception of the methods used will be found indispensable to the draftsman that desires to become proficient in his work. An effort should be made on the part of the student to trace each operation to its fundamental principle, when it will be discovered that the drawing practically resolves itself into a continuation of problems relating to the true lengths of foreshortened lines. Since such problems have occupied his attention during the drawing of the earlier plates, he should have no difficulty in following the constructions here given.

Case I.—*When the cutting plane is oblique to the base and intersects all the elements of the cone.*

The position of the cone, its dimensions, and the angle of the cutting plane are shown in Fig. 42.

CONSTRUCTION.—Draw a right plan and elevation and represent the cutting plane *m n* (see plate) by a line drawn at an angle of 45° with *B C*, cutting *A B* ½ inch from *B*. Divide the circle that represents the outline of the base of the cone into any convenient number of equal parts (in this case 16). Draw lines from these points (*b*, *1*, *2*, *3*, etc.) to the center; next, project these lines—or *elements*, as they are called—to the elevation. Project the intersections of the elements in the elevation with the cutting plane *m n* to the corresponding elements in the plan and trace a curve through the points thus obtained.

Attention is called to the fact that, although the distances between the points found in this manner are unequal, yet

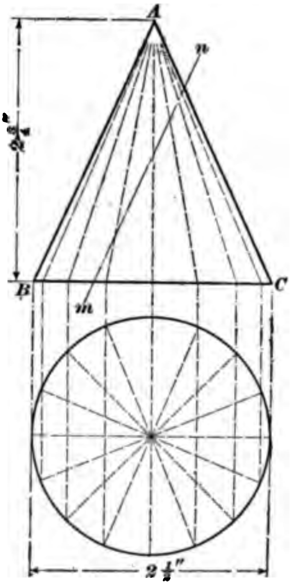


FIG. 43

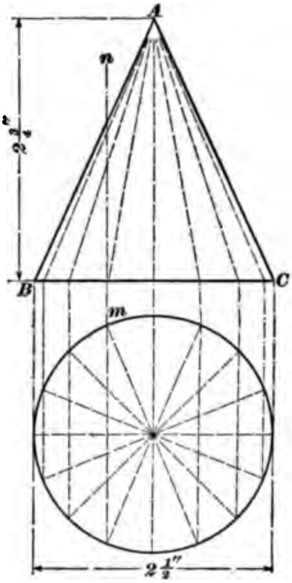


FIG. 44

the foreshortened view of the sectional surface thus shown in the plan is a true ellipse, as is also the full view of the section next to be drawn. Project a side elevation to the right (see Art. **61**), also showing a foreshortened view of the section—an ellipse of a different curvature.

Case II.—*When the cutting plane is parallel to one of the elements, the section being in this case a* **parabola.**

Explanation.—Fig. 43 is a right plan and elevation, giving the dimensions of the cone. The cutting plane $m n$ is in this case parallel to the side $A B$ of the cone and cuts $B C$ $\frac{5}{8}$ inch from B. The method of drawing these projections is precisely similar to that in the preceding case, and since corresponding points are similarly designated in the different views shown on the plate, the student should experience no difficulty in completing the drawing.

Case III.—*When the cutting plane is perpendicular to the base of the cone, the section being an* **hyperbola.**

Explanation.—To produce that section of the cone known as the hyperbola, the cutting plane may form with the base any angle included between a right angle and the angle formed by an element with the base (as the angle $A B C$, Fig. 44). The dimensions and position of the cone and the cutting plane are shown in Fig. 44, and since the method of projection is the same as in the two preceding cases, the student may complete the views without further instruction. In this case, the projection of a separate full view may be omitted, the latter being shown in the side elevation.

PROBLEM 21

89. To project sectional views of a scalene cone.

Explanation.—This solid, which is of varied form and of frequent occurrence in the metal trades, is an irregular geometrical figure. It is a cone whose axis is inclined to its circular base. All the elements of a regular cone are of equal length, but the elements of a scalene cone are necessarily of variable length, for, since its axis is inclined towards a portion of the base, the elements in that part of the surface must be shorter. It is to be noted in this case that the axis of the cone shown in Fig. 45 does not pass through the center of the circle that represents the base of the solid.

Construction.—To reproduce this drawing on the plate, draw first the horizontal line $b a$ in the plan $3\frac{5}{8}$ inches long, as called for by the dimension figures in Fig. 45; next, describe the circle *b 3 c* 2½ inches in diameter from a center located on the line *b a*, its circumference passing through the point *b*. Project the elevation according to the dimensions given in Fig. 45. The axial line is next drawn; bisect the angle *B A C* and draw the bisector *A O*; represent the cutting plane *m n* by a line drawn perpendicular to *A O* and cutting *A C* ⅝ inch from *C*.

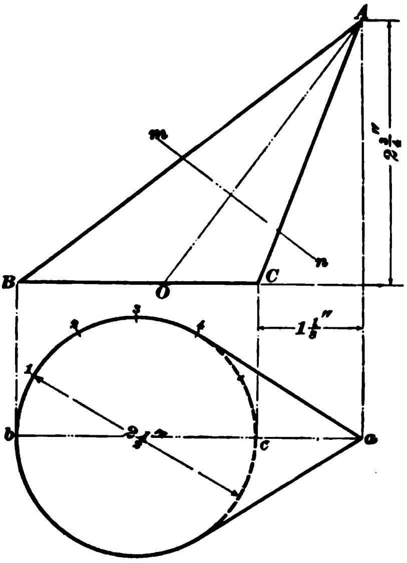
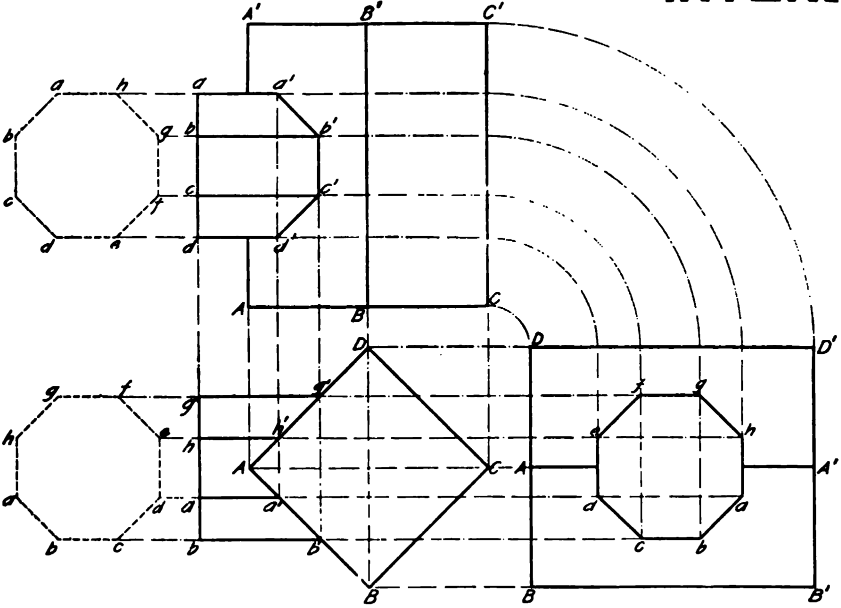


FIG. 45

The method of projection for the various views of this solid is the same as in former cases; that is, divide the outline of the base in the plan into a convenient number of spaces (12 in this problem), and from points thus located draw the elements to the apex *a* (see plate). Next, project these elements to the elevation and finally project their intersections with the cutting plane to the different views, as shown in the drawings on the plate. Note that the full view of the section, which in this problem is taken at right angles to the axis of the solid, is an ellipse. If the cone were in an upright position—that is, with its base at right angles to the axis (as that part of the cone above the cutting plane in Fig. 45)—the solid would be termed an "elliptical" cone. This, strictly, is not a geometrical solid, but its characteristics in projection drawing are somewhat similar to the regular cone, although the elements in each quarter of the base are always of unequal length.

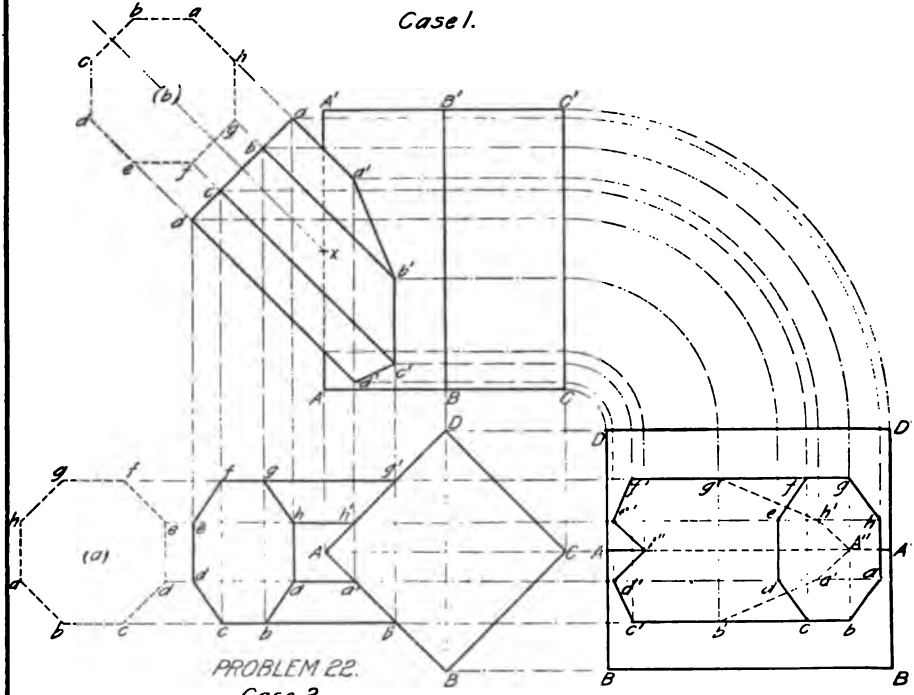
The student should now be able to project any sections that may be desired.

INTERS



PROBLEM 22.

Case 1.



PROBLEM 22.
Case 2.

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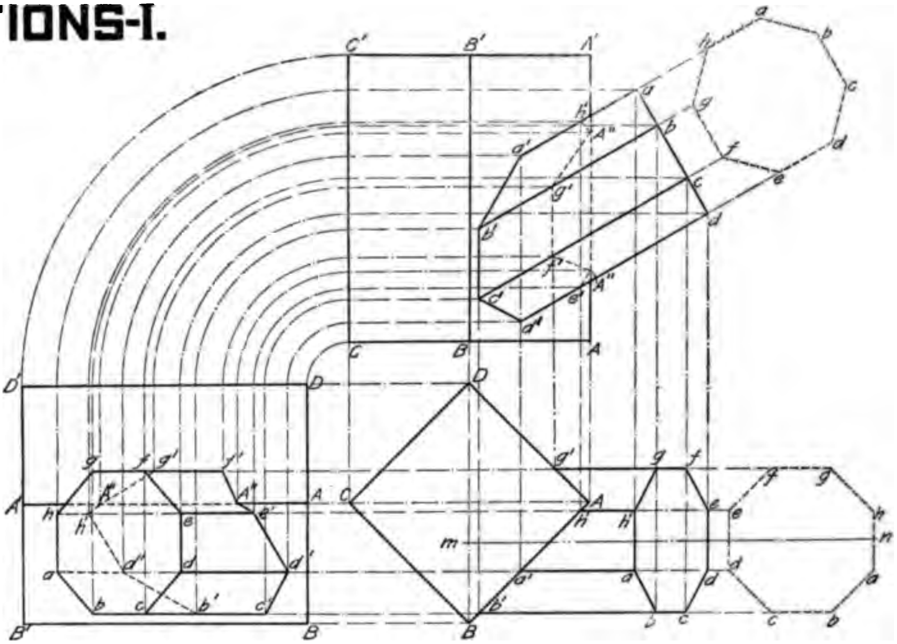
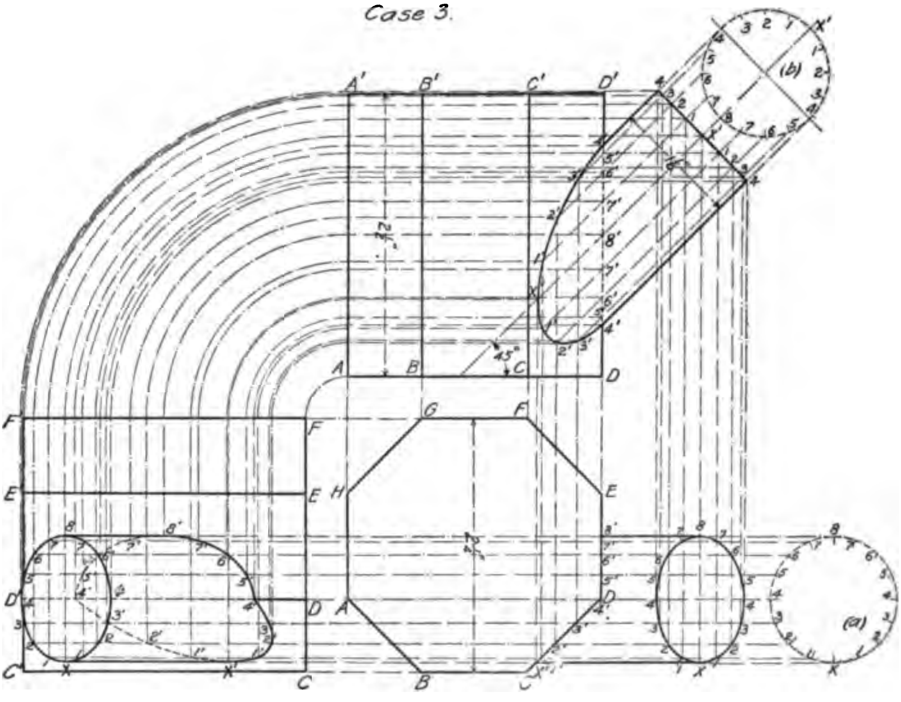


PROBLEM 22.

Case 3.



PROBLEM 23.

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DRAWING PLATE, TITLE: INTERSECTIONS I

90. The Miter Line.—To represent properly the intersections of the surfaces of solids—or to "draw the miter line," as it is commonly called—is the final process of projection. It has already been remarked that plane surfaces intersect in a line; the representation of the intersection of plane surfaces is therefore a very simple process, the draftsman merely having to define each surface by the application of the regular projection methods already explained. The intersection of curved surfaces is apparently more complicated, but only because it is necessary to locate a greater number of points than are required for the intersection of plane surfaces. The location of points for the representation of the intersection of curved surfaces is done in a manner somewhat similar to that already shown in connection with the projection of plane surfaces having curved outlines. There is, however, this important difference to be observed: in the case of the surfaces mentioned, their projection is accomplished by means of *points* located on their outlines; while in the case of the intersection of curved surfaces, it is necessary to locate *lines* in such positions on each surface that they will lie in the same plane, although drawn on different surfaces. It is possible to locate a number of these lines in such positions on the drawing that through their points of intersection a curve may be traced that will be the correct line of intersection of the surfaces.

91. Relation of the Miter Line to the Pattern.—No drawing of an object in which intersected solids are represented is complete unless the line of intersection is accurately produced. This is a very important part of the drawing, and the correct "fit" of the pattern in work of this class often depends entirely on the accuracy with which the line of intersection is drawn. In fact, a development, or pattern, cannot be made until the drawing is complete in this particular. The principles governing the use of these lines are clearly shown in the explanation accompanying the problems, which the student should study carefully; for if he thoroughly comprehends the principles governing their use and exercises due care to see that lines are drawn from the same corresponding points in each view, he will have no difficulty in producing the correct lines of intersection for the surfaces of the solids represented on these plates, or, in fact, for the surfaces of any solid. The problems for this plate consist of the projection of intersecting solids having plane surfaces. The solids are shown in perspective in the illustrations accompanying each problem, and reference to the projections on the plate will be sufficient to show the method of finding the lines of intersection.

PROBLEM 22

92. To project views of intersecting prisms.

When the plane or curved surfaces of any solid so intersect as to present one continuous surface—that is, so that the surfaces meet "edge and edge" in the same plane—no line of intersection is necessary, since such surfaces are relatively in (and a part of) the same plane. When, however, the surfaces of one solid intersect a central portion of the surface, or surfaces, of another solid, it is necessary that the line of intersection—that is, the boundary lines of the surfaces of the intersecting solid—should be accurately drawn.

Case I.—*When the axes of the prisms intersect at an angle of 90°.*

Explanation.—The solids for the projection of this problem are shown in perspective in Fig. 46, which represents also their position in the intersection of Case I. The figure consists of an upright shaft in the form of a quadrangular prism with a horizontal octagonal prism intersecting, or "mitering," at right angles along the axial lines of the two solids. The projection and arrangement of the solids is seen in the drawings for this problem on the plate. It will be noticed that the octagons

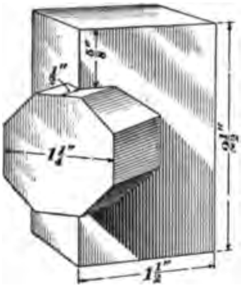

Fig. 46

drawn in dotted lines in the figure do not form a part of the finished drawing, but are thus drawn to facilitate the projection, as the edges of the solids may thus be determined before the side view is drawn. It is often convenient to place portions of views temporarily on the drawing in this manner, as much labor is thereby saved. The plan is completed first and the remainder of the drawing finished in the usual way, as may be seen from the reduced copy of the plate. It is thus shown that the correct line of intersection is found by simple methods of projection, the position of the points in the line being determined by projection from the different views.

CONSTRUCTION.—Draw the plan $A\ B\ C\ D$ in its proper position, as shown on the plate, and next construct the elevation $A\ A'\ C'\ C$, thus completing the projection of the quadrangular prism as though that solid alone were to be represented. Draw a horizontal axial line (not shown on the plate) for the octagonal prism through both views, and at the left of the views thus drawn construct a full view of the end of the octagonal prism, as shown by the octagons in dotted lines on the plate. Next, draw the lines that represent the edges of the octagonal prism in both views. From the points of intersection of such lines in the plan with the edges $A\ B$ and $A\ D$ of the quadrangular prism, draw primary projectors to corresponding lines in the elevation. Draw connecting lines through the points of intersection thus determined in the elevation, as shown at $a'\ b'\ c'\ d'$ on the plate. This completes the projection of that view; the side elevation may then be projected by means of secondary projectors, as in former problems.

This plate is divided in the same manner as the preceding plate, and the projections for this case are drawn in the upper left-hand space.

Case II.—*When the axes intersect at an oblique angle.*

This projection is completed as shown on the plate. In this, as in the preceding case, the plan is first drawn; but before it can be completed, a portion of the elevation has to be drawn in order to determine the outline of the octagonal surface in the plan (Problem 6).

CONSTRUCTION.—First draw the outline *A B C D* in the plan and then construct the octagon shown in dotted lines at (*a*), from the points of which draw horizontal lines to the plan of the quadrangular prism, in the manner shown. Next draw the elevation of the quadrangular prism and locate the point *x* midway on the line *A A'*; the angle of inclination of the octagonal prism is in this case 45°, and a line at that angle is then to be drawn through *x* for the axial line of that solid. Construct the full view of the end of the octagonal prism at (*b*), and from the points *e*, *f*, *g*, etc. in that view draw lines of indefinite length towards the right; intersect these lines at *a'*, *b'*, *c'*, and *d'* in the elevation by vertical primary projectors drawn from corresponding points in the plan, thus establishing the line of intersection in the elevation. The plan is then completed by drawing primary projectors vertically downwards from the edge view of the end of the octagonal solid in the elevation and tracing the outline of the inclined surface thus designated, as in Problem 6. The side elevation is next projected by the arc method of secondary projectors, as heretofore. Note that the intersection of the upper portion of the octagonal surface is to be represented in that view by dotted lines.

Case III.—*When the axes are at an oblique angle, but do not intersect.*

EXPLANATION.—The position of the solids is shown in the drawings on the plate, and the projections do not differ materially from those of the two preceding cases. The octagonal solid is in this case inclined at an angle of 30°, and its axial line *m n*, as shown in the plan, is drawn $\frac{5}{16}$ inch below the center of the quadrangular prism. The order of procedure is the same as that given for the drawing of Case II and will present no difficulties to the attentive student. It is necessary, however, to use extreme care in these projections, in order that the position of points in the different views may be located *in the same corresponding position with regard to one another.**

PROBLEM 23

93. To project views of a prism intersected by a cylinder.

Explanation.—Fig. 47 is a perspective view of an octagonal prism intersected by a cylinder at an oblique angle, the axes of the solids not intersecting. The arrangement of the views, the dimensions, and the angle of inclination are shown on the plate. The projection of this problem is very similar to the last case of Problem 22. Note that the position of the two solids is such that the axis of the cylinder intersects the edge *D* of the prism. The circles that represent the end surface of the cylinder in each view are divided into a convenient number of equal spaces, and from the points thus located lines parallel to the axis of the cylinder are drawn on its surface. These lines are then projected in the same way as the lines that represented the edges of the octagonal prism in Problem 22. Since the surface of the cylinder is a curved surface, the line drawn through the points of intersection of the two solids will be a curved line.

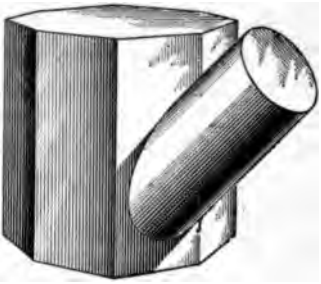

Fig. 47

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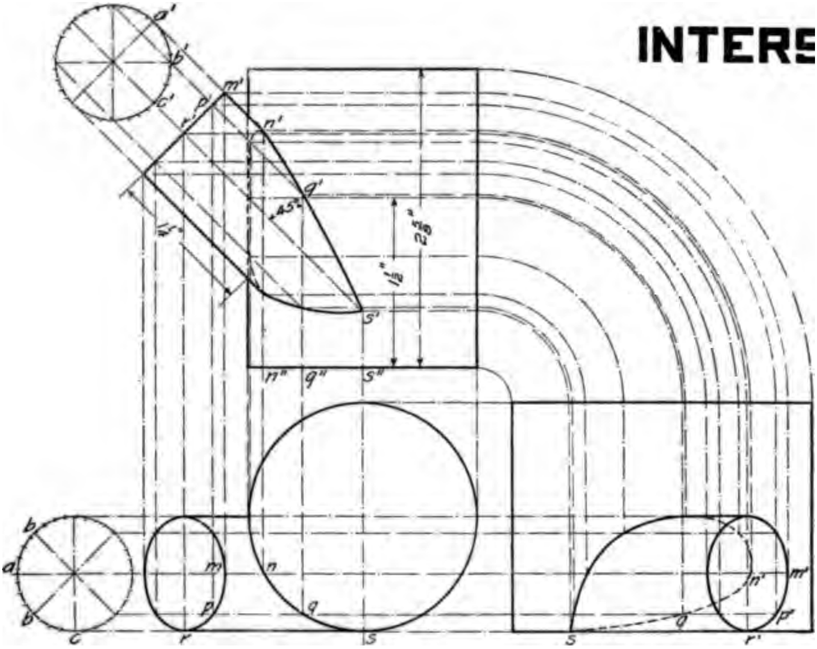
* Too much stress cannot be laid on this important statement; as the student progresses with the projection of the views, he will see the importance of this matter. The work must be done slowly, keeping the pencil points well sharpened; the position of the points in the drawing may then be accurately determined, if the views are constantly compared. If this is done, there will be no difficulty attached to any of the problems to follow in this section.

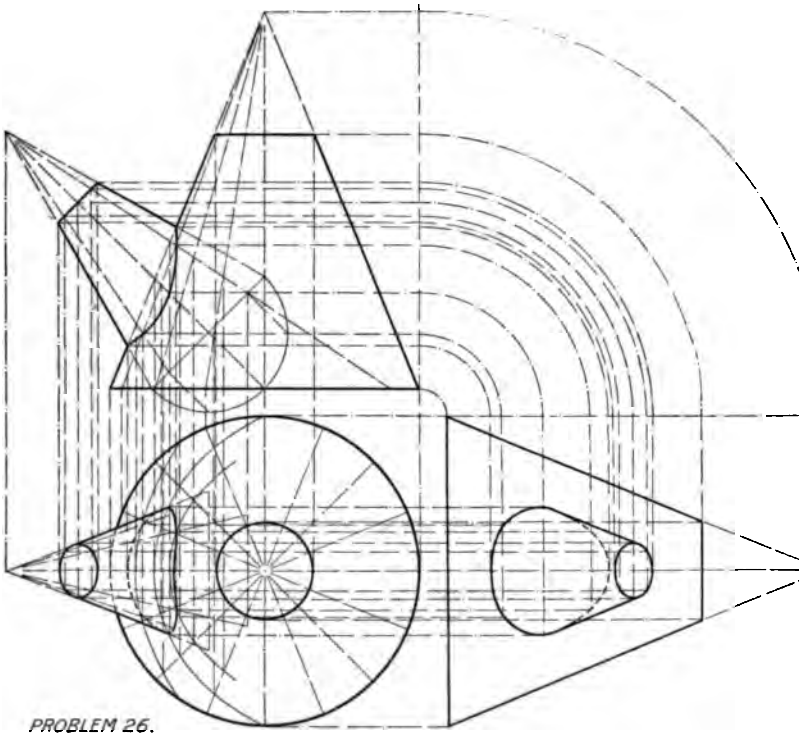
94. General Instruction Relating to Intersections. When points are located on the full view of a surface in a plan and elevation, as on the circles at (*a*) and (*b*) in the drawings on the plate for the last problem, care must be taken that the distinction between the different views is maintained; thus, the point *x*, at (*a*) and in the plan, is located at *x'* at (*b*) and in the elevation, both positions on the drawing representing the same position on the solid. Any other points thus located on an outline will be changed in a corresponding way with relation to one another. To project intersections of solids, all of whose surfaces are bounded by parallel lines or on whose surfaces parallel lines may be drawn that will also be parallel to the axis of the solid, it is necessary first to draw a view that will show the intersect*ed* surface, or surfaces, in one line—that is, "on edge." Such views have been drawn in the plans of the preceding problems. The lines of the intersect*ing* solid are then represented in this view, and the points of their intersection with the upright surfaces are projected to the elevation in the manner described.

It will be seen that, if all the surfaces of any intersecting solids are plane, their edges or outlines alone will suffice for finding the lines of intersection. If the surfaces are curved, it is merely necessary to locate a number of points on the outline of the full view, through which to draw parallel lines similar to those drawn on the cylinder in Problem 23. This practically changes, or reduces, the cylinder to a solid bounded by a number of plane surfaces. In the case of Problem 23, the cylinder has really been treated as though it were a prism having a number of sides equal to the number of spaces into which the circles were divided. This would actually have been the case had straight lines been drawn between the points thus located on the circles at (*a*) and (*b*). Had this been done, the line of intersection would also have been represented by a series of short lines drawn between the points located by projection methods.

INTERS



PROBLEM 24.



PROBLEM 26.

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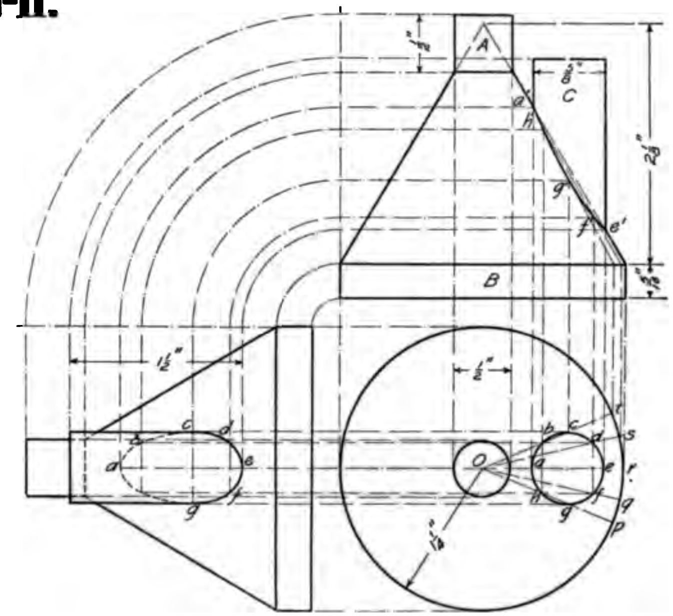
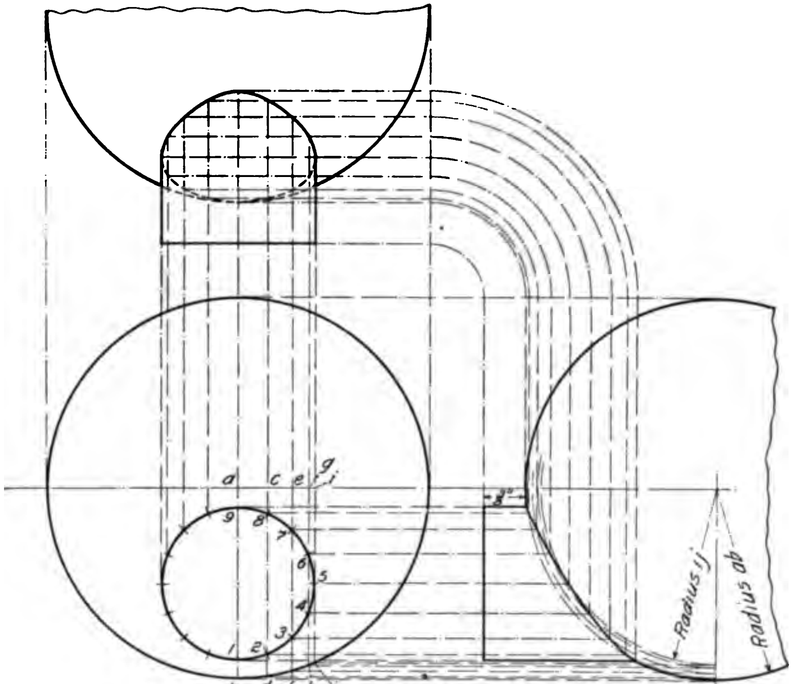


PROBLEM 25.



PROBLEM 27.

ENGINEER COMPANY.

JOHN SMITH, CLASS No 4529.

ed.

DRAWING PLATE, TITLE: INTERSECTIONS II

95. This plate is divided in the same manner as the preceding plate, and the problems occupy the same relative positions.

96. Intersections of Cylinders. — *The intersection lines of cylinders equal in diameter and intersecting one another in the same plane—that is, so that their axes also intersect—are always represented by straight lines in a view that shows the axes in their true length.* This is the case in the projections that are shown in Fig. 48, which represents objects that are familiar to all sheet-metal workers; namely, pipe angles (*a*), elbows (*b*), **Y**'s (*c*), and **T**'s (*d*). The same is also true of similar moldings "mitered" in a plane in which the true length of all members of such moldings may be shown. The line of intersection may always be found in such cases by bisecting the angle made by the pipe or moldings. Fig. 48 also illustrates an example of line shading often used by draftsmen to designate cylindrical surfaces on

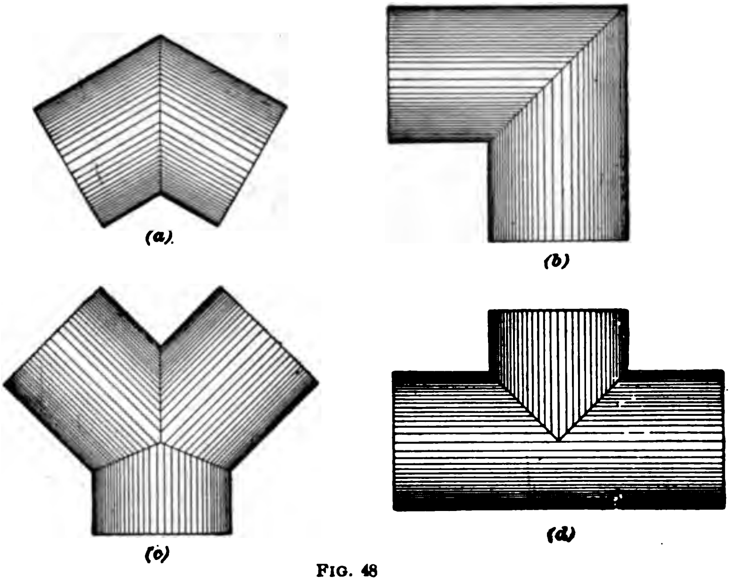


FIG. 48

working drawings. When cylinders of different diameters intersect (or cylinders of the same diameter whose axes do not intersect), the lines of intersection in any view are curved lines, and are found in a manner similar to the projection in the last problem. The lines, however, that are drawn on any one cylinder must be projected to the intersecting cylinder, for it is at the intersection of lines thus drawn that points are established through which the curve of intersection may be traced. This is illustrated in Problem 24.

PROBLEM 24

97. To project views of intersecting cylinders of unequal diameters.

EXPLANATION.—Fig. 49 is a perspective view of a branch Y of occasional occurrence in blowpipe work. The arrangement of the views and the method of projection are shown on the plate, and since it is similar to those of drawings that have been made on the preceding plate, no definite instructions are necessary. The diameter of the larger cylinder is 2 inches and that of the smaller 1 inch, while the angle of intersection shown in the elevation is 45°. The position of the two cylinders is such that the outline *r s* of the smaller cylinder in the plan is tangent to the circle that represents the large cylinder. Note the position of points on the circle that represents the end surface of the smaller cylinder in the plan, at *a*, *b*, and *c*, and their corresponding location at *a'*, *b'*, *c'*, etc. in the elevation (Art. **94**). Also observe that certain points on lines drawn on the surface of the smaller cylinder, as the lines *m n*, *p q*, and *r s* in the plan, are projected to the front elevation, where they are represented as lines at *n' n''*, *q' q''*, and *s' s''*. These corresponding lines are in the same plane and are the lines drawn on the surface of the larger cylinder, as above mentioned. Through the points of their intersection with lines drawn from *a' b'*, and *c'*

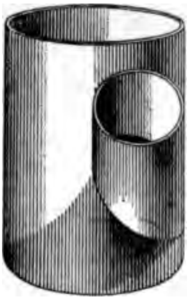


FIG. 49

in the elevation, the curved line of intersection of the two cylinders is to be traced, thus completing the problem. Project the side elevation as heretofore.

PROBLEM 25

98. To project views of a cone intersected by a cylinder, the axes of the two solids being parallel.

EXPLANATION.—Fig. 50 is a perspective view of a steam-exhaust head, a modification of which is in common use. The drawing on the plate is a complete projection of the same, in which the proportion of the cylinder *C* is increased, in order to show the method of projection to better advantage. Observe that the position of the object is reversed, as the drawing is thereby facilitated. The elevations on the plate show that the lines of intersection of the two cylinders *A* and *B* with the cone are represented by straight lines; this is always the case when a cylinder whose diameter is equal to a section of the cone intersects in this manner.

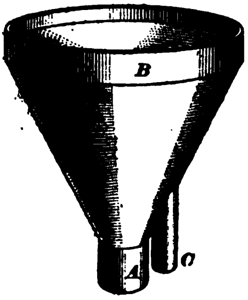


FIG. 50

CONSTRUCTION.—First, construct the elevation of the cone in its proper place on the drawing and in accordance with the dimensions given on the plate. The cylinders *A* and *B* are then drawn as shown. Next, draw the plan and represent the outline of the cylinder *C* in that view by a circle of the diameter indicated on the plate. Divide this outline into a convenient number of equal spaces (in this case 8), as shown at *a*. *b*, *c*, etc., and through each of these points draw elements of the cone, as *O p*, *O q*, etc. Project these elements to the elevation and locate thereon by projection methods the position of the points of intersection *a'*, *h'*, *g'*, etc. Complete the elevation by drawing the outline of the cylinder *C* in that view and tracing the line of intersection of the two solids through the points *a'*, *h'*, *g'* etc. Project the side elevation by the usual methods.

PROBLEM 26

99. To project views of intersecting cones.

Fig. 51 is a perspective view of an object that illustrates this problem. The cones are shown in a somewhat more convenient proportion for this problem in the projections on the plate. The construction of this problem must be followed very carefully, as a number of the operations are necessarily made over one another on the drawing and the student must be careful to distinguish each process.

FIG. 51

CONSTRUCTION.—Describe a circle $2\frac{3}{4}$ inches in diameter in the plan to represent the lower base of the larger, or intersected, cone in that view; and from the same center describe a circle $\frac{7}{8}$ inch in diameter, to represent the upper base. Project the front elevation of this cone and define the frustum $2\frac{1}{4}$ inches high, as shown on the plate, producing the outlines until they meet in the vertex. Next, through the point *F*, Fig. 52, draw the axial line of the smaller, or intersecting, cone (the line *A B*, Fig. 52) at an angle of 45° with the base of the larger cone; locate the point *A* 3 inches from *F*, and, after fixing the point *C* 1 inch from *F*, draw *C D* perpendicular to *A B*. Draw the outline of the upper base of the

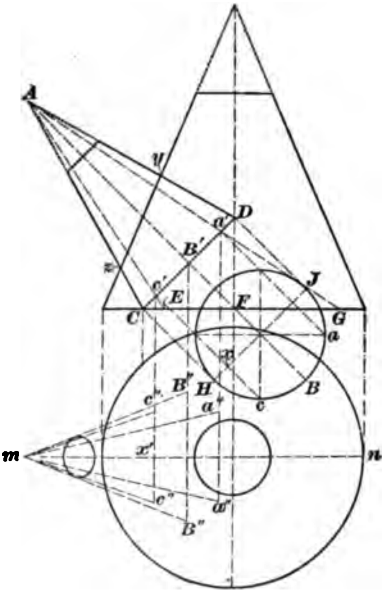


FIG. 52

smaller cone in the elevation parallel to *C D* and ¾ inch from *A*. Draw the full view of the base of the smaller cone at *J B H*; divide this outline into a convenient number of equal parts, as at *J*, *a*, *B*, *c*, etc., and project these points to the base *C D*. Draw the elements of the smaller cone, as shown in Fig. 52, and produce them until they intersect the base of the large cone at *E*, *F*, and *G*.* In the plan of this drawing, a series of sections of both cones are now drawn as the sectional curves would appear if each of the elements of the smaller cone shown in the elevation were considered as a cutting plane, as in Problem 20. The sections of the smaller cone will in each case be a triangle (Art. **86**), while the sections of the larger cone will be elliptical, parabolic, or hyperbolic curves, as the case may be. The point of intersection with the side of each triangle and its corresponding sectional curve of the larger cone is then projected to the elevation and the line of intersection of the two cones traced through these points. The sectional triangles of the smaller cone are shown in the plan of Fig. 52. Draw vertical primary projectors to the plan from points a', B', and c', and on these projectors set off distances from the horizontal center line of the plan similar to the distances from *H J* in the full view of the base; that is, make $x'\,c''$ equal $x\,c$, etc.

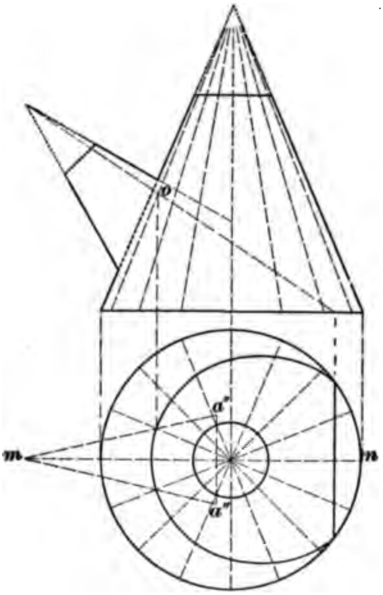


FIG. 53

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* In this case eight elements are represented, since it is not desirable to complicate the drawing by using more, although in practical work it will be found necessary to use a larger number of points in order that the line of intersection may be more accurately traced.

It is not necessary to develop the sectional curves of the larger cone in their entire length, since all that is required is to find a point on each curve that is at the intersection of the triangular sections of the smaller cone. This is better illustrated in Fig. 53, in which is shown the projection of the point *o* in the line of intersection. A study of this figure will show to the student that the operations are similar to those of Problem 20; the process in the case of each point is merely a repetition of that here indicated and need not be further explained. The extreme upper and lower points *y* and *z*, Fig. 52, are the points of intersection of the central section and may be projected directly to the plan from the elevation, since the outline of the figure in the elevation is really a section on the line *m n* in the plan of Fig. 52. Fig. 54 shows the three sections produced by the above method; that is, the irregular curve *p q r*, Fig. 54, is a section of the larger cone produced by the intersection of the cutting plane *A E*; *s s* and *t t* are found as above described, the section lines being indicated in the figure by short cross-hatching. Project the side elevation as in former problems.

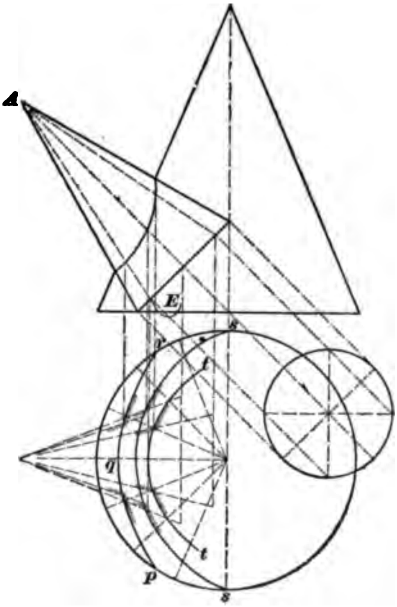

FIG. 54

The student may at the completion of the drawing on the plate erase all the construction lines except those projectors shown on the reduced copy of the plate, those lines only being inked in that are necessary to show the outlines of the figure and the line of intersection in each view, as well as the outer projectors.

PROBLEM 27

100. To project views of a sphere intersected by a cylinder.

When the position of these two solids is such that the axis of the cylinder passes through the center of the sphere, as shown in perspective in Fig. 55, the line of intersection is shown in a right elevation as a straight line. In the case of the solid shown in perspective in Fig. 56, the axis of the cylinder does not pass through the center of the sphere, and the line of intersection is an irregular curve, which may be found by the following method. Since the construction of this problem involves lines that must necessarily be drawn closely together in the small scale adopted for these drawings on the plate, a proportion is selected that does not admit of the entire figure being shown in the elevations; these projections are therefore finished by broken lines, as indicated on the plate.

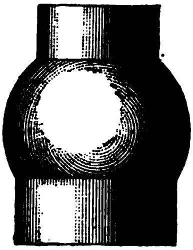

FIG. 55

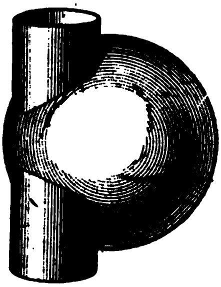

FIG. 56

CONSTRUCTION.—Draw the plan first and represent the view of the sphere by a circle 3½ inches in diameter. Draw a vertical diameter and from a point midway on the radius *a b* describe a circle 1⅜ inches in diameter, to represent the end view of the cylinder, on the outline of which locate a number of points by spacing with the dividers, as at *1*, *2*, *3*, *4*, etc. Through these points, in the manner shown on the plate, draw vertical lines, as *a b*, *c d*, *e f*, *g h*, and *i j*, each of which will now represent a cutting plane. Sections of the sphere and cylinder on these lines are now to be produced in the side elevation. These sections, as already stated, are circles and parallelograms, respectively, the diameter of the circles being ascertained from the view in which the cutting plane is shown on edge, as in the plan. In this problem, the side elevation is next projected. Describe arcs representing the sections in the side elevation, using the radii *a b*, *c d*, *e f*, *g h*, and *i j*, the radius *a b* being the great circle of the sphere. By the aid of primary projectors, project the points from the cylinder in the plan to the side elevation, intersecting corresponding arcs in the side elevation. Trace the irregular curve shown in that view on the plate through these points. Next project the front elevation, thus completing the problem.

101. Recapitulation.—The problems of this plate have afforded the student an opportunity for careful study. Owing to the number of points necessary to be found in each figure, the problems may have had the appearance of more or less complication, but if the student, as previously cautioned, will carefully locate the points required, *one at a time*, not hurrying his work nor trying to grasp the entire problem at once, but keeping in mind the different principles in the order presented, and by referring, if necessary, again and again to the primary principles, he will experience no difficulty in making the drawings. He will also be able intelligently to project any view of any object; in other words, he will be able to make any working drawing whatever, and, in addition to this, be able to read and understand any working drawing he may be called on to examine.

DEVELOPMENT OF SURFACES

INTRODUCTION

1. Definition of a Development.—A development is a drawing in which a full view of all the surfaces of a solid is represented. Whenever a development is to be drawn (except in the case of solids of very simple form), a projection drawing must first be made. This projection should show the solid in a *right* position. Since the location of the various points in a development is dependent on their corresponding position in the projection drawing, the importance of the projection and the necessity for accuracy in its construction are thus clearly seen. If a solid is bounded entirely by *plane* surfaces, its development can be accomplished by merely projecting their full views, as already explained in *Practical Projection.*

A solid is said to be developed when all surfaces composing it are represented on one plane and in such relation to one another that, if formed or bent up, they will constitute a solid similar to the one represented by the projection drawing from which the development was made. Such a representation is called a **development,** or a **pattern,** the process of laying out the pattern being termed *developing the surfaces of the solid.*

2. Relation of the Surfaces in a Pattern.—When it is desired to produce a pattern requiring a combination of several surfaces that are adjacent in a solid, such surfaces must be drawn in the same relation to one another in the

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development. The surfaces of a solid when thus combined in a pattern, or development, bear the same relation to one another that they would if they were considered as being *unfolded* or *unrolled*—the same relation that a paper wrapper would bear to the package from which it had been unfolded or unrolled. The paper wrapper is not always an apt illustration, as the metal worker seldom requires several thicknesses of his material. In the case of the familiar "square pan," however, the ends are folded on one another in precisely the same way as in the paper wrapper.

It will be seen from the foregoing that, were all solids bounded by flat, or plane, surfaces, the subject of developments would present no new problems; it would be necessary merely to study the relation of surfaces to one another, project their full views, and carefully redraw them in the pattern in the same relative position.

3. Projection Methods Used.—It has been shown in *Practical Projection* that a single surface is developed, or, as stated, its full view is drawn, by a modification of the same methods that are used to produce the different views of that surface. Many of the operations attendant on the development of solids are like those used in producing full views of single surfaces; or, if not, the principles involved may be traced to their origin in other methods used in projection drawing.

A thorough knowledge of projection is absolutely necessary that the student may understand the operations involved in developing the surfaces of a solid. The position of the several points located in a drawing and their corresponding location in an imaginary way on the object itself must be definitely fixed in the student's mind. Each line must be determined in its relation to the other lines of the drawing and its ideal, or imaginary, location definitely ascertained; the surfaces, also, must be treated in a similar way. The student must picture to himself the completed object as it will appear when the surfaces laid out on the drawing board in the development are formed up in their final relation to one another. This imaginary part of the study is of even greater importance in the case of developments than in projection drawing. As the student has already had some drill in this part of the work, the subject he is now studying should be found less difficult than would otherwise be the case. In projection drawing, the surfaces of the solid are represented as being in their proper position; in the development, the same surfaces are represented as being developed or spread out on the surface of the drawing board.

GENERAL CLASSIFICATION

4. General Classification of Solids.—An accurate development may be drawn for the plane surfaces of any solid, or for surfaces having, when related to a given line on such surface, a curvature in one direction only. In general, it may be stated that any solid may be developed on whose surfaces it is possible to lay a straightedge, in continuous contact, in any one direction. To use in this connection the illustration of the cylinder, it will be seen that, if the straightedge is resting on the surface parallel to the axis of the cylinder, it will remain in contact at all points. If, on the other hand, the straightedge is resting on the curved surface and is not parallel to the axis of the cylinder, the surface will be in contact at a single point only. However, the fact that it is possible to place the straightedge in continuous contact on the surface allows the inference that such surface is capable of accurate development.

The same rule applies to solids of irregular form. The methods of development, however, are not the same in certain variously formed solids, as will be explained later. There are certain forms whose surfaces, owing to their curvature in several directions, are not capable of being thus laid out on a flat surface, i. e., not capable of being developed. On the surfaces of solids of this class—the sphere, for example—it will be found impossible to lay the straightedge in contact in any direction. For, if placed on such a surface, there will be but one point of contact—that of the *tangential* point. Tangential contact indicates that development can be accomplished only in an approximate way. For purposes of development, then, it is convenient to separate all solids into two general classes according to the result obtained in developing their surfaces. These two classes are: solids whose surfaces admit of accurate development and solids whose surfaces admit only of approximate development. Approximate developments are, however, so nearly accurate for the purposes of the sheet-metal worker that the kind of solid is more clearly marked by the method of developing its surface than by the result obtained by the development. Therefore, in order to distinguish the kinds of solids, both accurately and approximately developed solids are divided into three main classes according to the method used in developing their surfaces. These classes are explained later.

5. Accurate Developments.—Solids whose surfaces are capable of accurate development are of frequent occurrence in the sheet-metal-working trades. To this class belong all prismatic, cylindrical, and conical forms, whether of regular or irregular geometrical form. It includes all articles or objects whose covering may be formed without being submitted to the operations known to trade workers as "raising," or "bumping." Any solid whose surfaces may be unrolled or spread out on a flat surface without "buckling" may be accurately developed. Although it is often necessary, especially when working metal of unusual thickness, to take into account the stretching of the material when producing patterns for many objects, these objects belong to accurately developed solids, providing that the metal does not have to be "raised," or "bumped," in order to form the object. It is, therefore, essential that the metal worker should thoroughly understand the nature of the material and be well informed as to the best manner in which to provide for all laps and edges used in the construction of the finished article.

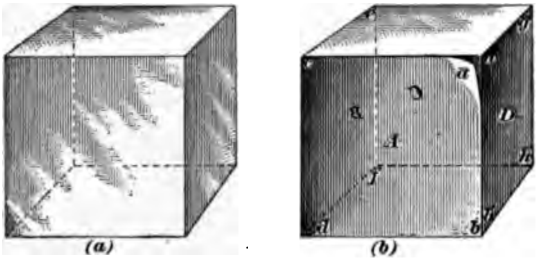
It is the purpose of *Development of Surfaces* to define and illustrate *theoretical developments* and the means used by the draftsman in their production.

6. Approximate Developments. — The sphere and other solids whose surfaces have a curvature in two or more directions are examples of objects capable of only approximate development. The test by the straightedge is (with the exception of the helicoidal surface) a positive indication of the class to which any solid may be assigned. Patterns for the surfaces of objects of this class may be *approximated*, because it is necessary for the metal to undergo the operations of "raising," or "bumping," before it will conform to the exact surface represented in the drawing. It is necessary in these cases to make allowance in the pattern for the stretching of the metal. Since this part of the subject does not belong to theoretical development, it is not treated here.

SOLIDS THAT MAY BE ACCURATELY DEVELOPED

7. There are three distinct methods in common use, by means of which patterns are produced for solids whose surfaces are capable of accurate development. It is advisable, therefore, to separate the different varieties of these forms into three general divisions, in order that their development may be studied in a systematic manner. This classification may be made by studying the manner in which the covering of these solids—to use again the illustration of a wrapper—would be unrolled or spread out if done by rolling the solid on a flat surface.

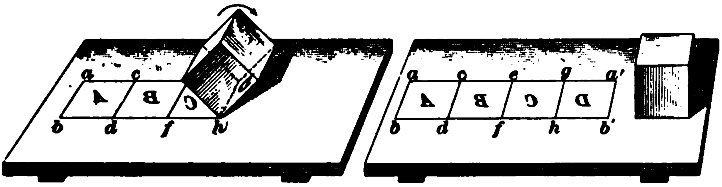
8. Solids Developed on Parallel Lines.—A convenient illustration of the manner in which the surfaces of a solid will appear when unrolled as above indicated may be found in the following example, which serves at the same time to define a property peculiar to solids of a certain form. Let the continuously adjacent surfaces of the prism shown in Fig. 1 (*a*) be carefully covered with thin paper, as at Fig. 1 (*b*). Denote each of the four surfaces by a letter, as *A*, *B*, *C*, and *D*, and further designate the edges of the prism by the letters *a b*, *c d*, *e f*, and *g h*. As the ends of the paper covering meet at the edge *a b*, that edge of the surface *D* may be denoted by the letters *a' b'*, as shown in Fig. 1 (*b*). Assume now that the prism is laid on the drawing board, the surface *A* face down, and the paper covering removed by turning the prism over and over, the paper remaining on the surface of the drawing board, as shown in Fig. 2 (*a*) and (*b*).



(*a*) (*b*)

FIG. 1

Two important principles relating to developments are demonstrated in these illustrations. *First*, as will be seen from Fig. 2 (*b*), the edges *a b*, *c d*, *e f*, *g h*, and *a' b'* are all parallel to one another. This is true both in the development and on the solid, as may be readily seen by the student, the only difference being that on the solid certain of the lines are in different planes, while in the development they are all in the same plane. *Second*, it will be noted

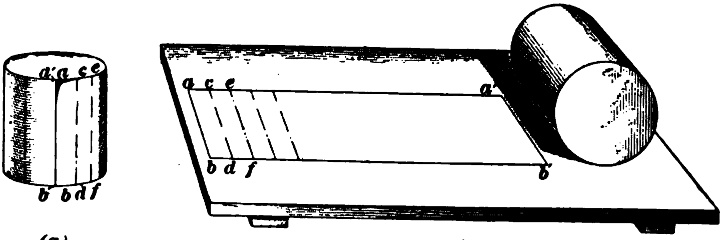


(*a*) (*b*)

FIG. 2

that, since the letters shown in Fig 2 are reversed, the *outer* surface of the paper covering in Fig. 1 (*b*) corresponds to the *under* surface in Fig. 2 (*b*). In a similar manner, it is learned from the second principle that positions indicated on any surface of a solid, as shown in a projection drawing, are reversed when shown in the development of the solid.

The same treatment of the cylinder is found to produce results closely resembling those shown in the case of the prism. The cylinder is represented in Fig. 3 (*a*) as covered



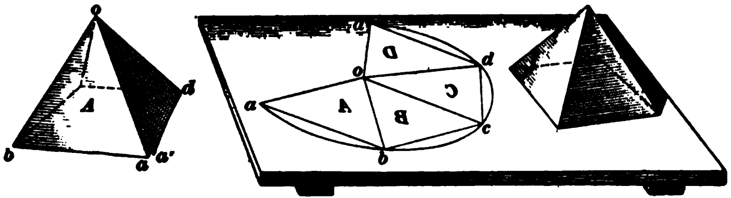
(*a*) (*b*)

FIG. 3

with paper, a number of lines being ruled on the covering parallel to its axis, as shown at cd, ef, etc. The paper is shown unrolled in Fig. 3 (*b*), and it will be observed that not only the outer edges ab and $a'b'$ are parallel to each other, but that all other lines parallel to the axis of the solid appear in the development parallel to one another and to the edge lines ab and $a'b'$. The student that has ever flattened out a piece of straight molding, as for cornice work, probably noticed a number of straight parallel lines on the metal where it had been bent in the brake. This is an illustration similar to that of the cylinder, the different members of the molding being considered as the various surfaces of an irregular solid.

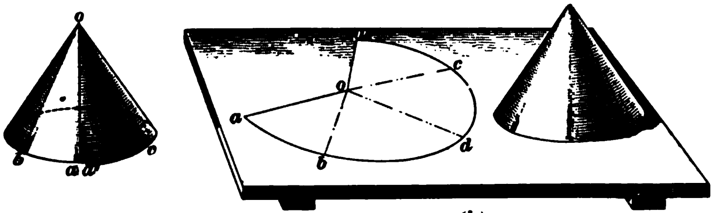
Such illustrations indicate that parallel lines bear some relation to certain forms, and it will be shown that the patterns for these forms are developed by a method whose principles are based on this fact. Many solids may be at once recognized as belonging to this division. In general, any solid whose edges are parallel may be located here. In the case of solids having curved surfaces, it may be stated that, if it is possible to draw a series of parallel lines on such surfaces, the development of the solid may be produced by the same methods given for this class. The first general division, therefore, comprises those solids whose surfaces may be developed on *parallel lines*.

9. Solids Developed on Radial Lines.—When the test given to the cube and the cylinder in Figs. 1 to 8 is applied to the pyramid, it is found that the lines indicated on the paper converge to a point, as shown in Fig. 4 (*a*) and (*b*). It is noticed, also, that this point *o*, Fig. 4, defines the position of the vertex of the pyramid. The same may be said of the cone, illustrated in Fig. 5 (*a*) and (*b*). If lines are first indicated on the surface of the cone corresponding to its elements, it will be found, when the covering is unrolled, that these lines also converge to a point, as in the case of the edges of the pyramid.



(*a*) (*b*)

Fig. 4



(*a*) (*b*)

Fig. 5

It was found possible to institute a system of obtaining developments based on parallel lines in the case of the prism and cylinder; in a similar manner, it is quite evident in this case that a system dealing with radial lines should produce like results. Since, in projection drawing, the elements of the cone are known to be useful factors in determining the position of points on its surface, it may readily be conceived that their use in a somewhat similar way may be adapted to developments. This is found to be the case; and a second general division of solids is thus made, consisting of those forms whose surfaces may be developed on *radial lines*. Included in this division are all regular tapering solids and such irregular forms as are derived from regular solids. The metal trades furnish many examples of solids belonging to this division; in fact, the writers of several works on patterncutting confine their instruction almost entirely to the development of solids of this character.

10. Solids Developed by Triangulation.—There are many forms of irregular surfaces to which the test of the straightedge may be applied and the conclusion thereby reached that their surfaces admit of accurate development. It may also be concluded that neither of the two former methods is applicable, for neither parallel lines nor a series of radial lines may be drawn on their surfaces. Many of these solids are not of such a shape as to admit of their being either turned or rolled on a plane surface. It is found, however, that on every such surface, series of two or more lines each may be drawn in certain directions, forming angles.

On such irregular surfaces it may happen that no two of the angles thus drawn on the solid, or represented—either correctly or foreshortened—in the projection drawing, will lie in the same plane or be equal to each other. Since it is possible thus to project these angles, evidently they may be reproduced on the flat surface of the drawing paper in their correct size. If this can be done, it may be reasonably assumed that the surfaces thus represented will be the same as the corresponding surfaces of the solid. An illustration of this principle, as pertaining to a plane surface, was given under another heading in *Practical Projection.*

In Fig. 6 an irregular solid of this kind is shown. It is the solid whose projection was drawn in Problem 19 of *Practical Projection.* This figure illustrates in a general way the method used in arranging the triangles on the irregular surface of such solids. The triangles are represented in the figure in a perspective way, but they are, of course, always drawn in connection with the usual methods of projection. The third general division, therefore, consists of those solids whose surfaces are developed by *triangulation*—that is, by means of triangles.

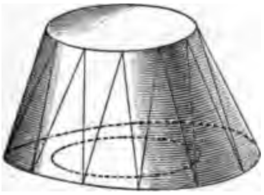


FIG. 6

11. How the Division of Solids Is Accomplished. It is not to be understood that the draftsman actually applies the test of the straightedge in reaching a conclusion as to whether the surfaces of a solid may be accurately or approximately developed. Nor does he roll the object on the drawing board in order to determine whether the method by parallel lines or one of the other methods is to be used. As a matter of fact, he seldom has a model to work from, and, therefore, could not apply such a test if he so desired. But as he studies the drawings and imagines the position of the surfaces as they will appear in the completed object, he is enabled to apply the tests as effectually, in an imaginary way, as though the tests were made with a straightedge. In the same imaginary way, also, he assigns the solid to the general division to which it properly belongs, and thus decides as to the method he will use in the development of its surfaces.

A little practice will enable the student to classify the variously formed objects in this way and to select the method that shall be applied in any given case. A very important part of the patterncutter's acquirements consists in being able to recognize in various irregular objects those forms that may be only a portion of some regular solid. In other words, the student must learn to establish in his own mind the connection between complete and perfectly formed solids and those objects in which only a portion of the solid may be represented. The method of development is, of course, the same in both cases, but as a matter of fact, the operations are usually more complicated in cases where the incomplete solid demands the patterncutter's attention. Especially is this true of conical forms, or those developed on radial lines. Frequent illustrations of this principle may be found in commonly occurring objects. The flaring pail shown in Fig. 7 is seen to be a part (or frustum) of a cone, the completed cone being indicated by the light shading in the illustration.

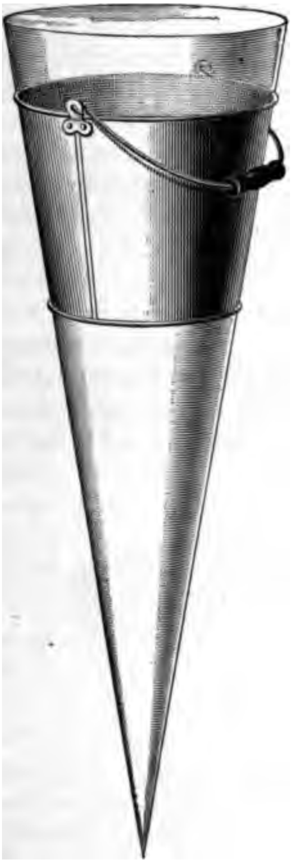


FIG. 7

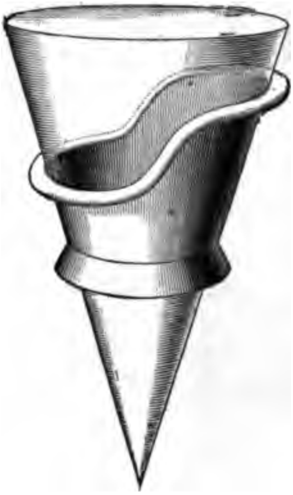


FIG. 8

The same is true of the sitz bathtub in Fig. 8. Here it is seen that the portion of the cone represented by the finished article is an irregular section of the cone; its development is, however, accomplished by the same methods to be shown for regular cones. Another instance is found in the measure shown in Fig. 9. Here are two intersecting cones: a regular frustum of one forms the body of the measure; and an irregular frustum of the other—an inverted cone—forms the lip of the finished article.

All the articles in Figs. 7 to 9 are thus shown to be frustums of regular cones, although varying in the regularity of their bases. In certain cases, as in the "oval" pan body represented by the heavily shaded band in Fig. 10, the surfaces may be portions of the surfaces of different cones or of cones differing in size. The bases of this article are elliptical in outline. The ellipses are drawn by circular

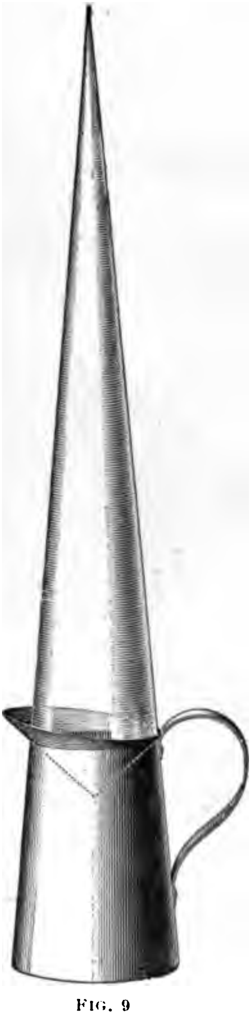


FIG. 9

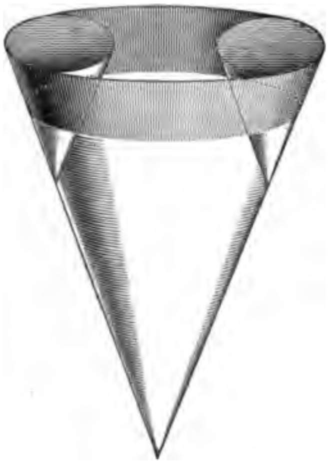


FIG. 10

arcs. The vertexes of the different cones would be represented in a plan view by the centers from which the different arcs are struck. These cones are partially shown in Fig. 10, and in the relation required by the portions of their surfaces that compose the sides of the pan.

It is essential, therefore, that the student should possess a certain familiarity with the forms of the regular solids, to assist him in the classification of the objects that he will be called on to develop. It is with this end in view that a series of plates is to be drawn by the student. The instruction is in the form of problems, and several of the drawings of *Practical Projection* are reproduced for the purpose of showing the development of the surfaces of the solids there represented. The student's attention will first be directed to a consideration of those solids whose development may be accomplished by means of parallel lines.

DEVELOPMENT BY PARALLEL LINES

12. Importance of Certain Views.—It has already been stated that the making of a working drawing is the draftsman's first step towards obtaining a pattern for the surfaces of any solid. The solid in question should be shown in this drawing in such a position that measurements may be taken of its surfaces in all their dimensions. In order to accomplish this, several views may have to be drawn, although a right plan and elevation will usually be sufficient. In some cases there may be given to the mechanic a drawing in which the object is so shown that these dimensions may not be readily obtained. In such cases, operations in projection are required; with these the student is already familiar.

For purposes of development it is important that the view shown should be that one in which the lines of the solid are given in their true length, or, in other words, a *right view* of the solid. In addition to this view there must be given that view also in which the surfaces thus partially bounded by these parallel lines are shown as *on edge.* In some instances, as in the case of a simple solid of which all the dimensions are known, the latter view may be omitted; it is, however, understood as being drawn, for the draftsman knows all its dimensions. Generally, therefore, before the pattern can be produced, a plan and an elevation showing the solid in a *right position* must be drawn.

In drawing the projections in *Practical Projection*, the right views have first been drawn and inclined views have then been projected from them. This has been done in order to familiarize the student with the appearance of such drawings; but in every case the development of the pattern is to be projected from a right view. It is possible for the draftsman to become so expert by practice that in certain cases he is enabled to obtain, from foreshortened views, patterns for some surfaces. The beginner should not attempt this, however, since the operations involved are confusing, and should be resorted to only by the experienced pattern-cutter that thoroughly understands the subject.

13. The Stretchout.—As stated in the preceding article, it is essential that in all cases where the development of solids may be accomplished on parallel lines, the view showing certain surfaces as on edge should either be given or assumed. From such a view, the width of each surface may readily be ascertained. The total width of all these surfaces—the distance around the solid—is called the **girth** of the solid. In case the solid has a curved surface, its girth is found by spacing with the dividers the outline in that view. The girth of the cylinder, for example, is equal to the length of the circumference of a circle that represents the base of the cylinder.

When a distance corresponding to the girth of any solid is represented by a straight line on a flat surface, such a line is called a **stretchout** for the development, or pattern. This line is then marked off by a series of points, the points representing the places at which the line would be bent if formed up to correspond with the outline of the solid represented in the view from which the distances were taken. In the case of curved surfaces, a number of points are located on the outline, as previously indicated. This is usually done by dividing the outline into a number of equal spaces in the same manner as in projection drawing; an equal number of spaces is then stepped off on the stretchout line, whose total length is in all cases equal to the girth of the solid. An important point to be observed is that the points thus located on the stretchout must be (although reversed) in a position on the line corresponding to that relatively occupied on the solid.

14. Position of the Development.—The position in which the stretchout is placed on the drawing determines the position of the development. This line is always drawn *at right angles to the parallel lines of the solid* and from a view in which these parallel lines are shown *in their true length.*

In making a projection, then, from which to produce the patterns of any object, it is important that a sufficient space be left on the drawing, to one side or the other of that view. It frequently happens that this cannot be done, and in such cases it is a common practice to lay an extra piece of paper over a portion of the drawing, on which the development may be produced. When the latter method is adopted, the paper on which the development is made may be used in transferring the outline of the pattern to the metal, and the original drawing may then be preserved in perfect condition.

15. Development of the Cube.—For the purpose of explaining to better advantage the use of the stretchout, the development of the cube is presented, step by step, in Figs. 11 to 14, inclusive. A reproduction of this development should be made by the student in accordance with the following instructions, although the drawing is not to be sent to the Schools for correction.

The plan and the elevation shown in the left-hand portion of the figures are drawn first. The development could be produced from either view in this case, since in any right view the dimensions of a cube are equal. For the purpose of the illustration, the development is drawn from the elevation. At right angles to the parallel lines in the elevation draw a line as *M N*, Fig. 11. On this line locate a point at any convenient distance, as at *w*. This point may correspond to any of the upright lines or edges of the solid represented by *A*, *B*, *C*, or *D* in the plan of the cube. Since it is necessary to begin from one of these edges to unfold in an imaginary way the surfaces of the cube, the point *w* will be considered as representing a position on the edge *A*. The surfaces are to be represented in their regular order in the development, that is, the order in which they appear on the solid itself; first, the surface represented in the plan by *A B*, then *B C*, *C D*, and *D A*, in their natural order as they are shown in the projection at the left of the line *M N*. The dividers may, therefore, be set at a distance equal to the length of the side *A B*, and since the sides of the cube are equal in length, the distances *w x*, *x y*, *y z*, and *z w'*—corresponding, respectively, to the sides represented in the plan by *A B*, *B C*, *C D*, and *D A*—are to be spaced off on the line *M N*.

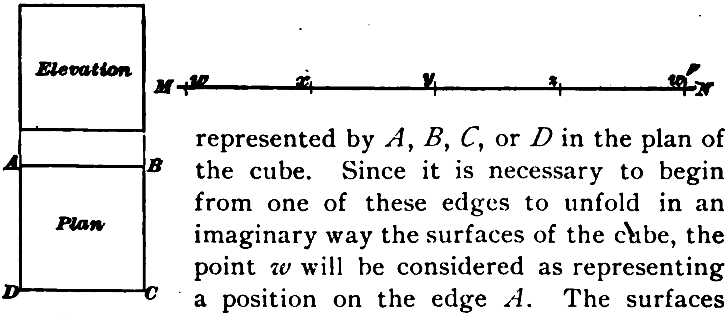


FIG. 11

16. Laying Off the Stretchout.—That portion of the line *M N* included between the points *w* and *w'* is called the *stretchout* of the cube. A stretchout may be drawn in any position on the drawing board, at the convenience of the draftsman, but it is invariably at *right angles to the parallel lines of the solid.*

Wherever the stretchout occurs in the drawings of this section, it is represented by a heavy line, as shown in Fig. 11. It is customary to draw a line of indefinite length quite near the view of the solid that is being developed, as in Fig. 11. When the stretchout is mentioned, the only part of the line referred to is that included between the extreme points *w* and *w'*, Fig. 11, located to define the total width of the adjacent surfaces. This operation is called *laying off*, or *developing*, the stretchout. It will be seen that if a string equal in length to *w w'* should be stretched around the cube in a horizontal direction, it would exactly reach the entire distance, and the ends would meet, in Fig. 11, at the edge *A*.

The next step is to erect perpendiculars to the stretchout *MN* that shall pass through the points *w*, *x*, *y*, etc. and be produced on both sides of the line. This is done by means of the triangle, in connection with the T square, as in Fig. 12. The lines thus drawn are called *edge lines*, since they represent, in the development, those portions of the surfaces that would form the edges of the solid if the pattern were to be cut out and formed up to the shape indicated by the projections. Edge lines are to be represented in these drawings by dash-and-dot lines, as shown in Fig. 12, similar to those used in *Practical Projection* for projectors.

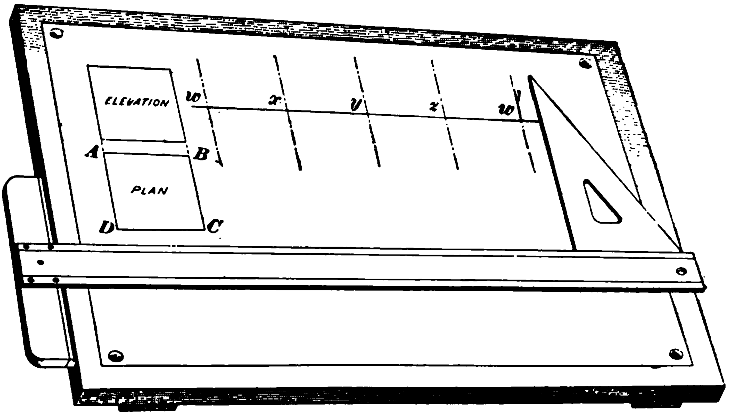


FIG. 12

Next, the length of each of the upright edges shown in the elevation is marked off on its corresponding edge line in the development. This is readily done with the T square, and, since each of the four edges represented in the elevation is of the same length, the T square may be brought even with the horizontal lines in that view, and a line drawn from each across the development, as shown in Fig. 13. The lines thus drawn are called *developers*, and will be represented in these drawings by broken lines, as shown in Fig. 13. This name is applied to them for the reason that the length of the edge lines is determined—developed—when a developer is drawn from each extremity of the edge shown in the elevation to its corresponding edge line in the development. In this case, the four upright edges of the cube, represented by the two vertical lines in the elevation, are of the same length, and their ends are in the same horizontal lines; therefore, the two developers *p q* and *r s*, drawn from the elevation, define the upper and lower boundaries of each of the four upright surfaces of the cube.

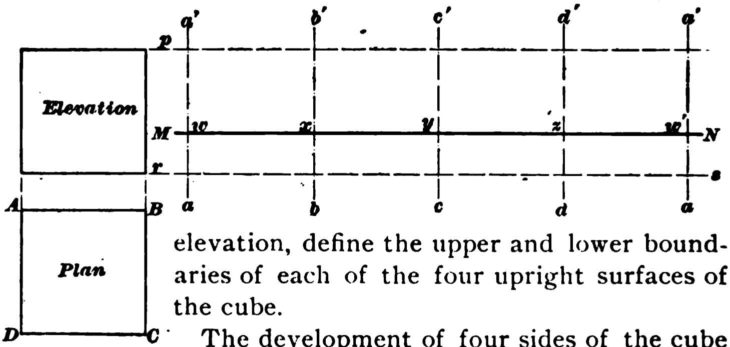


FIG. 13

The development of four sides of the cube is thus accomplished; and if desirable, the other two sides may be added to any one of the four that have been developed. Since the method of obtaining a development, as it is now called, or a full view of any side or surface of a solid is already familiar to the student, no further explanation will be given. In sheet-metal work it is generally preferable to make the ends (of forms similar to the cube) of separate pieces of metal, and, on account of the waste of stock, it will seldom be found desirable to combine all the surfaces of a solid in a single piece. Should such a case arise, however, the full view would be projected and afterwards copied into its proper place on the development.

The development of any solid of this class, whose bases are parallel and at right angles to its parallel lines, is always a parallelogram; and, as in the development of the cube in Fig. 13, this is divided into smaller parallelograms, each representing a surface of the solid.

17. Finishing the Drawing.—In order to enable the draftsman to distinguish the features of a development at a glance, it is customary to define the outer edges of such a drawing by means of full lines, as is shown in Fig. 14. The outer edges are further distinguished by small arrowheads, while the other edge lines of the pattern are marked near each extremity by a small circle drawn by freehand methods, in the manner shown, thus indicating to the mechanic that the sheet is to be bent along this line. It is sometimes desirable, as in detail developments for certain classes of sheet-metal work, to designate the stretchout by a line drawn with a blue pencil, thus readily attracting the draftsman's attention.

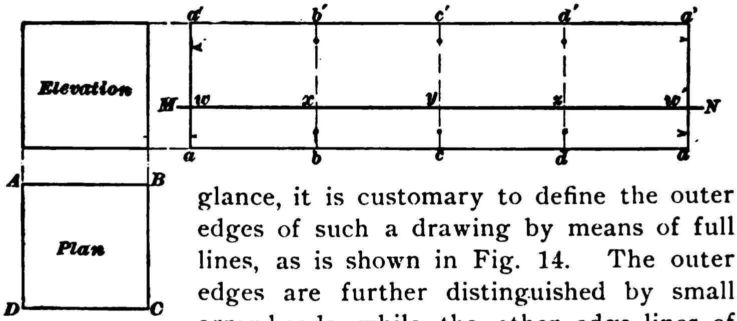


FIG. 14

The mechanic seldom resorts to the drawing board in order to produce a development of a simple solid such as the cube, since the same result may be accomplished with the steel square, the sizes being marked out directly on the metal. The development of the cube, however, has been shown in these illustrations, inasmuch as by the same principles any solid of this class may be developed. It may also be stated that the draftsman rarely represents developers or edge lines by the particular lines used for that purpose in this section. These distinguishing lines have been adopted here solely for the purpose of fixing clearly in the student's mind the principles on which these drawings are made. After these principles have been mastered, the use of such lines in practical work may be discontinued, and the student may then, by the use of light pencil lines only, proceed with the development of such other solids as he may be called on from time to time to lay out. These drawings, when inked in, should be completed in the manner shown in the illustrations.

18. Development of Intersected Solids.—In cases where the parallel lines of a solid are interrupted by the intersection of another solid or by a cutting plane, it becomes important to follow carefully the instructions just given; in such cases it is of extreme importance that an exact development should be made on the drawing board. For examples of the development of such intersected solids, we will refer to the figures illustrating the cutting planes in their effect on the cube, which figures have become familiar to the student from their use in *Practical Projection*. Fig. 15 is a reproduction of one of these projections, showing the development of the parallel surfaces. Here it will be seen that the development can be produced only from the elevation, since the cutting plane has altered the solid in such a manner as to admit of parallel lines being drawn in but one direction.

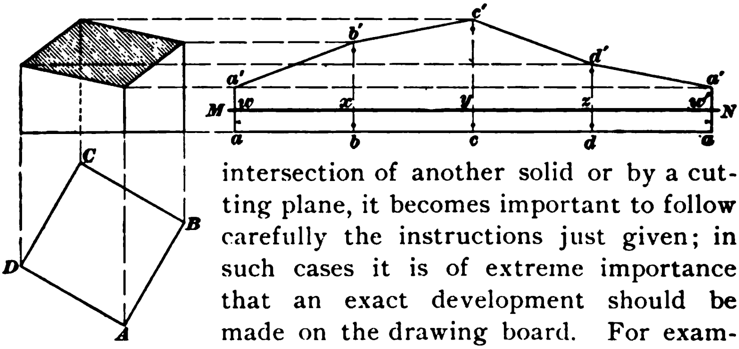


FIG. 15

The stretchout is drawn as before and the width of the surfaces spaced off in the usual manner. It will be further noticed that, since the edges of the cube are unequal in length, it becomes more important to observe the order of the surfaces as they are being unrolled from the solid. After the edge lines are drawn in the development, the developers are drawn in the same manner as in Fig. 13, but with this difference: the lower ends of the edge lines are defined by a single developer as before, but it becomes necessary to draw a developer from the upper end of each edge in the elevation to its corresponding edge line in the development. If this is done carefully, it will be seen, from a comparison of the surfaces in the development with those on the solid in the elevation, that they are in the same relative position with reference to one another, although reversed. This is clearly indicated in Fig. 15 by the use of similar capitals and small letters for the corresponding edges and edge lines, respectively, in the projection and development. Thus, the edge line *a a'* represents the edge *A*; *b b'* represents the edge *B*, etc. It will be noticed, also, that those parts of a development that come together and form edges or seams are indicated by the same letters. A similar principle of lettering these drawings will be continued throughout *Development of Surfaces*, since, when once understood by the student, he can study the drawings intelligently and with less reference to the descriptive text.

In this drawing of the cube, another fact is presented that demands care on the part of the draftsman; that is, the outer edge lines in the development must be of the same length. It may seem unnecessary to call attention to a fact so obvious, since it is very clear that, as the outer edge lines represent the same edge of the solid, they *must*, therefore, be of the same length in the development. It is, however, a cause of frequent error, and is due simply to carelessness in drawing the developer to the wrong edge line. Great care must be exercised in this respect, since the accuracy of a development depends in no small degree on this feature.

A development similar to the one given in Fig. 15 is shown in Fig. 16. This development is made from the front elevation of the figure in *Practical Projection* that shows how a plane that cuts a cube is represented. Fig. 15 shows a development made from the side elevation of the same figure. An excellent illustration is here furnished of the importance of all edges of a solid being defined in that view from which a development is made. The drawing in Fig. 15 represents the cube in such a position that the lengths of all its parallel edges are shown, while in Fig. 16, the length of the edge B is seen only by the aid of the dotted line. In drawings of this class, therefore, all edges should be indicated, whether on the side nearest the observer or not. In such drawings, however, it is customary to represent these hidden edges by dotted lines, in order to avoid confusion.

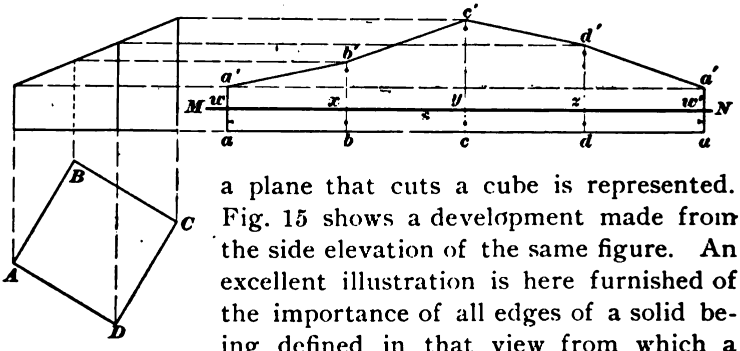


FIG. 16

It frequently happens that the cutting plane so intersects the surfaces of a solid as to produce angles at points other than at the vertical edges of the solid. An example of this is found in Fig. 17. The method of obtaining the development is, in the main, similar to that used in the preceding cases. From the plan of the cube in Fig. 17, however, it will be seen that points are indicated on the lines $B\ C$ and $C\ D$ denoting the corners, or points, at the extremities of the line $K\ L$. The distances $B\ K$ and $D\ L$ must, therefore, be indicated on the stretchout $M\ N$, as shown at $x\ k$ and $l\ z$, the points in each case being located from $b\ b'$ and $d\ d'$ towards $c\ c'$. This is because the points K and L approach C in the plan in their distances from B and D, respectively; according to the foregoing instruction, it is necessary to define them in a position in the development corresponding to that represented on the solid.

Parallel lines must be produced through the points *k* and *l* in the same manner as the edge lines were drawn. In a certain way, these lines serve a similar purpose, since they are the termination of certain developers; but as it is not necessary to bend the surfaces of the pattern on such lines, they are distinguished from the regular edge lines by the term *interedge lines.* Interedge lines are as essential in determining the outline of a development as the edge lines themselves, and are to be distinguished on the drawings for this section by the dash-and-double-dot line. A comparison of the projections with the development in Fig. 17, made by looking for similar capitals and small letters in the figure, will enable the student to see clearly the manner in which the additional spaces are located on the stretchout, and also how this development is produced from the clevation.

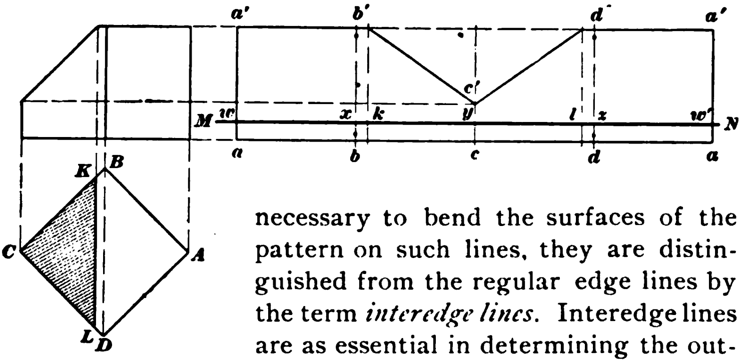


FIG. 17

19. Importance of Accuracy.—The attention of the student has frequently been called to the necessity for accuracy in his drawings. If this is necessary in the case of projection drawing, it is doubly important in the case of developments. Too much stress cannot be laid on this very important feature of the patterncutter's training. Unless his drawings are accurate they are of no value; it is, therefore, of the utmost importance that the patterns for any piece of sheet-metal work should be carefully and accurately described. The draftsman may thoroughly understand all the principles involved in projection drawing and the development of surfaces; but if the work on the drawing board has been done in a careless manner, the pattern is as liable to be incorrect as though it had been "guessed at," or "cut and trimmed."

There are few solids whose development may be accomplished by the aid of the limited number of lines required in the case of the cube. The same principles, however, govern all solids of this class, and it is necessary merely to be careful and observe the form of the solid as it is shown in the projection drawing. The fact that the drawing contains many lines should not deter the student from recognizing each surface independently of the others, although it may require more care on his part to select the correct lines in each case.

20. The Imagination a Great Help.—The student's imagination will be found to be his best assistant in this work, and by the aid of the projections he should picture to himself a model of the object represented. Further, he will find it a valuable help in being able to imagine the surfaces as they would appear if a covering of the solid were unrolled and spread out on the drawing board for the development. In this way, the corresponding surfaces on the solid and in the development may be compared, and the student may be able to detect any errors that might otherwise escape him.

GENERAL RULES FOR OBTAINING PARALLEL DEVELOPMENTS

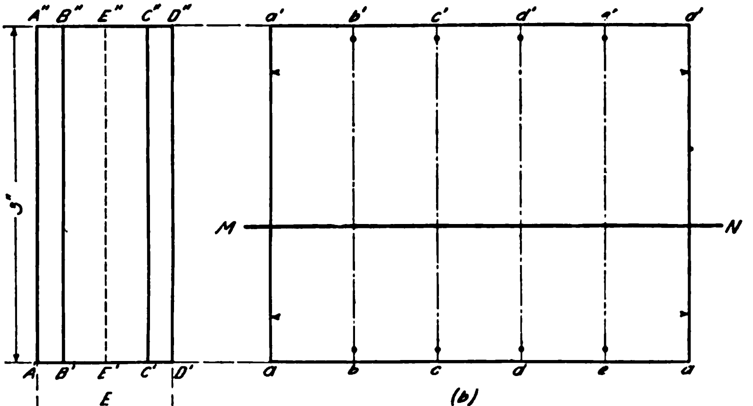
21. For the convenience of the student, and to aid him in the production of developments of solids by means of parallel lines, a general summary of the important features is here presented. This summary contains the substance of the foregoing instruction.

1. A projection must first be drawn, consisting of a plan and elevation, showing the solid in a right position.

2. The development is always obtained from that view in which the parallel lines are shown in their true lengths.

3. The stretchout is drawn at right angles to the parallel lines of the solid.

DEVEL



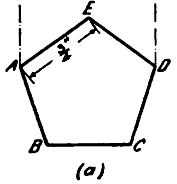
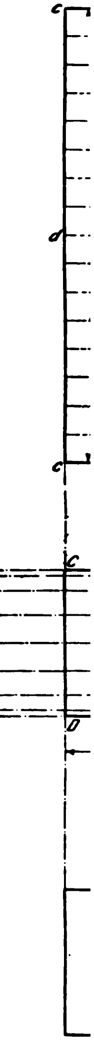
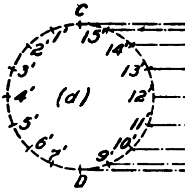


Fig. 1.





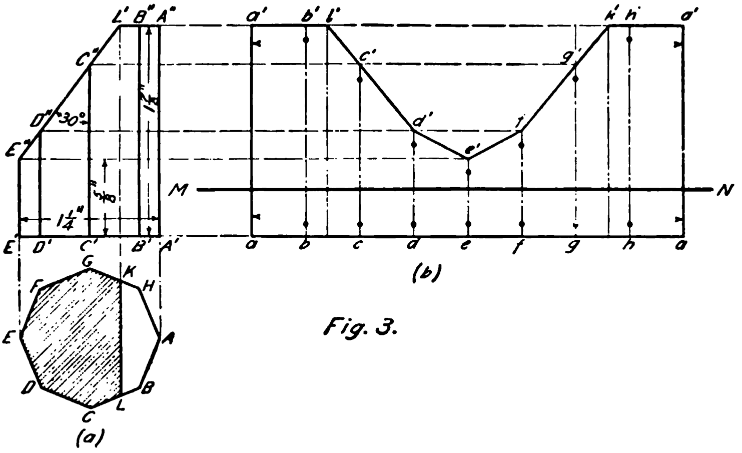


Fig. 3.

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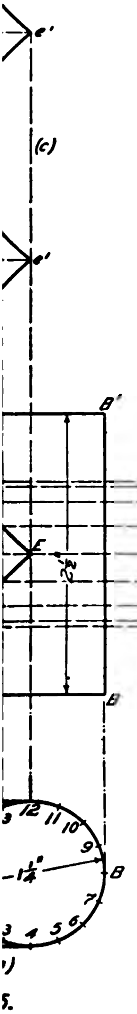

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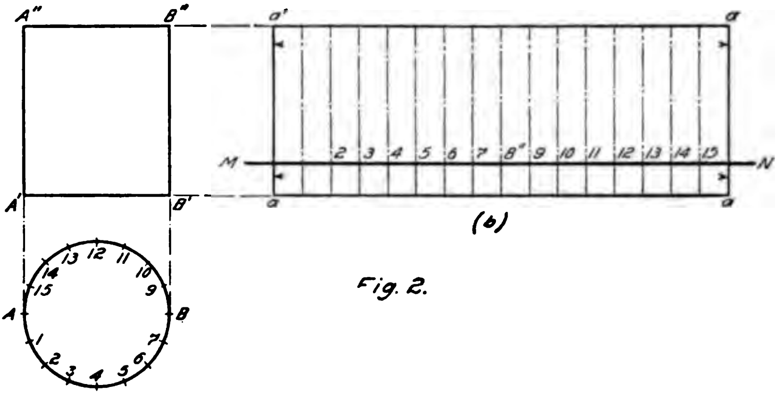
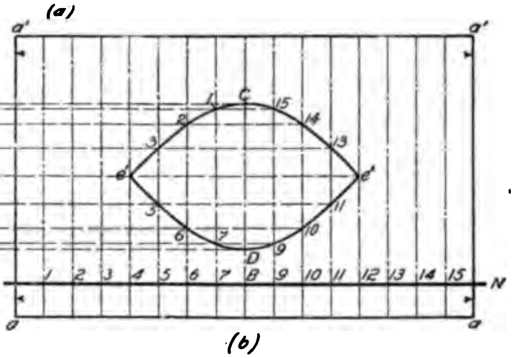


Fig. 2.



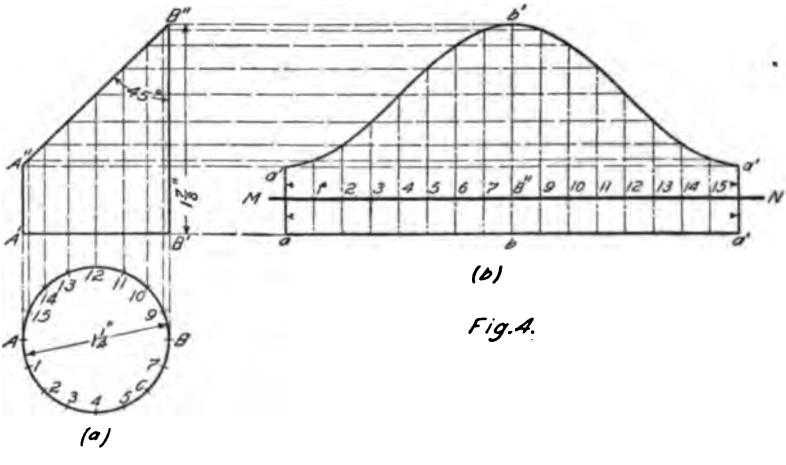


Fig. 4.

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served.

4. To indicate the width and relative position of the surfaces, points are located on the stretchout corresponding to the place of those points in a view that represents the surfaces on edge.

5. Edge lines and interedge lines are always drawn at right angles to the stretchout.

6. Developers are drawn from each edge or interedge represented in the projection drawing to the corresponding edge line or interedge line in the development. The position of points located on these lines is determined in a similar manner.

7. Interedge lines, when necessary for the development, must be indicated on the projection as well as on the development, and the same care exercised with the corresponding developers as with those drawn from edges to edge lines.

8. The length of the outer edge lines in a complete development must be defined by the same developers.

These instructions should be carefully observed by the student, and if the work involved in the accompanying problems is done in accordance with the principles just enumerated, no difficulties will be encountered that may not be readily overcome by careful study and a comparison of the drawings with these rules.

DRAWING PLATE, TITLE: DEVELOPMENTS I

22. For *Development of Surfaces* the student is required to draw five plates, which are the same size as those drawn for *Practical Projection* and in accordance with the same general instructions. The titles of the plates are given and are to be placed and lettered in the same manner as heretofore. The division lines between the problems are to be omitted, and in their place a general arrangement is to be followed which resembles the reduced copies of the drawings shown on the printed plates. The problems should first be drawn on separate paper to the given sizes. The developments may then be drawn in such positions on the plates as to present an appearance similar to that of the reduced copies.

These problems consist mainly of developments of solids whose projection occupied the attention of the student in the study of *Practical Projection*. They have been selected for this purpose because they are representative solids whose development affords an illustration of the principles involved in patterncutting. The student that desires to pursue the study at greater length may find convenient illustrations in other objects of frequent occurrence. Desirable practice may thus be obtained, and the practical application of the principles outlined will serve to fix them more definitely in the student's mind. No insurmountable difficulties are likely to be encountered in developments thus undertaken. Additional work of this kind should be of the same class as the developments explained in the text.

PROBLEM 1

23. To develop the surfaces of a pentagonal prism.

A perspective view of the prism is presented in Fig. 18. The drawing is to be made on the plate to the size indicated by the dimension figures there given. The completed development is shown at Fig. 1 (*a*) and (*b*) on the plate. First draw the plan in the position shown and then the elevation, according to the given sizes. Next, draw the stretchout *M N*, spacing off on its length, as previously instructed, the width of the surfaces; after this the edge lines are drawn, and finally the developers. As before, indicate the edge lines, and finish this and all drawings in the manner described in Arts. **16** to **18**.

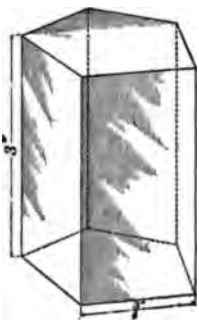

FIG. 18

PROBLEM 2

24. To develop the curved surface of a cylinder.

Explanation.—No perspective figure is given for this problem, the cylinder being 1¼ inches in diameter and 1½ inches high. Since there are no parallel edges presented on the curved surface of the cylinder, it is necessary to assume them. When edges are thus assumed for the curved surface of any solid, the corresponding edge lines in the development are represented in the same manner as for the prism. It is unnecessary, however, to designate assumed edge lines by small circles, since there is no angular bending on such lines when the surface is formed to the shape indicated in the projections.

When it becomes necessary, on account of the intersection of another solid or plane with the cylinder, to represent intermediate lines, they are then treated as interedge lines, the same as for the prism. Edge lines and interedge lines, therefore, bear the same relation to the development of curved surfaces as to plane surfaces. But it is necessary to exercise more care in the development of curved surfaces, since these lines are not so readily distinguished from one another as in the case of prisms. Indicators may be marked on the outer edge lines; but, since the others are assumed merely for the purposes of development, the small circular indicators are omitted. In the case of a regular cylinder, as in this problem, it is unnecessary to project the edge lines to the elevation, and this work may be omitted in the construction of the problem on the plate.

Construction.—Draw first the plan and elevation shown on the plate at Fig. 2 (*a*), giving the figure the required dimensions. Next, divide the outline in the plan into a convenient number of equal spaces (in this case 16). Draw the stretchout *M N* as heretofore, and lay off on this line an equal number of spaces similar to those on the plan of the cylinder; draw the edge lines as shown, and complete the development by drawing the two necessary developers from the elevation. Then finish the drawing in the usual mnaner, as shown at Fig. 2 (*b*).

PROBLEM 3

25. To develop the surfaces of an intersected octagonal prism.

Fig. 19 is a perspective view of the prism projected in *Practical Projection* in order to show an octagonal prism cut by a plane; the dimensions of the figure, however, are slightly changed, as may be seen from the plate. The drawing of the projections is similar to that in the problem referred to and may be completed as shown on the plate at Fig. 3 (*a*). This problem requires interedge lines, as in the case of the cube in Fig. 17; more developers, too, are needed than are used for preceding problems. Aside from these features, the drawing does not differ materially from those that have preceded it. After developing the stretchout, as in previous problems, and drawing the edge lines and interedge lines at right angles to the stretchout, developers are drawn from the several points indicated in the elevation; viz., E'', D'', C'', etc., as shown. Care must be used to terminate each developer on the line corresponding to each edge or interedge, as the case may be.

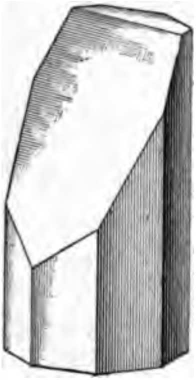

FIG. 19

After this development is completed, the student may derive some assistance by cutting out a paper model according to the lines indicated and bending it to conform to the octagonal prism; he should then understand exactly what is implied by the operations that have been explained.

PROBLEM 4

26. To develop the curved surface of an intersected cylinder.

Fig. 20 is a perspective drawing of the intersected cylinder required for this development; the dimensions in Fig. 4 on the plate indicate the size the drawing must appear thereon.

This problem is very similar to Problem 2, the only difference being that it is necessary to project the edge lines to the elevation in order to obtain the points of their intersection with the cutting plane. From these points developers are drawn to their corresponding edge lines in the development. The curve traced through points of intersection in that portion of the drawing is the upper outline of the development. This drawing being fully shown on the plate at Fig. 4 (*a*) and (*b*), the student should have no difficulty in completing the problem. Since the cutting plane in this problem is at an angle of 45°, the development may be used as a pattern for a two-pieced elbow.

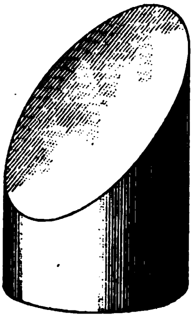

FIG. 20

PROBLEM 5

27. To develop the surfaces of two intersecting cylinders.

EXPLANATION.—Fig. 21 is a perspective view of intersecting cylinders. Since the two cylinders are of the same diameter, their axes intersecting, the lines of intersection are represented on the drawing by straight lines.

CONSTRUCTION.—The projections shown on the plate at Fig. 5 (*a*) are first completed and an end view of the shorter cylinder projected as shown at (*d*). The outline of each cylinder in that view in which it is represented as on edge is then divided into a similar number of equal spaces (16 in Fig. 5). The points thus located for the purpose of representing the assumed edges are then projected from each view to the elevation. Stretchouts MN and $M'N'$ are then developed, and edge lines are drawn perpendicular to the stretchout in each development. Developers may now be drawn from the ends of all assumed edges in the elevation

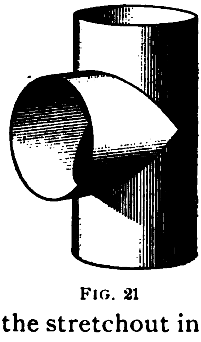

FIG. 21

to their corresponding edge lines in the developments. Note that in the development of the vertical cylinder the outline of the intersection of the horizontal cylinder is projected to any set of edge lines desired, the position of the outline of the opening being optional with the draftsman.

The development of the intersecting cylinder could be drawn from either the plan or the elevation in this case, since the true length of the parallel lines of that solid is shown in either view; for the sake of the appearance of the drawing, it is developed from the elevation. The development of the intersected solid is here shown to be a parallelogram having an irregularly curved portion outlined in the central part of the figure. It may be seen that the plan of the cylinders is not absolutely necessary for this development, since the edge lines from each circle intersect in the same points on the line of intersection of the two cylinders. In the practical work of such developments, therefore, one of the full views may hereafter be omitted.

28. Recapitulation.—These five problems are to be copied on the plate in the relative positions indicated on the printed copy. Care should be exercised that the figures when completed occupy central positions on the plate, and an equal distance should be left on all sides of the drawing. The plate will then present a neat appearance, and the developments may be easily distinguished from one another. When finishing these drawings on the plate, the various features are to be represented as follows:

Represent the boundaries of figures and all visible parts of solids by light full lines. ————————————

Hidden edges and hidden intersection lines should be represented by light dotted lines. – – – – – – – – – – – – – –

Projectors, as heretofore, are to be indicated by dot-and-dash lines. ——·——·——·——·——·——·——

Edge lines are indicated by dot-and-dash lines, used also for projectors.

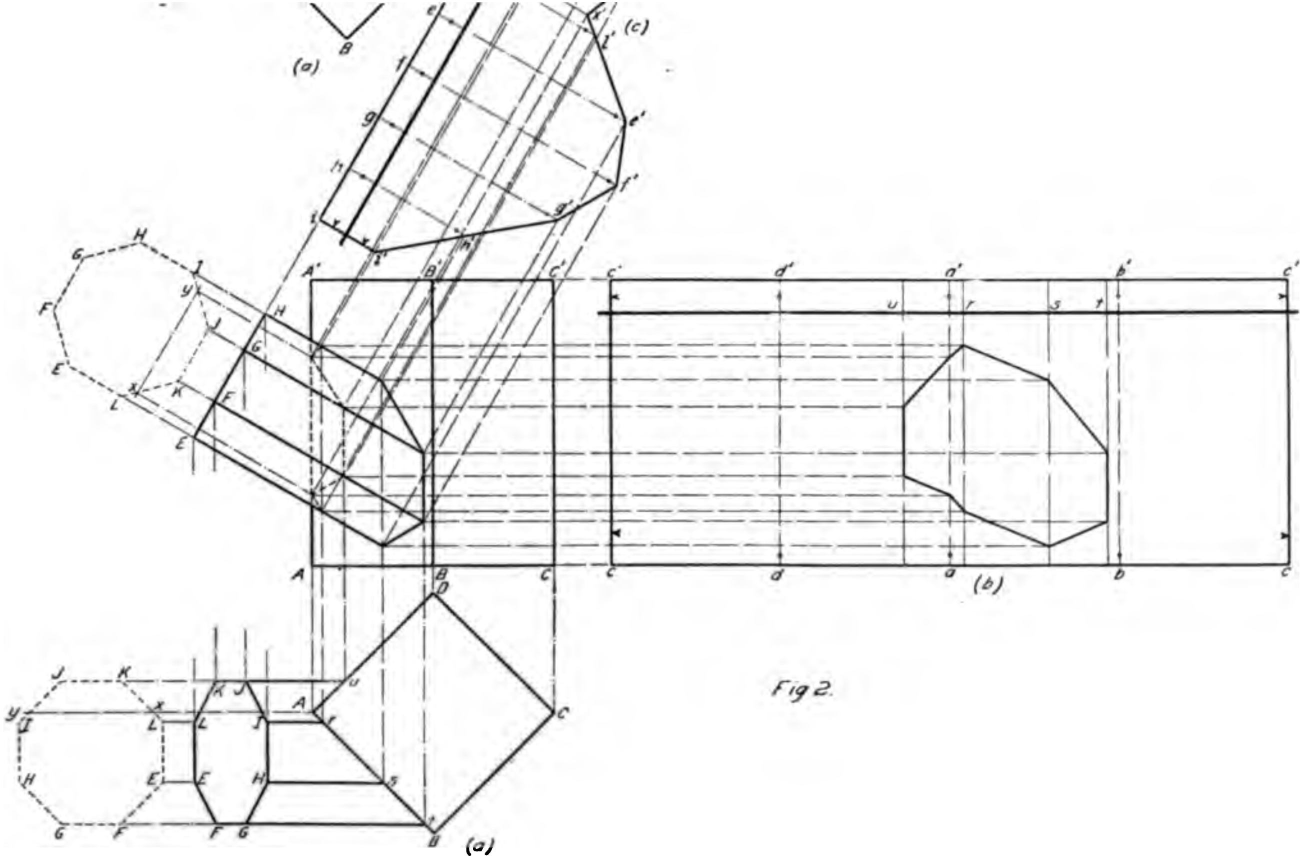

Fig 2.

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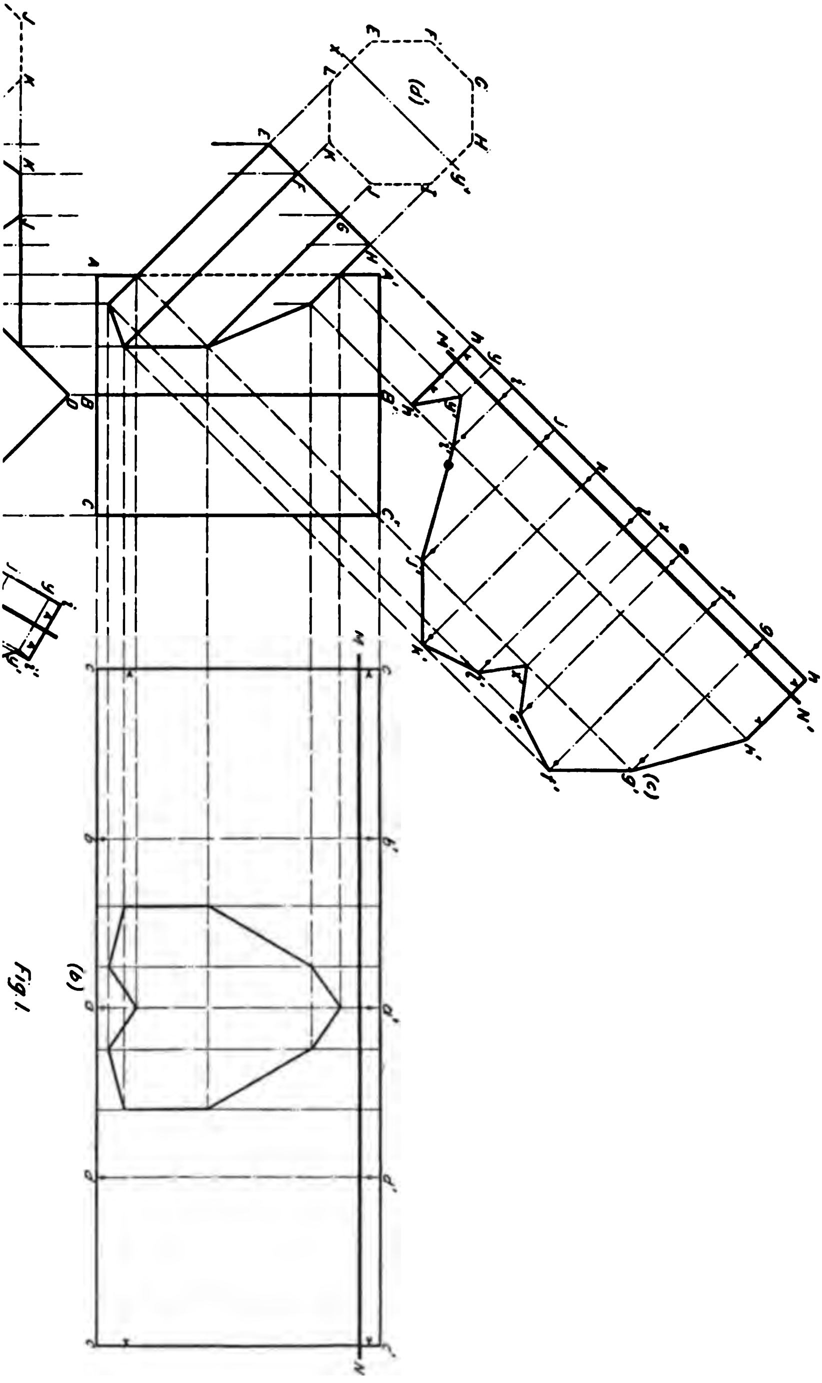
DEVELOPMENTS-II.



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xd.

Interedge lines are indicated by dash-and-double-dot lines. —··—··—··—··—··—··—

Represent developers by broken lines. — — — — — — —

Represent the stretchouts by heavy full lines. ————

DRAWING PLATE, TITLE: DEVELOPMENTS II

PROBLEM 6

29. To develop the surfaces of two intersecting prisms.

The prisms developed in this problem are the octagonal and quadrangular solids whose projection was given in *Practical Projection* in order to show views of intersecting prisms.

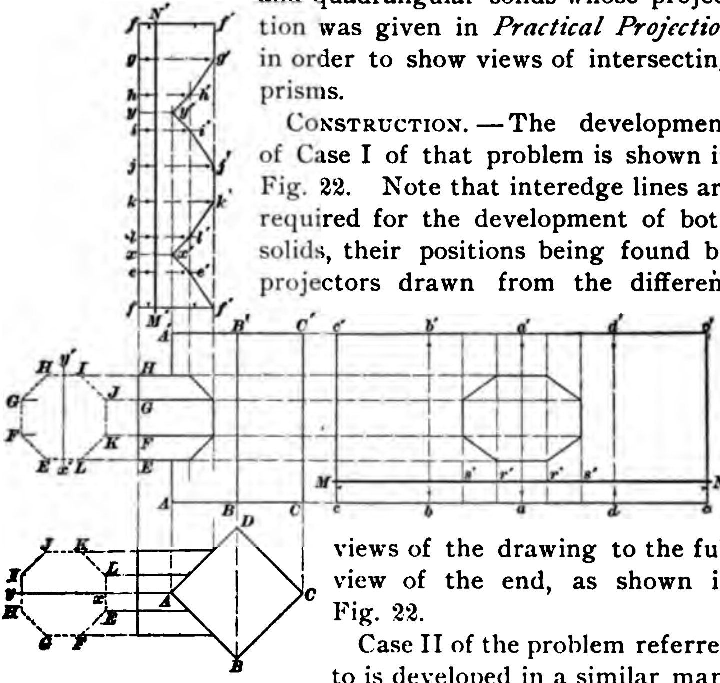

FIG. 22

CONSTRUCTION.—The development of Case I of that problem is shown in Fig. 22. Note that interedge lines are required for the development of both solids, their positions being found by projectors drawn from the different views of the drawing to the full view of the end, as shown in Fig. 22.

Case II of the problem referred to is developed in a similar manner. A drawing of the latter will form the first development for this plate, and the projections are accordingly reproduced as shown at Fig. 1 (*a*) of this plate. In order to avoid confusion of lines, projectors are sometimes drawn as shown on this plate; that is, only their starting points are indicated, as between the two views of the octagonal prism in this drawing. Next, develop the stretchouts for the two solids, as at *MN* and *M' N'*, and draw edge and interedge lines through their respective points. The positions of the interedge lines are found by projecting the point *A* in the plan across to the full view, as shown at (*d*), afterwards locating the points *x* and *y* at *x'* and *y'* in (*d'*); thence, they are projected to the elevation and carried to the development in the usual manner, the resulting figures at (*b*) and (*c*) completing the development of the solids.

The development of Case III of the problem in *Practical Projection* forms the second part of this problem. These projections are reproduced in Fig. 2 on the plate at (*a*). In this case, the axes of the solids do not coincide, and the problem has an appearance of greater complication than the drawing in Fig. 1. It is necessary to trace out the line of intersection very carefully, and if this is done, the drawing of the remainder of the problem will be comparatively easy. The interedge lines are determined in the same manner and are shown on the plate by similar letters. The method of procedure is precisely the same as already explained, the resulting developments at (*b*) and (*c*) completing the problem.

In drawings of this class, the student will perceive that the true length of the parallel lines is found by projection drawing. The application of the rules found under the heading "General Rules for Obtaining Parallel Developments" is then sufficient for the development of any solid; and if this application is carefully made, the student will meet with no obstacles that may not be readily overcome.

30. Development of Intricate Solids.—In the process of developing intricately formed solids, it becomes necessary to exercise diligence and caution in the observance

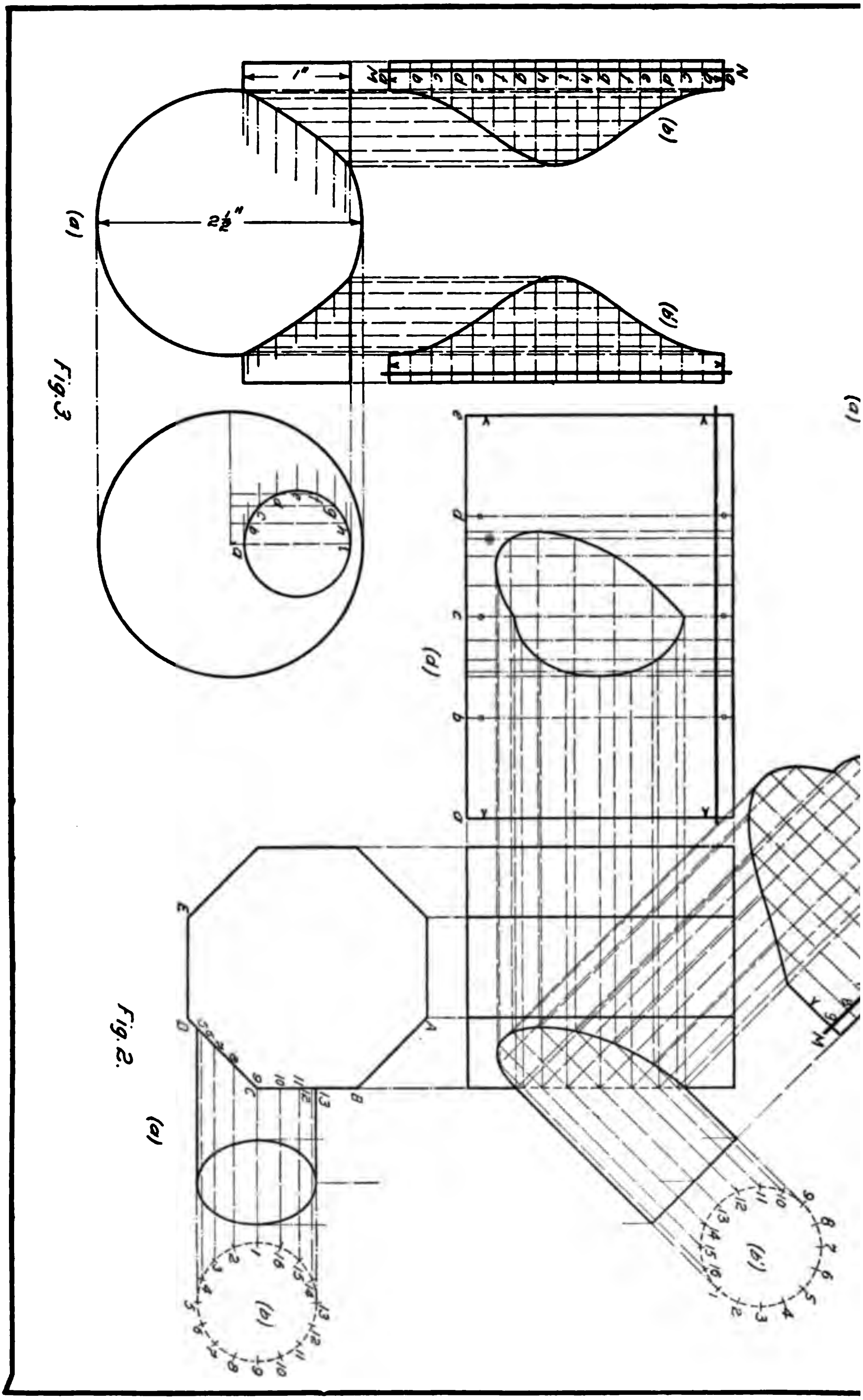

Fig. 3.

Fig. 2.

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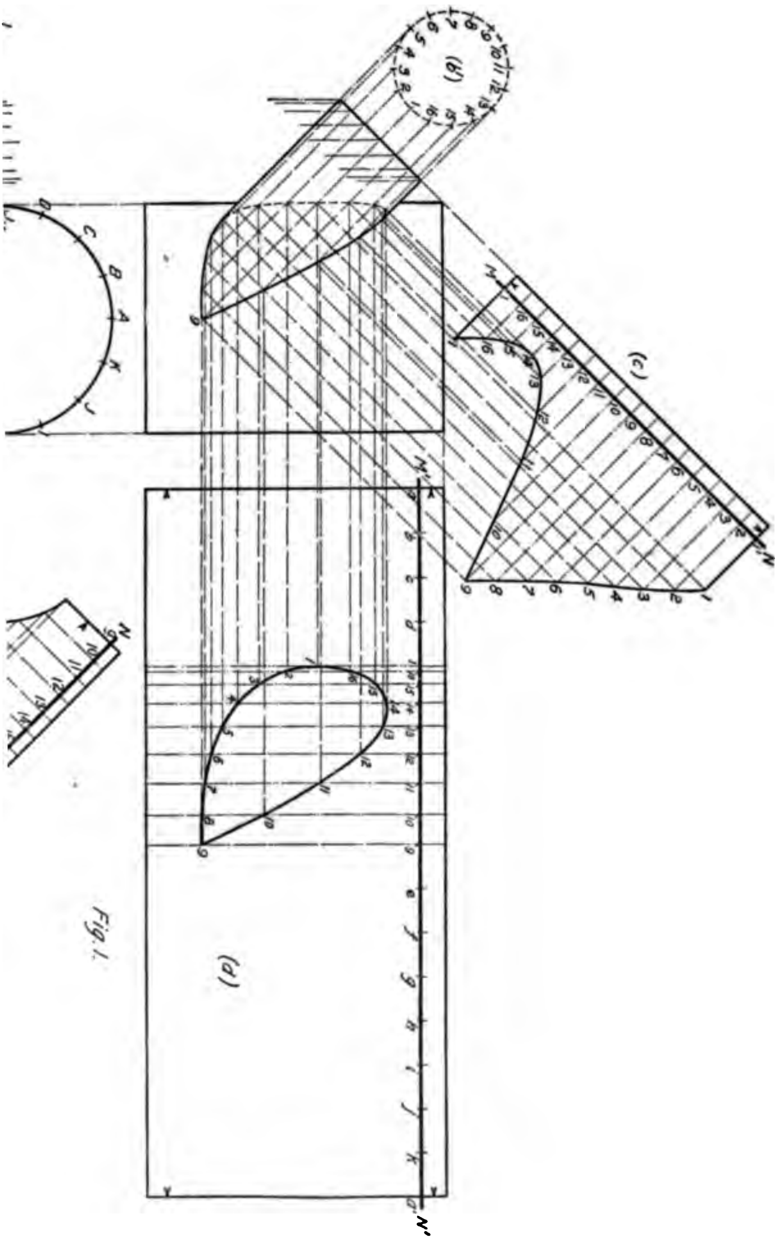

Fig. 1.

DEVELOPMENTS-III.

of the regular order of unrolling the surfaces on the stretchout and carefully to draw the developers to their corresponding edge lines in the development. Note that in order to obtain the outline of the development for the intersecting solid at Fig. 2 (*c*), a separate developer is required in the case of each edge or interedge of the solid.

DRAWING PLATE, TITLE: DEVELOPMENTS III

PROBLEM 7

31. To develop cylinders that intersect irregularly.

This problem is a development of the cylinders projected in *Practical Projection* to show views of intersecting cylinders of unequal diameters.

CONSTRUCTION.—The principles are the same as those governing previous developments, but since many lines are required for the drawing, the student should carefully follow the directions in the order stated. Draw, first, the projections as at Fig. 1 (*a*) of this plate and carefully indicate the line of intersection of the two solids. The development of the smaller cylinder is then made. Space the outline of the full view as at (*b*) and (*b'*), using 16 spaces; number these points in a corresponding manner, as shown, and project them to their adjacent views. Designate the intersections in the different views by similar numbers, to avoid confusion. Develop the stretchout MN in the usual way; draw edge lines and, afterwards, developers, as shown on the plate. The development at (*c*) is thus completed, the irregular outline there shown being the pattern for the surface of the smaller cylinder. This part of the work may be completed without reference to the larger cylinder. In fact, it may be considered as a separate drawing, and the student should take no notice of any other lines on the drawing except those pertaining to the smaller solid. In this way he will accustom himself to working on drawings that overlap one another.

A saving of the draftsman's time is often effected by thus making developments over other portions of the drawing, for if this were not done, a separate projection would be required for each development. The large number of lines on the drawing is, however, apt to be confusing to the beginner, and is frequently a cause of error when the drawing is transferred to the metal. Unusual care must therefore be taken in such cases.

The development of the smaller cylinder being completed, attention is directed to the larger solid. It is evident that the outline of the development of the larger cylinder will be a parallelogram having an irregularly outlined portion in some part of the figure. Reference to the plan indicates that the edges assumed for the smaller cylinder intersect the surface of the larger at *9*, *10*, *11*, *12*, *13*, *14*, *15*, *16*, and *1*, the total distance from *9* to *1* being one-quarter of the length of its outline. These points may now be assumed as the edges for that portion of the surface of the larger cylinder, the remainder being spaced off in the usual manner, as *A*, *B*, *C*, etc. Any convenient point on the outline may be taken as the starting place for the stretchout, the extreme upper point of the plan at *A* being selected in this case. Since the intersections occur only on the assumed edges that are indicated by the *numbered* points, the drawing of the edge lines in the development may be omitted through the *lettered* points, the outer edge lines alone being necessary to define the size of the parallelogram. Developers are then drawn from the same points of intersection in the elevation used for the development of the smaller cylinder, care being exercised that they extend to their corresponding edge lines in the development of the larger cylinder, as shown on the plate at Fig. 1 (*d*). *

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* It will be noted during the drawing of this development that the stretchouts of these cylinders are measured as chord distances. The lengths in (*d*) for the large cylinder, therefore, will be slightly different as measured from *a* to *1* when comparison is made with the length *1* to *9*, and, correspondingly, from *A* to *H*. The student should understand that in actual shop practice these chord distances are equally spaced in all quadrants in order to overcome this difficulty. The difference is, however, so slight in this drawing that no allowance is necessary.

PROBLEM 8

32. To develop an octagonal prism and an intersecting cylinder.

Explanation.—The projections given in *Practical Projection* to show views of a prism intersected by a cylinder are redrawn for this problem. The development of the cylinder is very similar to that of the smaller cylinder in the preceding problem, as shown at Fig. 2 (*c*). Note that the edges *1* and *9*, assumed on the surface of the cylinder, coincide with the edge *C* of the prism ; this simplifies the development, and the student will have no difficulty in completing the work at (*c*), it being similar to that described in the preceding problem.

Construction.—Since the development of one-half of the prism will serve to illustrate the principles of this problem, four sides only need be laid out on the stretchout. In this case, the edge lines are to be drawn and indicated as such, in order that the distinction between them and the inter-edge lines may be clearly marked, in accordance with the instructions given under the heading "Development by Parallel Lines." After developing the stretchout as shown at (*d*), the completion of the development is made in the same manner as heretofore.

PROBLEM 9

33. To develop the surfaces of a cylinder intersecting a sphere.

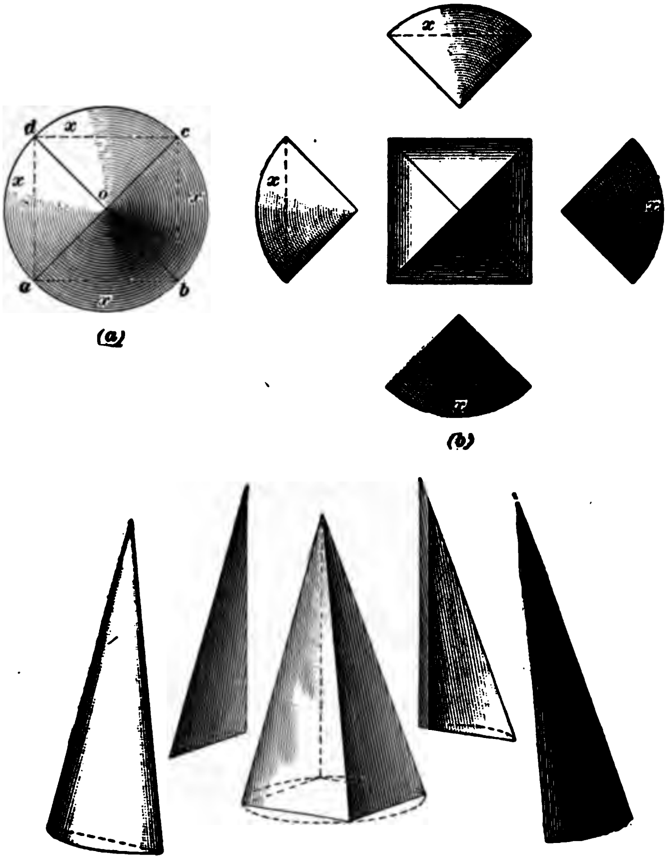
Explanation.—The projections of the sphere intersected by a cylinder that were drawn for *Practical Projection* are here reproduced at Fig. 3 (*a*) on the plate, but to a smaller scale. This problem introduces no new element in the development of solids by parallel lines. It is given a place in this instruction merely to show the student that, while the task of finding the line of intersection of parallel-lined solids with solids of other classes may depend on various principles of projection drawing, their development after this line has been produced is the same in all cases. After the completion of the projections, the outline of the cylinder, as shown in the plan, is divided into spaces by locating a convenient number of points; this number is always optional with the draftsman, it being customary to take as many as are required for accurate development; for, of course, the more points there are, the greater will be the accuracy.

CONSTRUCTION.—In this case, as in the case of all symmetrical solids, it is necessary to locate points on but one-half of the outline, since their projectors, if produced, will fix corresponding points on the remaining portion of the outline. The assumed edges are then projected to the elevation and their intersections with the sphere carried by developers to the development at (*b*). A similar development may be completed at (*b'*) by producing the edge lines to that figure and drawing developers as shown, completing the problem.

The stretchout line has been prominently used in these problems. In many cases it may be convenient to develop the stretchout on one of the lines used for developers, thus avoiding an additional line on the drawing. Generally, in sheet-metal work, it will be found preferable to give the stretchout line the same prominence as in these problems, for reasons that will be apparent in the student's later work.

DEVELOPMENT BY RADIAL LINES

34. Relation of the Pyramid to the Cone.—We now come to the second general division of those solids whose development may be accurately accomplished—that is, solids developed on *radial lines*. It is here proposed to show the relation borne by the pyramid to the cone, since the methods of development of both solids are similar. In the development of the cylinder, its curved outline was divided by points into a number of equal parts. These points in the plan were then projected to the elevation and there considered as assumed edges; in other words, the cylinder was treated as a many-sided polygonal solid, this solid appearing inscribed within the cylinder. In a similar manner we shall now consider the pyramid as an inscribed solid in its relation to the cone. This may be better understood by reference to Fig. 23 (*a*), (*b*), and (*c*).



(a)

(b)

(c)

FIG. 23

The removal from the cone at (*a*) of the pieces marked *x* leaves a solid that may be recognized as a quadrangular pyramid—better shown at (*b*), the pieces being removed and shown in an adjacent position. The illustration at (*c*) is a perspective elevation of the parts shown at (*a*) and (*b*), and is introduced for the purpose of showing the relation between the cone and the pyramid to better advantage. Comparing these two solids in the figure, it will be seen that the edges of the pyramid at (*b*) correspond to certain elements of the cone at (*a*). Further, it will be seen that, if the cone is covered with paper, as in Fig. 24, and the position of the elements noted, the paper afterwards being unrolled as shown, the elements may be imagined as leaving their imprint on the paper, as at *o a*, *o b*, etc., Fig. 24. The position of these imprints will correspond relatively to the location of the elements on the surface of the solid. A figure similar to the unrolled covering may be described by the aid of the compasses, the boundaries of the development being defined by the position of any element chosen at pleasure, notice being taken of the first and last contact of such element with the drawing during a single revolution of the cone. The *radius* of the arc thus described is always equal to the true length of an element, while its *length* is equal to the circumference of the base of the cone.

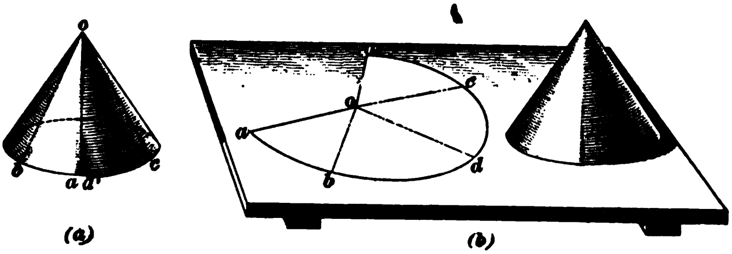
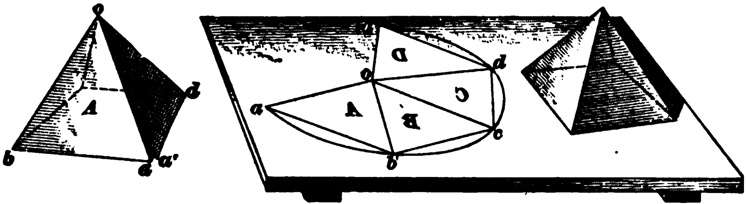


FIG. 24

Suppose, now, that the surface of the pyramid shown at Fig. 23 (*b*) is covered in a similar manner, the covering being afterwards unrolled as shown in Fig. 25. It will be found that an arc described with a radius equal to that used for the development of the cone in Fig. 24—that is, equal to the true length of an edge of the pyramid—will pass through the points *a*, *b*, *c*, *d*, and *a'*, representing the lower extremities of the upright edges of the pyramid. These points are equally distant from one another as measured on the arc. This may be proved by setting the dividers at a distance equal to the length of the base edge of the pyramid and stepping off the spaces on the arc. The only difference between the developments for these two solids lies in the fact that, for the *cone*, the development is defined by the circular arc, while in the case of the *pyramid*, straight lines are drawn between the points *a*, *b*, *c*, etc., as shown in Fig. 25. These developments will be completed in a later problem.



(a) (b)

FIG. 25

35. Stretchouts for Radial Solids.—Since the distance around the bases of the cone and the pyramid may be measured on an arc whose radius is equal to the length of one of the elements of the cone (or, in the case of the pyramid, to the length of one of its upright edges), such an arc may be described for the stretchout of these solids. The measurement around the solid being taken on a particular line, or base plane, it may here be observed that any real or assumed base plane of a cone or pyramid may be treated in a similar manner. The length of the radius by which the stretchout is described must in all cases be equal to the true length of the elements in that portion of the solid.

This is illustrated in Fig. 26, the cone *O A B* being developed along a stretchout described with the radius *O B*. An assumed base may be taken at *C D*; an arc is then described from the center *O* with the radius *O D* (*O D* being the true length of the elements of the cone *O C D*). If the width of a space between the elements on the assumed base *C D* (measured on the plan) is taken in the dividers and spaced off on the arc DF, the spaces will be found to coincide with the intersection of the elements in the development drawn from the arc BE. The same will be found true for any right base that may be assumed for the cone. As a matter of precaution, it is customary, when drawing developments of the radial solids, to describe the stretchout with as long a radius as possible, usually not exceeding the length of the elements or edges shown in the drawing. For reasons that will be shown during the construction of the problems, it is convenient to locate the center for describing the stretchout at the vertex of the solid.

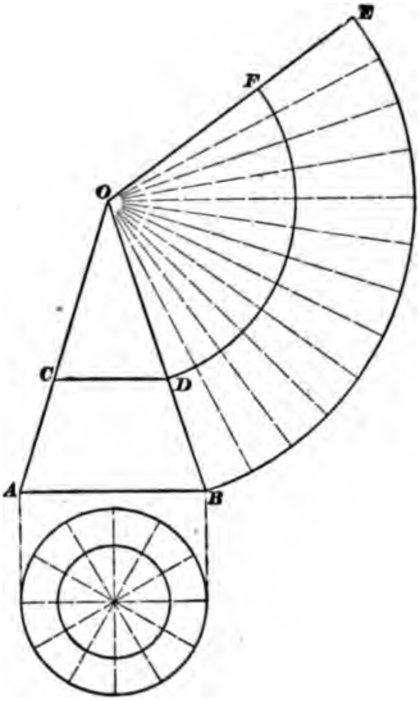


FIG. 26

36. Revolution of Radial Solids.—During the study of projection drawing, it was learned that measurements for all distances and the position of points on the surfaces of cones and pyramids—radial solids—are determined by means of their radial lines, or elements. The same principle must be adhered to when developments of such solids are produced. It is essential, therefore, that these lines should be shown in their true lengths on drawings from which such developments are produced. This will necessitate much work on the part of the draftsman, especially if the surfaces are in any way irregular or are intersected by other solids. To overcome the necessity for drawing a number of views, advantage is taken of a principle that may be observed during the revolution of the solid. This revolution is effected in a very simple way on the drawing board, and an illustration of the method used is furnished in Fig. 27. The student will understand, from an inspection of that figure, that if the cone $O\ A\ B$ is revolved on its axis in the direction of the arrow, the motion of any point on its surface will be indicated on the elevation by a horizontal line. When any elements of the cone are in the positions occupied by the elements $O\ A$ and $O\ B$, their true lengths are shown on the elevation, and measurements may, therefore, be taken from such lines or from any points located on these lines. Lines in this position may be called *true edge lines*, their use under this name being peculiar to the radial solids.

When developing the surfaces of pyramids shown in certain positions, it is sometimes necessary to draw this true edge line independently of the figure. A problem illustrating this principle is given on the following plate. Since the elements of a cone are equal in length, the position of any point that may be located on any of these elements (as x, located on the element $O\ C$, Fig. 27) may be projected in a horizontal direction to the true edge line at x'', and its correct distance from the vertex O be there ascertained. The point x, therefore, could be projected to either element $O\ A$ or $O\ B$, but the supposed revolution is generally represented as being made towards the true edge line that is nearest the point whose location it is desired to determine, although in Fig. 27 the point x is moved in the opposite direction. Thus, in Fig. 27, if it is desired to determine the exact distance of the point x from the vertex O, the

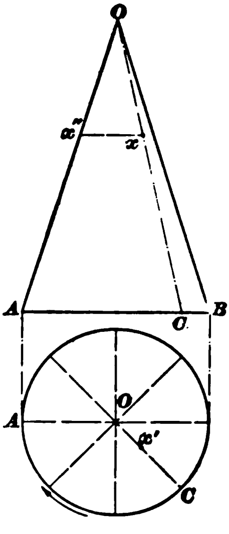


FIG. 27

horizontal line $x x''$ is drawn in the elevation, and the distance from the point O to the point x'' is then the exact, or true, distance between the two points. The same result would be obtained if an elevation were drawn showing the element $O C$ in its true length; but as seen from the foregoing explanation, the method here explained is much shorter and better adapted to the wants of the draftsman.

37. Use of Construction Lines.—In the development of radial solids, the same construction lines are used as in obtaining the developments for parallel solids, although in a slightly different manner. Since the stretchout line has been shown in its adaptation to the development of these solids, it may be represented in a similar manner on the drawings. Developers also are indicated by the same kind of broken lines used in the preceding class, but, like the stretchout, they are described in the form of arcs, and must be produced from an element or edge that is shown in its true length. In a similar manner, these developers are drawn to the development, arcs being described extending from points on edges or interedges, as the case may be, to their corresponding edge lines or interedge lines in the development. When edge lines and interedge lines occur, they converge to a point, and are the radial lines by which this class of solids is distinguished. Such edges and interedges are represented by lines similar to those used for the same purpose in parallel developments and refer to the same corresponding portions of the solid.

GENERAL RULES FOR OBTAINING RADIAL DEVELOPMENTS

38. A slight modification of the rules for obtaining developments on parallel lines is here applied to the development of radial solids. In connection with the foregoing instruction, a comparison of the two sets of rules will enable the student to understand the principles by which these developments may be accomplished.

1. A projection must first be drawn, consisting of a plan and elevation and showing the solid in a right position.

2. The development is always obtained from that view in which the axis of the solid is shown in its true length (since the revolution of the solid may not readily be shown in any other view).

3. The stretchout is described with a radius equal to the length of the true edge of the solid. Its center may be conveniently located at the vertex.

4. To indicate the width and relative position of the surfaces, points are located on the stretchout corresponding to the position of those points on the outline of a sectional or base view. This view must be taken at right angles to the axis of the solid, the distance from the vertex being determined by the length of the true edge lines in the elevation.

5. Edge lines and interedge lines are always radii of the stretchout arc.

6. Points located on the surface of the solid must be projected to the true edge line by projectors drawn at right angles to the axis.

7. Developers are described with radii equal to the distances on a true edge line from the vertex to the points projected to such edge line. Each developer extends thence to its corresponding edge line or interedge line in the development.

8. Interedge lines, when necessary for the development, must be indicated on the projection as well as on the development. Points located on such lines are projected to the true edge lines and thence developed in the usual manner.

9. The lengths of the outer edge lines in a complete development of a solid must be defined by the same developers.

The application of these rules will be made apparent to the student in the construction of the following problems, which involve the development of radial solids.

DRAWING PLATE, TITLE: DEVELOPMENTS IV

PROBLEM 10

39. To develop the surface of a cone.

CONSTRUCTION.—Draw a plan and an elevation of the cone, as shown at Fig. 1 on the plate. The cone is there shown in a right position, the dimensions being 1½ inches in diameter at the base and 2 inches high. The true length of its axis is shown in the elevation, and the development is therefore made from that view. Divide the outline of the base in the plan into a convenient number of equal parts (in this case 12); from the vertex *O'* of the cone as a center, describe the stretchout arc *B' a'* with a radius equal to the true length of the elements of the cone (that is, the distance *O' B'* in the elevation). With the dividers, take the length of one of the equal spaces in the plan, and, starting at a convenient point on the stretchout, as at *a*, step off spaces equal in number to those on the plan, thereby making the length of the stretchout equal to the circumference of the base of the cone. From each of the points thus located on the stretchout, an edge line may be drawn to the vertex *O'*; but since there are no points on the surface of the cone that it is desirable to locate in this instance, only the outer edge lines *a O'* and *a' O'* need be inked in on the drawing. These lines are to be further indicated by means of the small arrowheads (as in the case of parallel solids) illustrated on the plate in Fig. 1. This completes the development.

PROBLEM 11

40. To develop the surfaces of a quadrangular pyramid.

CONSTRUCTION.—This development is shown on the plate at Fig. 2, the right plan and elevation being first drawn according to the dimensions given in the figure. It will be seen that the true length of the edge is shown in the

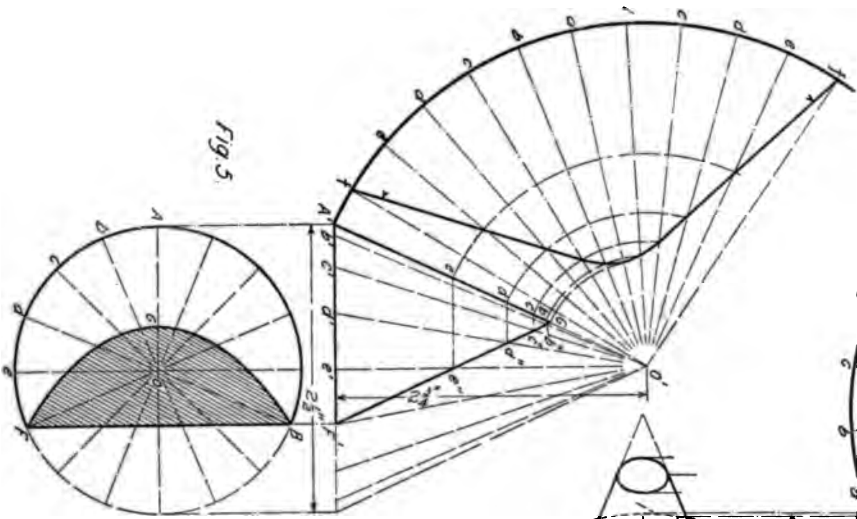


Fig. 5.

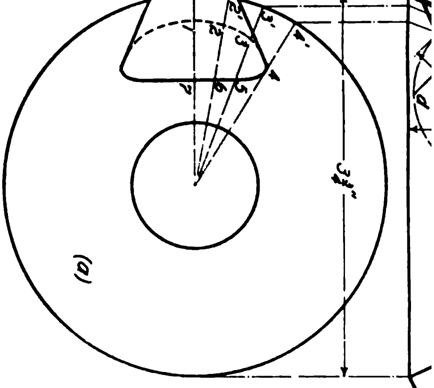


Fig. 7.

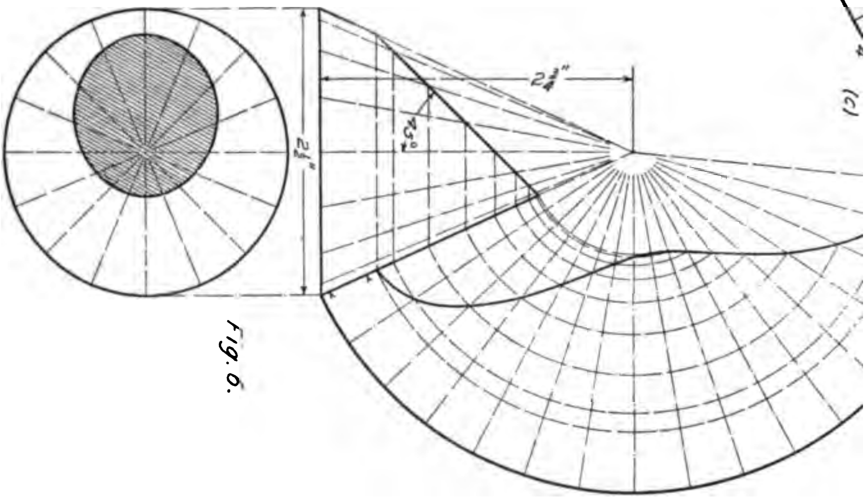


Fig. 6.

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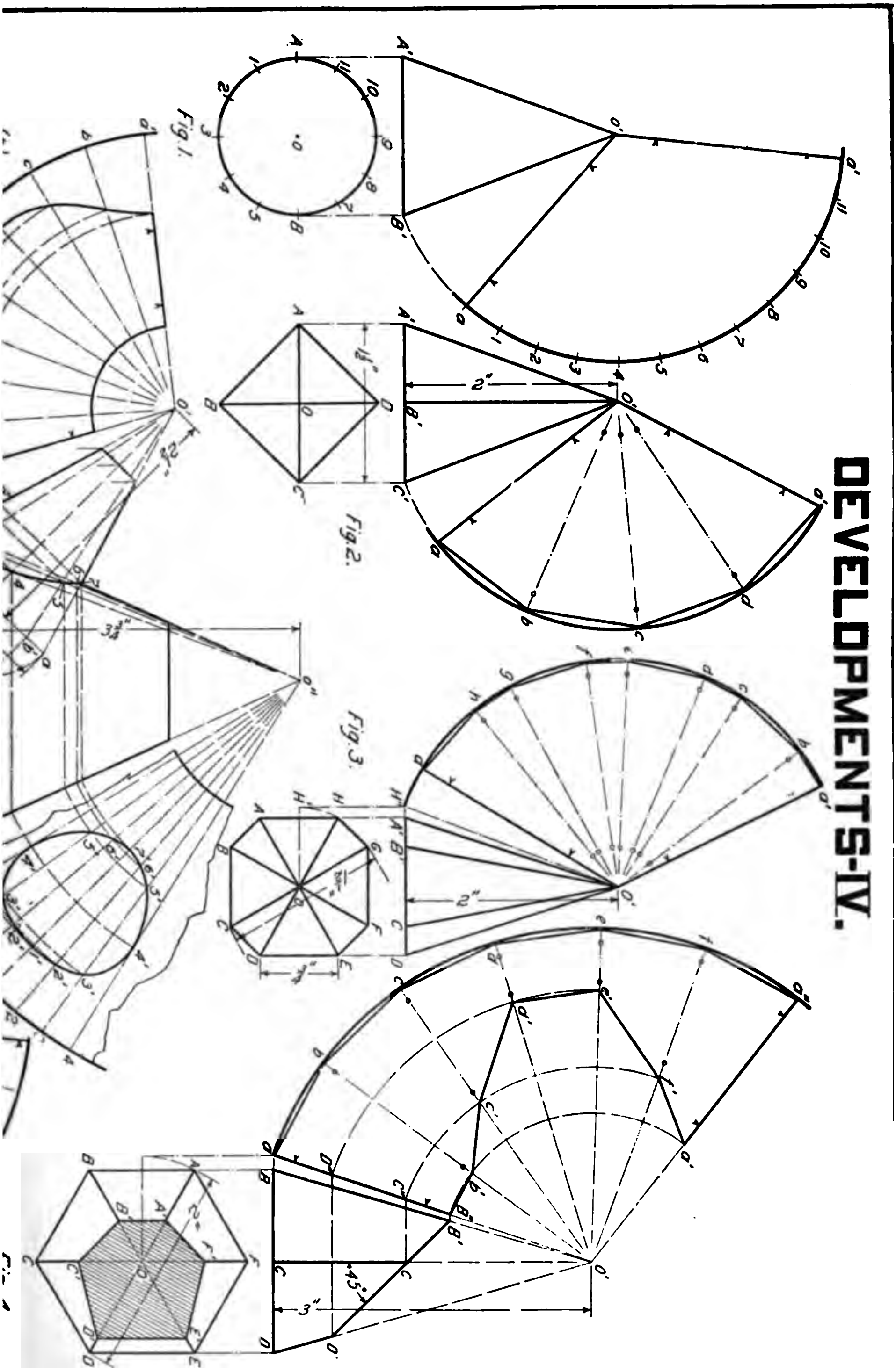

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elevation; the stretchout may, therefore, be described as in the case of the cone in the preceding problem. After setting the dividers to the width of one of the base edges shown in the plan at *A B*, Fig. 2, begin at *a* and step off on the stretchout line spaces equal in number to the base edges of the pyramid. Thus, points are located at *a*, *b*, *c*, *d*, and *a'*. Draw lines connecting these points in the manner shown, and draw other lines from each of these points to the vertex *O'*. In this case the edge lines must be drawn in the development, since there are actual edges on the solid; besides, it is necessary to define those portions of the development as indicated on the plate. Complete the drawing in the manner shown, the outline *O' a b c d a'* being the development of the pyramid.

PROBLEM 12

41. To develop the surfaces of an octagonal pyramid.

Explanation.—The base of the pyramid whose dimensions are given in Fig. 3 on the plate is not a true octagon, the alternate sides only being equal. It will be seen, however, that the octagon may be circumscribed by a circle; that is, a circle may be described in the plan from the center *O* with a radius *O A*, whose outline will pass through all the points *A*, *B*, *C*, *D*, etc.; therefore, the development of the solid may be accomplished by this method.

Construction.—The true length of the edge lines is not shown in either view presented, and it is therefore necessary to draw a line that will represent the true edge in the elevation. This is found as follows: From *O* as a center, with the radius *O H*, describe the arc *H H'*, intersecting a line *O H'* (drawn parallel to the base line of the front elevation) at *H'*; project the point *H'* to the base line of the front elevation (extended) at *H''*; *O' H''* is then the true length of an edge of the pyramid, and is, at the same time, the length of the stretchout radius. Next, describe the stretchout with this radius from the vertex *O'* as a center; then space off the width of the surfaces shown on the base in the plan, as at *a*, *h*, *g*, *f*, etc., and complete the development as directed in the preceding problem.

PROBLEM 13

42. To develop the surfaces of an irregular frustum of a hexagonal pyramid.

CONSTRUCTION.—The projections shown in Fig. 4 on the plate are first drawn in accordance with the dimensions there given, thus producing a right plan and elevation of the frustum. In this and all similar developments, it is desirable to extend the edges of the pyramid to the vertex of the solid. Since the drawing does not show the true length of the edge, this must first be found by the method described in the preceding problem, and produced as shown in the elevation at *O' a*. The points in the upper portion of the solid, at *B'*, *C'*, *D'*, are then projected to the true edge line at *B''*, *C''*, and *D''*. The stretchout is next described from *O'* as a center with a radius *O' a*; the widths of the surfaces are then laid off at *a*, *b*, *c*, *d*, etc., and the corresponding edge lines are then drawn. Developers are now described from *B''*, *C''*, and *D''*, as previously directed, the intersections with their corresponding edge lines being noted at *b' a'*, *c' f'*, and *d' e'*. Complete the development by drawing its full outline and adding the indicators in the manner shown.

PROBLEM 14

43. To develop an irregular frustum of a cone.

Two developments are required for this problem, one of them being fully described and the other to be drawn by the student as a test of his advancement.

Construction.—The projection drawings for the first development are shown on the plate at Fig. 5, that portion of the cone representing a parabolic section being presented for development. In this, as in the preceding case, the completion of the figure, as shown by the broken lines, must first be made. Proceed then as if the complete cone were to be developed; that is, divide the outline of the base in the plan into a number of equal spaces, observing that certain of the points fall on the ends of the parabolic curve (as at F and B in the plan, Fig. 5). The elements of the cone are then produced in the elevation, their intersections with the edge of the frustum at b'', c'', d'', etc., being then projected as before to the true edge line. As in former problems, the stretchout is next described and spaced off, and edge lines are drawn. Developers may now be described as shown on the plate and the development completed in the usual manner. The irregular curve is now traced through the points thus determined. The projection drawings for the second development of this problem are shown on the plate at Fig. 6, and, since the methods used are the same as in the case just described, the development may be completed by the student without further instruction.

PROBLEM 15

44. To develop the surfaces of intersecting cones.

The projections for this problem are shown on the plate at Fig. 7. They should first be carefully drawn by the student, the line of intersection being accurately determined by the method for finding the intersection of two cones that was given in *Practical Projection*. After this line has been found, all construction lines used in the projection should be erased from the drawing; if this is not done, confusion is liable to result.

Construction.—The development of these surfaces does not differ materially from those in the preceding problem, as an inspection of Fig. 7 will indicate. First draw the development of the surface of the smaller cone; extend the edge lines in the elevation to the vertex *O'*, as shown; extend them also to the assumed base *a e* and produce the full view of the base in the manner indicated by the broken lines at (*n*), using only one-half of the circumference. The semi-circumference is then spaced off as at *a*, *b*, *c*, *d*, and *e*; *b*, *c*, and *d* are then projected to the base line *a e*; and the elements *b' O'*, *c' O'*, and *d' O'* are drawn as shown. The intersections of these elements with the line of intersection of the two solids are then projected to the true edge of the smaller cone, at *a''*, *b''*, etc. A stretchout for this solid is next described from the vertex *O'* as a center with a radius *O' e*; the points *a*, *b*, *c*, etc. at (*b*) are located by spacing the stretchout; and the distances *a b*, *b c*, etc. are taken from the full view at (*n*). Next, draw the edge lines *a O'*, *b O'*, *c O'*, etc. at (*b*); and from points *a''*, *b''*, etc. describe developers in the manner shown. The irregular curve at (*b*) is then traced, completing the development for this solid.

As a matter of convenience in this case, the development of the larger cone, or, rather, as much of its surface as will show the opening made by the intersection of the smaller cone, may be drawn to the right of the projections, as at (*c*). Now draw the horizontal center line in the plan and from the center of the plan draw a line tangent to the line of intersection of the cones, marking the point where this line meets the base *4'*. Divide *1'*–*4'* into a convenient number of parts (in this case 3) and project the points *1'*, *2'*, *3'*, *4'* to the elevation. Draw elements from *O''* to the projected points and mark the points where these elements cross the line of intersection of the cones *1*, *2*, *3*, *4*, *5*, *6*, and *7*. The points *1'*, *2'*, *3'*, *4'* in the plan establish the width of the spaces that shall be stepped off on the stretchout. Describe the stretchout from the center *O''*, as shown at (*c*), and on this line set off the spaces determined in the plan at *1'*, *2'*, *3'*, and *4'*, and repeat them on both sides of the edge line *1 O''*, as indicated at (*c*) Project the points *1*, *2*, *3*, etc. to the right-hand true edge line of the elevation and carry them thence, by developers, to the drawing at (*c*), completing the development as there shown.

The student will readily perceive that if the line of intersection of the two cones is not correctly drawn, the true development cannot be produced, since the drawing depends entirely on the line thus determined. If these points are incorrectly located, the development is necessarily wrong. The importance of having a correct projection is thus evident.

45. How to Recognize Radial Forms.—Since many objects of frequent occurrence in the trades are portions of the cone or pyramid, the student should familiarize himself with the appearance of such frustums. Many objects represent a combination of the surfaces of cones of unequal sizes, and at first sight it might appear that their development should be produced only by triangulation. But an experience in the projection of cones and conic sections will often enable the student to refer such sections to the appropriate form, in many cases thereby avoiding a tedious process of development.

The representation of the elements of the cone in both plan and elevation will determine the method to be used in the development of a given solid. In a certain case, a number of vertexes may be defined; each cone is then to be traced out carefully and the development of its surface made, or as much of that surface as is required for the object in view, the operation being similar to that of the preceding problems.

TRIANGULATION

46. Elementary Principles. — **Triangulation** — the process by which the development of solids belonging to the third general division is accomplished—is generally regarded by sheet-metal workers as particularly intricate and difficult. It is, on the contrary, a very simple method of development, and should, to the student that thoroughly understands the principles of projection, present no serious obstacles. This process depends for its results

eral principles, both of which have been men section: *first*, to find the true length of all lines, real or assumed, appearing on the surfaces of the solid; *second*, having determined the true length of such lines, to construct triangles similar in form and relation to those shown on the solid.

Certainly, the construction of a triangle whose three sides are given is not a difficult problem; and the task of finding, from the projection drawing, the true lengths of its sides involves nothing but the elementary principles of that study. Having found the true lengths of the sides of such triangles as are involved in a development, nothing apparently remains but to show the method of arranging the triangles in their proper relation to one another on the different surfaces of the solids. Since this is naturally suggested by the shape of the solid itself, several solids are presented in the accompanying problems, from the study of which the student should learn to apply the principles to the development of any solid of this class.

47. Illustration of Methods Used.—This method of development has been previously mentioned, and while this principle of patterncutting is usually applied to solids having curved surfaces, it is best illustrated by its application to a solid having plane surfaces. Such a solid is shown in Fig. 28, where a perspective view is given of what may be termed a *transition piece*—that is, a piece used to connect openings of different sizes, as in pipework. Both bases are rectangular and in this case parallel, but diagonally arranged in their relation to each other, as may be seen from the figure.

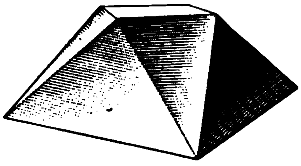
FIG. 28

It is at once seen that none of the methods described under the headings of parallel- or radial-lined solids will apply to the development of the lateral surfaces of this solid, although it is possible to project a full view of each of the surfaces shown in the figure. Since this would require an unusual amount of work on the part of the draftsman, the following process, admitting of more rapid application, is presented.

48. Determining the Triangles.—Clearly, a reproduction of these triangular surfaces on the flat surface of the drawing board, in the same corresponding relation with reference to one another, will be a development of the solid. Apparently, the only difficulty that presents itself is the fact that, in certain cases, the sides of the triangles are not shown in their true lengths. It is necessary to determine the true lengths of all lines, in order that the triangles may be constructed of the same size as they are on the surfaces of the solid.

As in all cases where a development is desired, a right plan and elevation must first be drawn. This is shown in Fig. 29, and from that illustration it is seen that all lines of the solid that appear on either base are shown in their true lengths. It is therefore necessary, before the triangles may be produced, to determine the true lengths of the remaining lines of the solid. As mentioned in *Practical Projection*, this is most readily accomplished by constructing in each case a right-angled triangle whose base is equal to the length of any foreshortened line in the plan, and its altitude to the vertical height of the same line, as shown in the elevation. The hypotenuse of such a triangle will then be equal to the true length of the line. In this case, the lines *A H*, *D H*, *A E*, *B E*, etc., foreshortened in the plan, are all represented by lines of the same length. The vertical height *M*, Fig. 29, is the same in the case of each line.

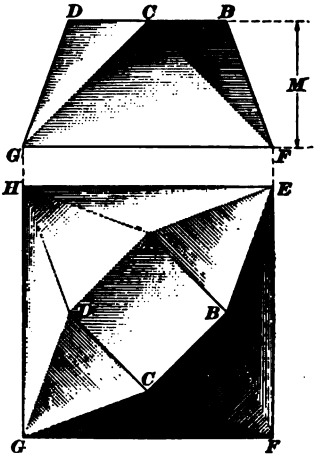

FIG. 29

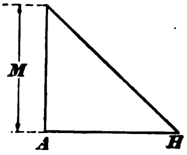

FIG. 30

A single triangle constructed by the above method, therefore, will be sufficient to indicate the true length of all lines not shown in their true length in Fig. 29. Such a triangle is constructed in Fig. 30; the base of the triangle *A H*, Fig. 30, is equal to the length of *A H*, Fig. 29, the altitude *M* being the same as *M*, Fig. 29. The hypotenuse of this triangle is therefore the true lengths of the lines *A H*, *D H*, etc., as shown in Fig. 29. The true lengths of all lines bordering the triangular surfaces of the solid shown in Fig. 29 having been found, the triangles may now be constructed on the drawing, care being observed that the adjacent triangles are completed *in the same order* as they are shown on the solid. Any edge of the solid may be assumed as a starting place for the operation; the true length of such an edge is then laid off on a line as at *A E*, Fig. 31. The triangle *A E B*, Fig. 31, is first constructed; the length of the side *A E* being laid off, and *B E*, Fig. 29, being of the same length, an arc may be described in Fig. 31 from *E* as a center, with a radius equal to *E A*. Intersect this arc at *B*, Fig. 31, with one described from *A* as a center, with a radius equal to *A B*, Fig. 29. Draw *A B* and *E B*, thus

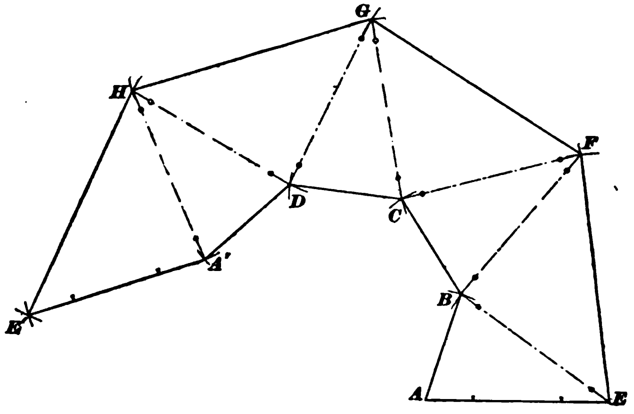

FIG. 31

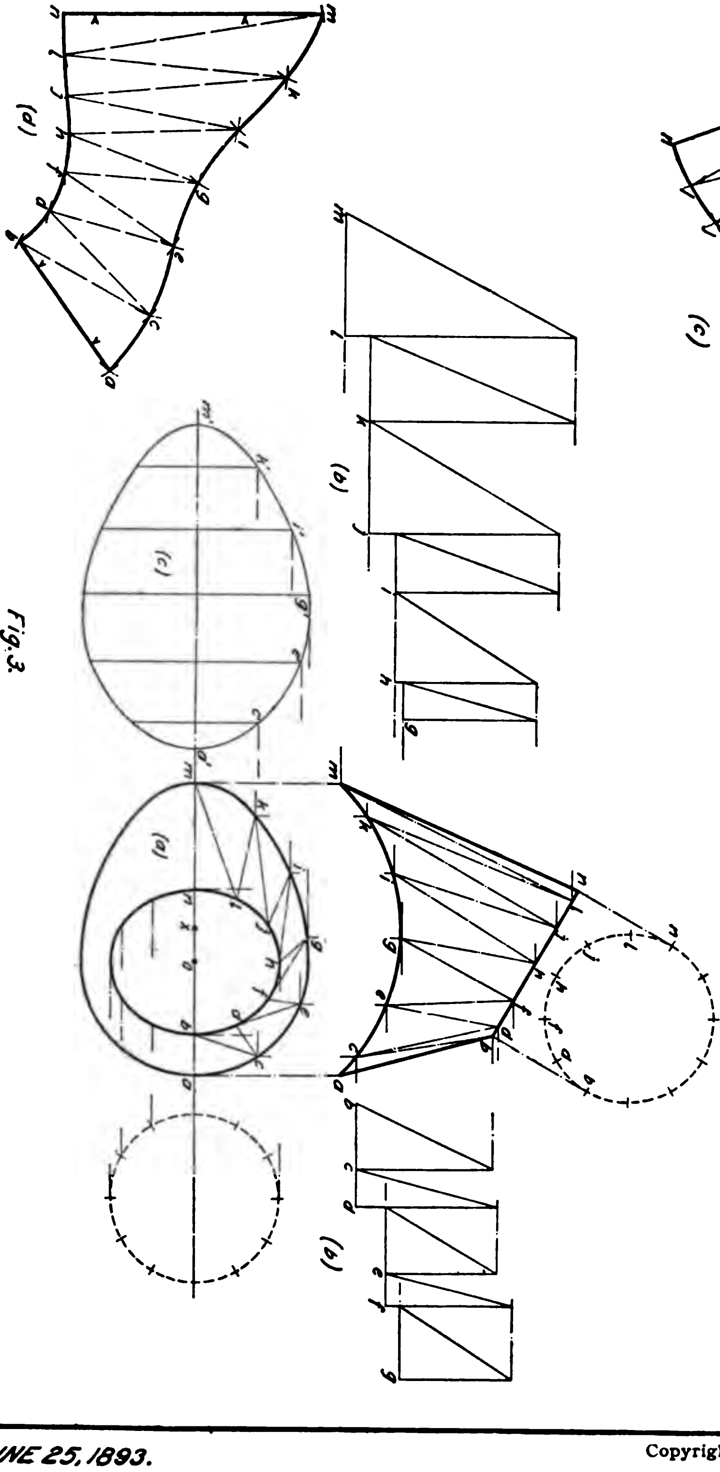

Fig. 3.

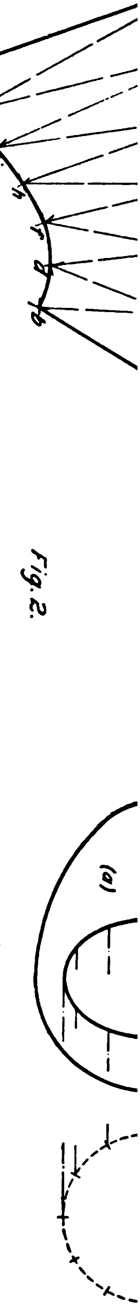

Fig. 2.

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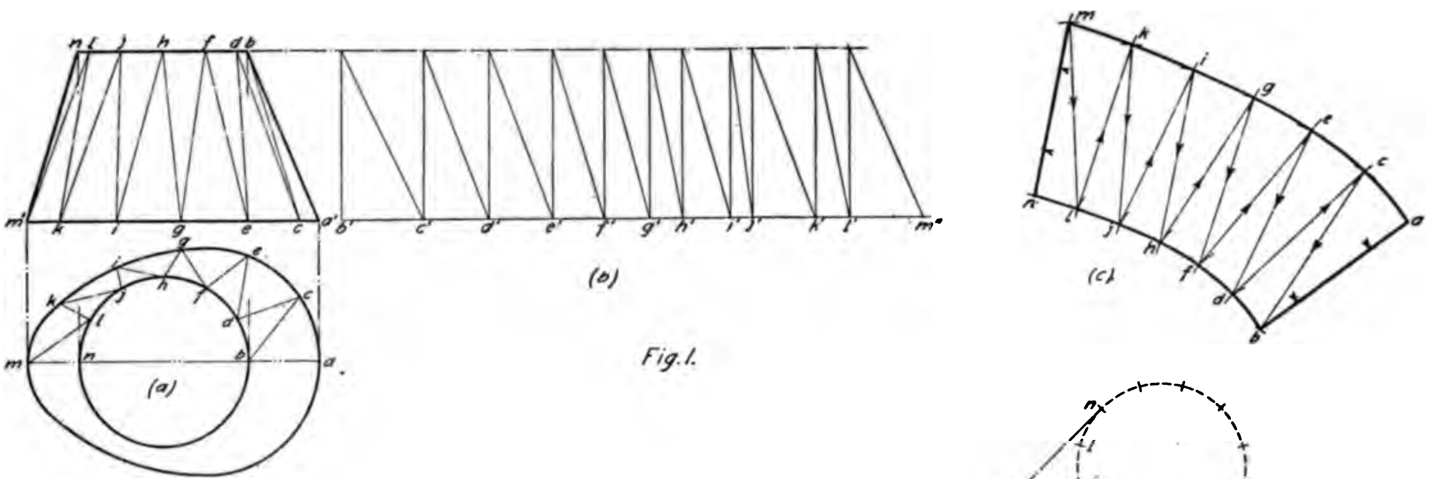
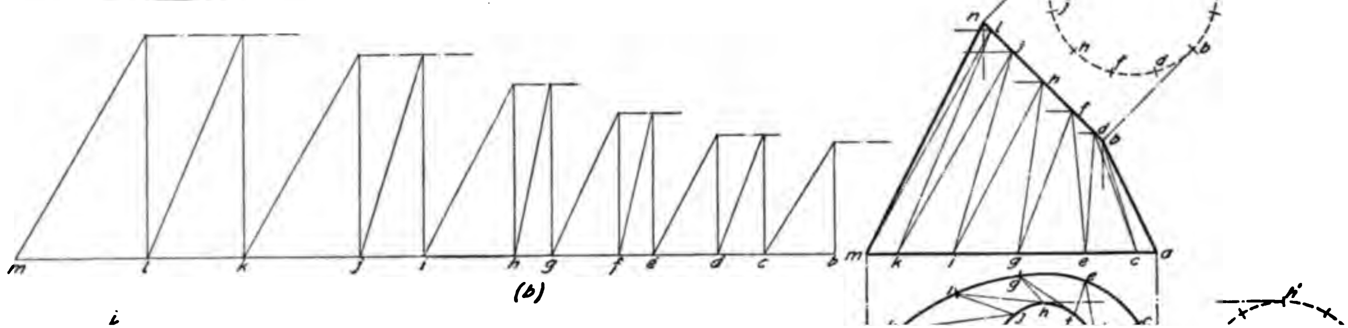

n l j h f d b
m' k i g e c a'
b' c' d' e' f' g' h' i' j' k' l' m'
m k i g e c a n l j h f d b
(a)
(b)
(c)
m k i g e c a n l j h f d b

Fig. 1.



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developing the triangle *A E B*, Fig. 31, which is the correct development of the surface *A E B*, Fig. 29.

The adjacent triangle *E B F*, Fig. 31, may next be constructed. Since *B F* is equal to *B E*, Fig. 29, an arc may be described in Fig. 31 from *B* as a center, with a radius *B E*; this arc is then intersected by an arc described from *E* as a center, with a radius equal to the length of *E F*, Fig. 29, thus developing the triangle *E B F*, Fig. 31, which corresponds to the surface *E B F*, Fig. 29. In like manner, each surface of the solid is developed, due care being observed that adjacent triangles are placed in corresponding positions in the development.

49. Completion of the Drawing.—The completion of the drawing is made in a manner somewhat similar to parallel and radial developments; that is, the edge lines may be indicated as in those methods, the outer edges being denoted by full lines and those lines on which bends are to be made when the flat surfaces are formed up being designated by the customary indicator circles. It will thus be seen that no new principles are required to produce developments by this method. A careful observance of the different portions of the drawing is required, since an object as simple as that shown in Fig. 29 is seldom met in practice. The same methods are used, however, and should be readily understood by the student and applied to the drawing in the same manner as has been shown.

DRAWING PLATE, TITLE: DEVELOPMENTS V

50. Several problems relating to the development of solids by triangulation are given on this plate. The same principles that were shown in connection with the development of the transition piece in Figs. 28 to 31 are used in these drawings; but, owing to the different shapes assumed by the solid selected for each problem, slightly different constructions are given in each case.

Particular attention should be paid to the manner in which the triangles are located on the irregular surfaces, care being exercised that similar points in each view shall be taken as a basis for finding the true length of each line.

PROBLEM 16

51. To develop the surface of an irregular solid having parallel bases.

The solid shown in perspective in Fig. 32 is the same as that used in *Practical Projection* for the projections of views of an irregularly formed solid and a sectional view from a given cutting plane.

CONSTRUCTION.—The first step is to draw a right plan and elevation, as shown at Fig. 1 (*a*) on the plate. Draw the horizontal diameter *a m* through the plan, thus dividing the solid into symmetrical halves. It will now be seen that, if a development is made of the upper portion of the solid, as seen in the plan, a duplication of the resulting figure will be the complete development. In order to locate the sides of the triangles that are to be assumed on the surface of the solid, the outline of the bases is divided into the same number of equal parts (in this case 6), as at *a*, *c*, *e*, etc. on the lower base and *b*, *d*, *f*, etc. on the upper base.

Draw, in succession, lines alternately from the points on the upper and lower bases; *a b* being on the line of the diameter, draw *b c*, *c d*, *d e*, etc. Project these points and lines to the elevation, and represent them by dot-and-dash lines, as used in preceding developments to designate edge lines. The surface of the solid is thus divided into a number of triangles that may be better understood by the student from an inspection of Fig. 32. This is a perspective view of the solid, showing in the full lines the triangles that have been assumed on its surface. The

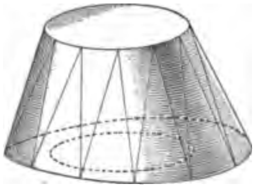

FIG. 32

lengths of the sides *b d*, *d f*, *a c*, *c e*, etc. may be taken as chord distances directly from the plan, since they are there shown in their full length. A construction of right-angled triangles is necessary, however, in order to find the true lengths of the lines *b c*, *c d*, *d e*, etc., Fig. 1 of the plate; and in order to construct them, the base line *a' m'* of the solid in the elevation, Fig. 1, is extended indefinitely towards the right of the drawing to *m''*, as shown at (*b*).

At a convenient distance from the elevation locate a point on this line, as at *b'* in Fig. 1 (*b*) on the plate. Take the distance *b c*, as shown in the plan, and set it off with the dividers, as at *b' c'*; in like manner, make *c' d'* equal to *c d*, as shown on the plan, and proceed to copy all the distances there shown until the point *m''* is reached. It will be seen from an examination of the projections that the lines *a' b* and *m' n* are shown in their true length in the elevation, and a triangle is, therefore, not required for those lines.

Since the bases of the solid are parallel, the vertical height of the triangles at (*b*) is the same in all cases, and may be projected from the elevation as shown on the plate. The true lengths of all lines now being determined, the triangles may be constructed as shown at Fig. 1 (*c*). Draw the line *a b*, making it equal in length to the corresponding line in the elevation—that is, *a' b*; next, describe an arc from *a* as a center, with a radius *a c*, taken from the plan at (*a*); intersect this arc by an arc described from *b* as a center, with a radius equal to the length of the hypotenuse of the triangle whose base is *b' c'* in (*b*). This completes the triangle *a b c* at Fig. 1 (*c*). The triangle *b c d* is next constructed in a similar manner, and the completion of the development is accomplished by a continuation of the methods described. Small arrows are introduced in Fig. 1 (*c*) to indicate the location of the centers of the corresponding arcs. Thus, the arrowhead on the line *b c* is pointed towards *b*, and indicates that the center of the arc, by means of which the point *c* is determined, is located at *b*; *c* in like manner is similarly shown to be the center of the arc described through (*d*). Since there are no edges to be bent angularly when the surface is formed to the shape shown in the plan and elevation, the circular indicators are omitted; but arrowheads indicating the boundary edges of the development may be added as heretofore, completing the problem.

PROBLEM 17

52. To develop the surface of an irregular solid having inclined bases.

A perspective view of this solid is shown in Fig. 33; it is a modification of the solid used for the preceding problem, and may be drawn as shown on the plate at Fig. 2 (*a*). The outline of the lower base is drawn as in Fig. 1, and the center of the semicircle at *o* is also the center of the circle that represents the inclined upper base, the angle of inclination being 45°.

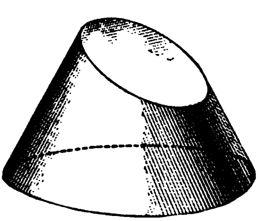

FIG. 33

CONSTRUCTION.—Draw a line (not shown on the plate) vertically upwards from *o* and fix the point *h* 1½ inches above the lower base of the solid. Through this point draw the line *b n* at the given angle; and at right angles to *b n* describe the circle representing the full view of the upper base, as shown. Next, locate the points shown on the semicircle and, by means of the temporary full view shown at *b' h' n'*, project the inclined view of the upper base in the plan. These points are then used, as in the preceding problem, for the purpose of defining the triangles. Divide the lower base of the solid, as shown in the plan, into an equal number of parts, and draw lines representing the sides of the triangles, as in Fig. 1, producing them in the plan and elevation, as *b c*, *c d*, *d e*, etc. Fig. 2 (*a*).

The manner of determining the true lengths of the lines *b c*, *c d*, *d e*, etc., is slightly different from that used in the preceding problem, since the vertical distances are not the same in all cases. Produce the lower base *a m* to the left, as shown at (*b*) on the plate, and on this line set off the lengths of the lines *bc*, *cd*, *de*, etc. as they appear in the plan at Fig. 2 (*a*). The vertical heights are then projected from the elevation in the manner shown, taking similar points in each case.

The true lengths of all lines now having been determined, the development may be constructed as shown at Fig. 2 (*c*). In this case, as in all instances where the solids are uneven and irregular in their form, it is preferable to begin the development from the longest edge that is shown in its true length. The line *mn* is, therefore, copied as shown at (*c*), and the triangle *n m l* constructed as in the preceding development, taking the lengths of the radii *nl*, *lj*, *jh*, etc. from the full view of the upper base, and the lengths of the radii *ml*, *lk*, *kj*, etc. from their respective triangles as formed at (*b*), while *mk*, *ki*, *ig*, etc. are taken from the full view of the lower base as shown in the plan at (*a*).

PROBLEM 18

53. To develop the surface of an irregular solid whose upper base is rightly inclined and whose lower base is a portion of a cylinder.

This solid is shown in perspective in Fig. 34, and the triangles that are to be located by the student in the projections are represented in the drawings shown on the plate.

Construction.—As may be seen from Fig. 3 (*a*) on the plate, the projections do not differ materially from those of the preceding problems. First draw the oval in the plan as in the two preceding problems, noting that, in this case, the outline is a foreshortened view of the real surface. Next, draw a line vertically upwards from *o*, the center of the semicircle, to the point *h* in the elevation, which is 3 inches distant from *o*. Through *h* draw *b n* at an angle of 30° with the horizontal; at right angles to *b n* project the full view of the upper base, which, as in Problem 17,

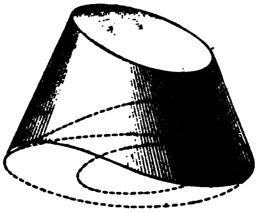

Fig. 34

is a circle. Through the plan draw *a m* and bisect it at *x*; with *x* as a center and a radius of $1\frac{13}{16}$ inches describe in the elevation the arc *a g m*, representing that view of the lower base.

As in the preceding problem, project the foreshortened view of the upper base, and, as before, designate the positions of the six spaces. Since the view of the base in the plan is foreshortened, it is necessary to project its full view, in order to ascertain the true distance around the outline. Divide the foreshortened outline of the lower base shown in the plan into six equal spaces, at *a*, *c*, *e*, etc. First, however, draw the center line *a m*, as in previous cases, and then project to the elevation the points thus located. To produce the full view, as at Fig. 3 (*c*), extend the line *a m* in the plan towards the left as far as to *m'*, and on this line lay off the stretchout of the lower base, as shown in the elevation; thus that portion of the solid is treated as a surface developed by means of parallel lines. Draw edge lines perpendicular to the stretchout, as on the plate, and produce developers from the points *a*, *c*, *e*, *g*, etc. in the plan to *a'*, *c'*,*e'*, *g'*, etc. at (*c*). The light curve shown at (*c*) is then drawn and represents the true outline of the lower base of the solid; measurements may now be taken from points on this outline, as *a' c'*, *c' e'*, etc., for the radii of the arcs required in that portion of the development at (*d*), their true lengths thus being shown. The true lengths of the lines shown in the projections at *b c*, *c d*, *d e*, etc. are obtained as before by constructing diagrams of triangles at (*b*), (*b*). Their projection on both sides of the elevation is done to avoid confusion from having a number of lines cross on the drawing.

Note the varied heights of the triangles, hence the need of extreme care; for if corresponding points are not taken from both plan and elevation, it will be difficult to trace the resulting errors. The true lengths of all lines having been determined in (*b*), (*b*), and (*c*), the development at (*d*) may be constructed by methods precisely like those used in Problems 16 and 17.

Since *m n* is the longer true edge shown in the elevation, the development should begin from that line by making *m n* at (*d*) equal to *m n* in the elevation at (*a*). With a radius *n l* (taken from the full view of the upper base), and with *n* in (*d*) as a center, describe an arc as shown, and intersect it at *l* with an arc described from *m* as a center, with a radius equal to the hypotenuse of the triangle *m l* at (*b*)—the true length of the line *m l*, as shown at (*a*).

This completes the triangle *m n l* at (*d*). Next, describe an arc from *m* as a center, with a radius *m' k'* taken from the full view of the lower base at (*c*), and intersect this arc at *k* with an arc described from *l* as a center and a radius equal to the hypotenuse of the triangle *l k* at (*b*). The triangle *m l k* is thus completed, and the remainder of the figure at (*d*) is constructed in a similar manner. The development at (*d*) is one-half of the irregular surface of the solid shown in Fig. 34.

It should be noted that, in the practical work of laying out patterns by this method, a sufficient number of points should be located on both bases of the solids to insure accuracy in tracing the curved line of the development.

54. Importance of a Correct Projection.—It will be seen from the foregoing problems that in each case the first operation is the drawing of a correct projection. The true lengths of all real or assumed lines that may have been shown in such a drawing of the solid are thus ascertained, and the draftsman is then enabled to determine which of the three general methods is to be applied in order that a development may be produced. The views to be drawn are in all cases those that will represent the solid in a *right* position, since the true length of any line is most easily obtained from such drawings. The ease with which this is accomplished is clearly shown in the foregoing examples; and if the student will devote his attention to the projection of a variety of commonly occurring trade subjects, he will quickly acquire a facility obtained only by constant practice.

Particular attention is directed to the imaginative feature of the projection, as mentioned in *Practical Projection;* for a correct conception of the actual position of the various lines as they will appear in the completed object can be acquired in no other way, and it is of extreme importance to the draftsman desiring to become proficient in the production of developments. If a student is expert at "reading" drawings, he will experience no difficulty in applying the various principles that have been given for producing developments of surfaces.

55. Employment of Modified Methods.—In producing the developments given in the last three problems, the triangles are not always projected from the views in the manner shown on the plate. Short methods are often used; but since in such cases there is more chance for error, the student will do well not to attempt any other method than that just described—at least, not until the principles are firmly fixed in his mind. As the drawings from which such developments are made are often too large to permit the projection of the triangles in the manner shown, the construction given in Fig. 35 is often applied. In this figure the triangles for Problem 16 are shown at (*a*), and those for Problems 17 and 18, at (*b*) and (*c*), respectively. A single

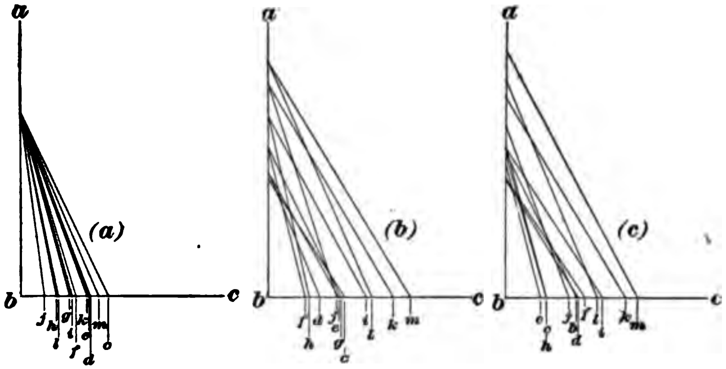

FIG. 35

right angle is first drawn, as *a b c* ; the lengths of all lines shown in the plan are then set off with the dividers on the side *b c*, while the vertical distances taken from the elevation are set off on the side *a b*. A number of slanting lines are then drawn between the points thus located, each forming the hypotenuse of its respective right-angled triangle. Unusual care must be observed when this method is adopted, since the points are located close together and are very apt to be mistaken for one another.

When many lines thus appear in the drawing, the student is very apt to become confused and to consider the drawing as complicated. If due care is used and ample time taken, this confusion will be avoided, for the drawings themselves depend on the simplest of principles—principles that may readily be understood by any one willing to give the time necessary for mastering this work.

56. Scalene Cone.—The development of this solid illustrates a short method of triangulation applicable to a number of solids, particularly to those represented by transition pieces one of whose bases is a polygon. A right view of such a cone is presented in Fig. 36 (*a*), and an inspection of its elements will show that they are of unequal length. This inequality, combined with the fact that a section taken at right angles to the axis of a scalene cone is not a circle, precludes its development by the method applied to radial solids, although the process is a combination of that method and triangulation. In accordance with instructions given hereafter, a drawing of this development should be made by the student, although it is not required to be sent to the Schools for correction.

After drawing the right plan and elevation as in Fig. 36 (*a*), the base is divided into a convenient number of equal parts (12 in Fig. 36). Next, the elements are drawn in both views, and it may then be seen that the surface of the cone is divided into a number of triangles whose common apex is at *o*, the vertex of the cone. In the elevation, the elements *o g* and *o a* are the only ones shown in their true length. In order to determine the true lengths of the remaining elements, the drawing shown at Fig. 36 (*b*) is constructed.

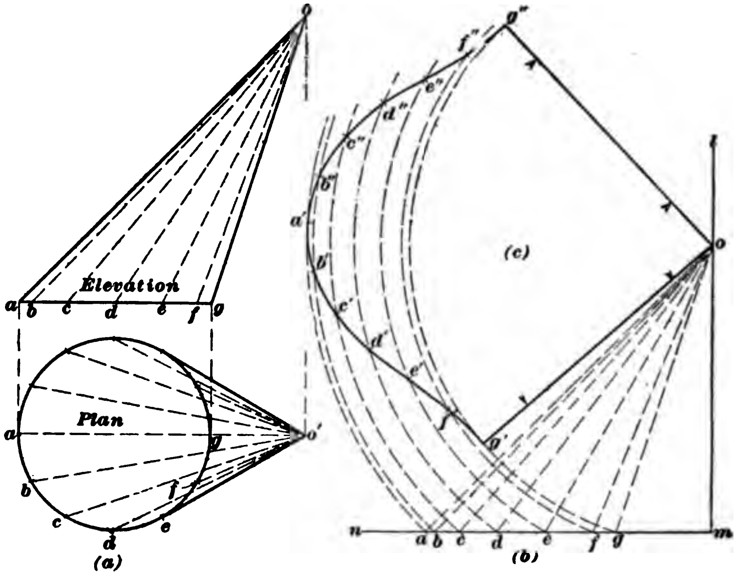

FIG. 36

Draw the right angle *l m n* and set off *m o*, the vertical height of the cone.

The length of each element shown in the plan at (*a*) is now set off on the line *m n*; thus, make *m g* at (*b*) equal to *o' g* in the plan at (*a*), *m f* at (*b*) equal to *o' f* at (*a*), etc. Draw *o g*, *o f*, *o e*, etc. at (*b*), thus producing the elements of the scalene cone in their true length, the method used, as will be seen by the student, being similar to that used in the other triangulation problems.

From *o* as a center, with *o g*, *o f*, *o e*, etc. as the respective radii, describe arcs in the manner shown in Fig. 36 (*b*). At a convenient point locate *g'*, and draw *g' o* as indicated at (*c*). This line (*g' o*) is one edge of the development, which may be completed by making *g' f'* at (*c*) equal to *g f* in the plan at (*a*), *f' e'* at (*c*) equal to *f e* at (*a*), etc., the compasses being set at this distance and similar arcs described as the successive points are located. Proceed in the manner shown until the point g'' is reached, when the development is completed by drawing $o g''$ and tracing the curve through points at the intersection of the arcs. The outer edge lines of the development may be further noted by the use of indicators as before described.

There are many irregular solids to which this method of development may be applied. The student should therefore work out this drawing very carefully, that the construction may be well understood and the method of its application readily seen. Where convenient, models should be made of these developments. These models may be cut out of paper or thin sheet metal and afterwards formed up to represent the solids described in the particular projections accompanying each problem. A better idea of the solids and their respective developments will thus be obtained. Should any errors arise in the course of the work, they may frequently be detected by this means.

APPROXIMATE DEVELOPMENTS

57. The Sphere.—The sphere is the most prominent example of the many solids whose surfaces admit of approximate development. The methods used with solids capable of accurate development must be applied to solids of approximate development. Since solids of approximate development may be resolved into parts resembling those capable of accurate development, it is clear that the same general methods are easily applicable. Thus, in the case of the sphere, that solid is resolved into a number of frustums of cones, in the manner described in the next article.

As already stated, it is necessary to submit the covering for these solids to the operations of "raising," in order to conform the metal to the profile of the solid itself; an allowance is therefore required for the consequent stretching of the stock. This allowance varies with the thickness and quality of the material; hence, no general rules can be given here. The mechanic must be acquainted with the nature of the material in order to determine the allowance required in each case.

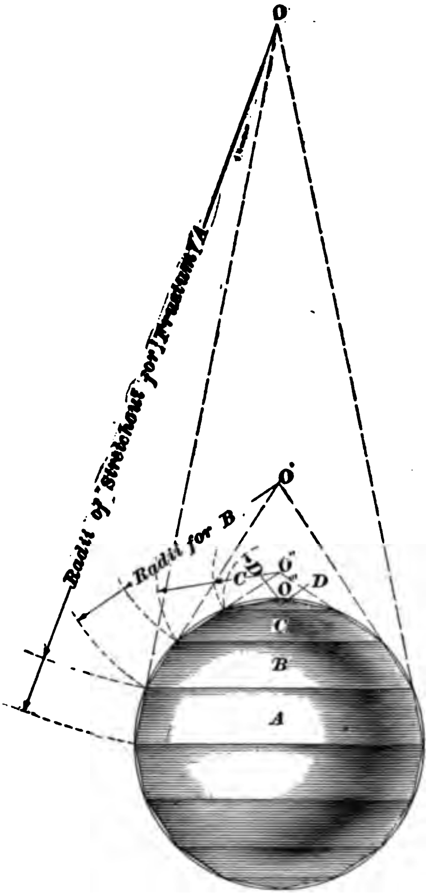

FIG. 37

58. Development of the Sphere.—It has been stated that each solid of *approximate* development must be referred to one of the three classes of solids capable of *accurate* development. It is first necessary, therefore, to determine which of the regular solids the irregular solid to be developed most resembles. The blanks, or patterns, produced by these methods are, as has been observed, flat, or plane, surfaces; and since it is necessary to "raise" these surfaces by hammering, the draftsman must imagine the surface desired in the final form. The principle by which these solids are classified is illustrated in Fig. 37 by the perspective of the sphere and its resolved cones. After the sphere has thus been resolved into cones—or rather, frustums of cones—their developments may be produced in the regular way, each frustum being separately treated, in the manner shown. This is called *development by zones*, and may be applied to a number of irregular solids resembling the sphere.

Another method of treatment is shown in Fig. 38, certain sections of the sphere being here considered as portions of the surface of the cylinder. This method is referred to as the *development by gores;* the patterns are developed, as with any portion of the cylinder, by means of parallel lines.

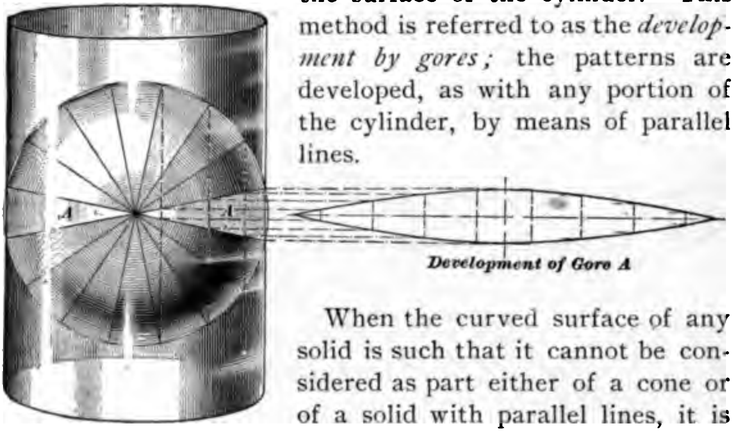

FIG. 38

When the curved surface of any solid is such that it cannot be considered as part either of a cone or of a solid with parallel lines, it is customary to determine a series of sections in a regularly occurring order and apply the method of triangulation as shown in Problems 16 to 18. Such surfaces are then formed up as nearly as possible to the shape of the solid and are then submitted to the operations referred to for "raising" to their proper shape.

A TEXTBOOK

ON

MECHANICAL ENGINEERING

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

TABLES AND FORMULAS

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
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TABLES AND FORMULAS.

This volume contains all the principal Tables and Formulas which are likely to be used by the student in practice. They have been collected and placed in this volume in order to make them convenient for ready reference, so that the student will not be obliged to hunt them out in the preceding volumes. The number after each formula is the same as the number following the same formula in the text.

TABLE

OF

COMMON LOGARITHMS

OF NUMBERS

From 1 to 10,000.

N	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
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12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
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P. P.

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4	17.6	17.2	16.8
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6	26.4	25.8	25.2
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8	28.0	27.2	26.4
9	31.5	30.6	29.7

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9	28.8	27.9	27.0

P. P.

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194	780	803	825	847	870	892	914	937	959	981
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N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

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9	26.1	25.2

	27	**26**
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5	13.5	13.0
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7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
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2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	**23**
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3	7.2	6.9
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5	12.0	11.5
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8	19.2	18.4
9	21.6	20.7

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6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
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201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	22	**21**
1	2.2	2.1
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8	17.6	16.8
9	19.8	18.9

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8	14.4
9	16.2

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9	15.3

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.

	18
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9	13.5

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8	11.2
9	12.6

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	**15** 1 | 1.5 2 | 3.0 3 | 4.5 4 | 6.0 5 | 7.5 6 | 9.0 7 | 10.5 8 | 12.0 9 | 13.5
306	572	586	601	615	629	643	657	671	686	700	
307	714	728	742	756	770	785	799	813	827	841	
308	855	869	883	897	911	926	940	954	968	982	
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49 136	150	164	178	192	206	220	234	248	262	
311	276	290	304	318	332	346	360	374	388	402	
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	**14** 1 | 1.4 2 | 2.8 3 | 4.2 4 | 5.6 5 | 7.0 6 | 8.4 7 | 9.8 8 | 11.2 9 | 12.6
317	50 106	120	133	147	161	174	188	202	215	229	
318	243	256	270	284	297	311	325	338	352	365	
319	379	393	406	420	433	447	461	474	488	501	
320	515	529	542	556	569	583	596	610	623	637	
321	651	664	678	691	705	718	732	745	759	772	
322	786	799	813	826	840	853	866	880	893	907	
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	**13** 1 | 1.3 2 | 2.6 3 | 3.9 4 | 5.2 5 | 6.5 6 | 7.8 7 | 9.1 8 | 10.4 9 | 11.7
328	587	601	614	627	640	654	667	680	693	706	
329	720	733	746	759	772	786	799	812	825	838	
330	851	865	878	891	904	917	930	943	957	970	
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52 114	127	140	153	166	179	192	205	218	231	
333	244	257	270	284	297	310	323	336	349	362	
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	**12** 1 | 1.2 2 | 2.4 3 | 3.6 4 | 4.8 5 | 6.0 6 | 7.2 7 | 8.4 8 | 9.6 9 | 10.8
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	
341	275	288	301	314	326	339	352	364	377	390	
342	403	415	428	441	453	466	479	491	504	517	
343	529	542	555	567	580	593	605	618	631	643	
344	656	668	681	694	706	719	732	744	757	769	
345	782	794	807	820	832	845	857	870	882	895	
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	**13**
355	55 023	035	047	060	072	084	096	108	121	133	1 | 1.3
356	145	157	169	182	194	206	218	230	242	255	2 | 2.6
357	267	279	291	303	315	328	340	352	364	376	3 | 3.9
358	388	400	413	425	437	449	461	473	485	497	4 | 5.2
359	509	522	534	546	558	570	582	594	606	618	5 | 6.5
360	630	642	654	666	678	691	703	715	727	739	6 | 7.8
361	751	763	775	787	799	811	823	835	847	859	7 | 9.1
362	871	883	895	907	919	931	943	955	967	979	8 | 10.4
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9 | 11.7
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	**12**
367	467	478	490	502	514	526	538	549	561	573	1 | 1.2
368	585	597	608	620	632	644	656	667	679	691	2 | 2.4
369	703	714	726	738	750	761	773	785	797	808	3 | 3.6
370	820	832	844	855	867	879	891	902	914	926	4 | 4.8
371	937	949	961	972	984	996	*008	*019	*031	*043	5 | 6.0
372	57 054	066	078	089	101	113	124	136	148	159	6 | 7.2
373	171	183	194	206	217	229	241	252	264	276	7 | 8.4
374	287	299	310	322	334	345	357	368	380	392	8 | 9.6
375	403	415	426	438	449	461	473	484	496	507	9 | 10.8
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	**11**
379	864	875	887	898	910	921	933	944	955	967	1 | 1.1
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	2 | 2.2
381	58 092	104	115	127	138	149	161	172	184	195	3 | 3.3
382	206	218	229	240	252	263	274	286	297	309	4 | 4.4
383	320	331	343	354	365	377	388	399	410	422	5 | 5.5
384	433	444	456	467	478	490	501	512	524	535	6 | 6.6
385	546	557	569	580	591	602	614	625	636	647	7 | 7.7
386	659	670	681	692	704	715	726	737	749	760	8 | 8.8
387	771	782	794	805	816	827	838	850	861	872	9 | 9.9
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	**10**
391	218	229	240	251	262	273	284	295	306	318	1 | 1.0
392	329	340	351	362	373	384	395	406	417	428	2 | 2.0
393	439	450	461	472	483	494	506	517	528	539	3 | 3.0
394	550	561	572	583	594	605	616	627	638	649	4 | 4.0
395	660	671	682	693	704	715	726	737	748	759	5 | 5.0
396	770	780	791	802	813	824	835	846	857	868	6 | 6.0
397	879	890	901	912	923	934	945	956	966	977	7 | 7.0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	8 | 8.0
399	60 097	108	119	130	141	152	163	173	184	195	9 | 9.0
400	206	217	228	239	249	260	271	282	293	304	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	**11** 1 1.1 2 2.2 3 3.3 4 4.4 5 5.5 6 6.6 7 7.7 8 8.8 9 9.9
407	959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	
409	172	183	194	204	215	225	236	247	257	268	
410	278	289	300	310	321	331	342	352	363	374	
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	**10** 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0
423	634	644	655	665	675	685	696	706	716	726	
424	737	747	757	767	778	788	798	808	818	829	
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	236	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	**9** 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1
438	147	157	167	177	187	197	207	217	227	237	
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	10 1 1.0 2 2.0 3 3.0 4 4.0 5 5.0 6 6.0 7 7.0 8 8.0 9 9.0
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66 087	096	106	115	124	134	143	153	162	172	
459	181	191	200	210	219	229	238	247	257	266	
460	276	285	295	304	314	323	332	342	351	361	
461	370	380	389	398	408	417	427	436	445	455	
462	464	474	483	492	502	511	521	530	539	549	
463	558	567	577	586	596	605	614	624	633	642	
464	652	661	671	680	689	699	708	717	727	736	
465	745	755	764	773	783	792	801	811	820	829	
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	9 1 0.9 2 1.8 3 2.7 4 3.6 5 4.5 6 5.4 7 6.3 8 7.2 9 8.1
473	486	495	504	514	523	532	541	550	560	569	
474	578	587	596	605	614	624	633	642	651	660	
475	669	679	688	697	706	715	724	733	742	752	
476	761	770	779	788	797	806	815	825	834	843	
477	852	861	870	879	888	897	906	916	925	934	
478	943	952	961	970	979	988	997	*006	*015	*024	
479	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
488	842	851	860	869	878	886	895	904	913	922	
489	931	940	949	958	966	975	984	993	*002	*011	
490	69 020	028	037	046	055	064	073	082	090	099	
491	108	117	126	135	144	152	161	170	179	188	
492	197	205	214	223	232	241	249	258	267	276	
493	285	294	302	311	320	329	338	346	355	364	
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	9 1 | 0.9 2 | 1.8 3 | 2.7 4 | 3.6 5 | 4.5 6 | 5.4 7 | 6.3 8 | 7.2 9 | 8.1
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	8 1 | 0.8 2 | 1.6 3 | 2.4 4 | 3.2 5 | 4.0 6 | 4.8 7 | 5.6 8 | 6.4 9 | 7.2
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	7 1 | 0.7 2 | 1.4 3 | 2.1 4 | 2.8 5 | 3.5 6 | 4.2 7 | 4.9 8 | 5.6 9 | 6.3
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	**8** 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	**7** 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	**8** 1 | 0.8 2 | 1.6 3 | 2.4 4 | 3.2 5 | 4.0 6 | 4.8 7 | 5.6 8 | 6.4 9 | 7.2
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	
609	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
611	604	611	618	625	633	640	647	654	661	668	
612	675	682	689	696	704	711	718	725	732	739	
613	746	753	760	767	774	781	789	796	803	810	
614	817	824	831	838	845	852	859	866	873	880	
615	888	895	902	909	916	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	**7** 1 | 0.7 2 | 1.4 3 | 2.1 4 | 2.8 5 | 3.5 6 | 4.2 7 | 4.9 8 | 5.6 9 | 6.3
622	379	386	393	400	407	414	421	428	435	442	
623	449	456	463	470	477	484	491	498	505	511	
624	518	525	532	539	546	553	560	567	574	581	
625	588	595	602	609	616	623	630	637	644	650	
626	657	664	671	678	685	692	699	706	713	720	
627	727	734	741	748	754	761	768	775	782	789	
628	796	803	810	817	824	831	837	844	851	858	
629	865	872	879	886	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	**6** 1 | 0.6 2 | 1.2 3 | 1.8 4 | 2.4 5 | 3.0 6 | 3.6 7 | 4.2 8 | 4.8 9 | 5.4
638	482	489	496	502	509	516	523	530	536	543	
639	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
641	686	693	699	706	713	720	726	733	740	747	
642	754	760	767	774	781	787	794	801	808	814	
643	821	828	835	841	848	855	862	868	875	882	
644	889	895	902	909	916	922	929	936	943	949	
645	956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
661	82 020	027	033	040	046	053	060	066	073	079	
662	086	092	099	105	112	119	125	132	138	145	
663	151	158	164	171	178	184	191	197	204	210	
664	217	223	230	236	243	249	256	263	269	276	
665	282	289	295	302	308	315	321	328	334	341	
666	347	354	360	367	373	380	387	393	400	406	
667	413	419	426	432	439	445	452	458	465	471	
668	478	484	491	497	504	510	517	523	530	536	
669	543	549	556	562	569	575	582	588	595	601	
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	6 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
685	569	575	582	588	594	601	607	613	620	626	
686	632	639	645	651	658	664	670	677	683	689	
687	696	702	708	715	721	727	734	740	746	753	
688	759	765	771	778	784	790	797	803	809	816	
689	822	828	835	841	847	853	860	866	872	879	
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	**7** 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
708	85 003	009	016	022	028	034	040	046	052	058	
709	065	071	077	083	089	095	101	107	114	120	
710	126	132	138	144	150	156	163	169	175	181	
711	187	193	199	205	211	217	224	230	236	242	
712	248	254	260	266	272	278	285	291	297	303	
713	309	315	321	327	333	339	345	352	358	364	
714	370	376	382	388	394	400	406	412	418	425	
715	431	437	443	449	455	461	467	473	479	485	
716	491	497	503	509	516	522	528	534	540	546	
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	**6** 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
723	914	920	926	932	938	944	950	956	962	968	
724	974	980	986	992	998	*004	*010	*016	*022	*028	
725	86 034	040	046	052	058	064	070	076	082	088	
726	094	100	106	112	118	124	130	136	141	147	
727	153	159	165	171	177	183	189	195	201	207	
728	213	219	225	231	237	243	249	255	261	267	
729	273	279	285	291	297	303	308	314	320	326	
730	332	338	344	350	356	362	368	374	380	386	
731	392	398	404	410	415	421	427	433	439	445	
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	**5** 1 0.5 2 1.0 3 1.5 4 2.0 5 2.5 6 3.0 7 3.5 8 4.0 9 4.5
738	806	812	817	823	829	835	841	847	853	859	
739	864	870	876	882	888	894	900	906	911	917	
740	923	929	935	941	947	953	958	964	970	976	
741	982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87 040	046	052	058	064	070	075	081	087	093	
743	099	105	111	116	122	128	134	140	146	151	
744	157	163	169	175	181	186	192	198	204	210	
745	216	221	227	233	239	245	251	256	262	268	
746	274	280	286	291	297	303	309	315	320	326	
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
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754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
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761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	6 1 | 0.6 2 | 1.2 3 | 1.8 4 | 2.4 5 | 3.0 6 | 3.6 7 | 4.2 8 | 4.8 9 | 5.4
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
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771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
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778	098	104	109	115	120	126	131	137	143	148	
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780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	5 1 | 0.5 2 | 1.0 3 | 1.5 4 | 2.0 5 | 2.5 6 | 3.0 7 | 3.5 8 | 4.0 9 | 4.5
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783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
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791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
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795	90 037	042	048	053	059	064	069	075	080	086	
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797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
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800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
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818	275	281	286	291	297	302	307	312	318	323	
819	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	
832	92 012	018	023	028	033	038	044	049	054	059	
833	065	070	075	080	085	091	096	101	106	111	
834	117	122	127	132	137	143	148	153	158	163	
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839	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
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7	4.2
8	4.8
9	5.4

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852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	**6** 1 0.6 2 1.2 3 1.8 4 2.4 5 3.0 6 3.6 7 4.2 8 4.8 9 5.4
858	349	354	359	364	369	374	379	384	389	394	
859	399	404	409	414	420	425	430	435	440	445	
860	450	455	460	465	470	475	480	485	490	495	
861	500	505	510	515	520	526	531	536	541	546	
862	551	556	561	566	571	576	581	586	591	596	
863	601	606	611	616	621	626	631	636	641	646	
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871	94 002	007	012	017	022	027	032	037	042	047	
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873	101	106	111	116	121	126	131	136	141	146	
874	151	156	161	166	171	176	181	186	191	196	
875	201	206	211	216	221	226	231	236	240	245	
876	250	255	260	265	270	275	280	285	290	295	
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879	399	404	409	414	419	424	429	433	438	443	
880	448	453	458	463	468	473	478	483	488	493	
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
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885	694	699	704	709	714	719	724	729	734	738	
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888	841	846	851	856	861	866	871	876	880	885	
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891	988	993	998	*002	*007	*012	*017	*022	*027	*032	
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902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
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910	904	909	914	918	923	928	933	938	942	947	**5**
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918	284	289	294	298	303	308	313	317	322	327	8 | 4.0
919	332	336	341	346	350	355	360	365	369	374	9 | 4.5
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932	942	946	951	956	960	965	970	974	979	984	
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959	182	186	191	195	200	204	209	214	218	223	
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963	363	367	372	376	381	385	390	394	399	403	
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966	498	502	507	511	516	520	525	529	534	538	
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971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
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977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
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982	211	216	220	224	229	233	238	242	247	251	
983	255	260	264	269	273	277	282	286	291	295	
984	300	304	308	313	317	322	326	330	335	339	
985	344	348	352	357	361	366	370	374	379	383	
986	388	392	396	401	405	410	414	419	423	427	
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988	476	480	484	489	493	498	502	506	511	515	
989	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
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997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

TABLES

OF

NATURAL SINES, COSINES, TANGENTS, AND COTANGENTS

GIVING THE VALUES OF THE FUNCTIONS FOR
ALL DEGREES AND MINUTES FROM
0° TO 90°

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

NATURAL SINES AND COSINES.

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

NATURAL SINES AND COSINES.

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53051	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

NATURAL TANGENTS AND COTANGENTS.

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infin.	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03402	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81812	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.30560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		65°		

'	25°		26°		27°		28°		29°		'
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
'	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	'
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59149	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.35567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1 48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

NATURAL TANGENTS AND COTANGENTS.

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.03252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

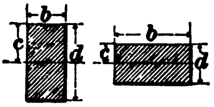
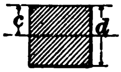
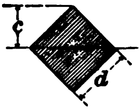
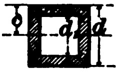
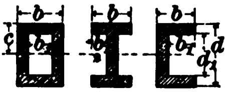
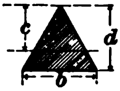
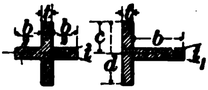
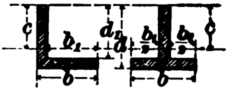
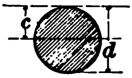
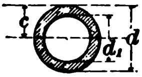
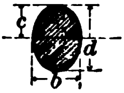
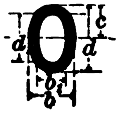
THE PROPERTIES OF SATURATED STEAM.

Pressure above Vacuum in Pounds per Square Inch.	Temperature, Fahrenheit Degrees.	Quantities of Heat in British Thermal Units.			Weight of a Cubic Foot of Steam in Pounds.	Volume.	
		Required to Raise Temperature of the Water from 32° to *t*°.	Total Latent Heat at Pressure *P*.	Total Heat above 32°.		Of a Pound of Steam in Cubic Feet.	Ratio of Vol. of Steam to Vol. of Eq. Weight of Dist. Water at Temp. of Maximum Density.
1	2	3	4	5	6	7	8
p	*t*	*q*	*L*	*H*	*W*	*V*	*R*
1	102.018	70.040	1043.015	1113.055	.003027	330.4	20623
2	126.302	94.368	1026.094	1120.462	.005818	171.9	10730
3	141.654	109.764	1015.380	1125.144	.008522	117.3	7325
4	153.122	121.271	1007.370	1128.641	.011172	89.51	5588
5	162.370	130.563	1000.899	1131.462	.013781	72.56	4530
6	170.173	138.401	995.441	1133.842	.016357	61.14	3816
7	176.945	145.213	990.695	1135.908	.018908	52.89	3302
8	182.952	151.255	986.485	1137.740	.021436	46.65	2912
9	188.357	156.699	982.690	1139.389	.023944	41.77	2607
10	193.284	161.660	979.232	1140.892	.026437	37.83	2361
11	197.814	166.225	976.050	1142.275	.028911	34.59	2159
12	202.012	170.457	973.098	1143.555	.031376	31.87	1990
13	205.929	174.402	970.346	1144.748	.033828	29.56	1845
14	209.604	178.112	967.757	1145.869	.036265	27.58	1721
14.69	212.000	180.531	966.069	1146.600	.037928	26.37	1646
15	213.067	181.608	965.318	1146.926	.038688	25.85	1614
16	216.347	184.919	963.007	1147.926	.041109	24.33	1519
17	219.452	188.056	960.818	1148.874	.043519	22.98	1434
18	222.424	191.058	958.721	1149.779	.045920	21.78	1359
19	225.255	193.918	956.725	1150.643	.048312	20.70	1292

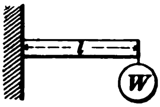
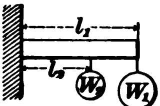
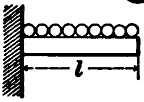
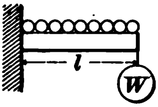
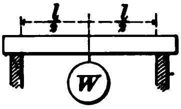
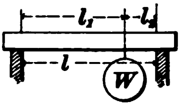
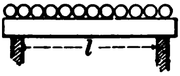
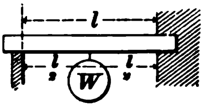
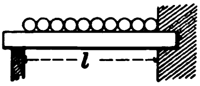
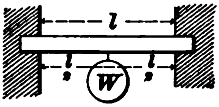
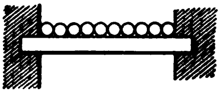
1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
20	227.964	196.655	954.814	1151.469	.050696	19.73	1231.0
22	233.069	201.817	951.209	1153.026	.055446	18.04	1126.0
24	237.803	206.610	947.861	1154.471	.060171	16.62	1038.0
26	242.225	211.089	944.730	1155.819	.064870	15.42	962.3
28	246.376	215.293	941.791	1157.084	.069545	14.38	897.6
30	250.293	219.261	939.019	1158.280	.074201	13.48	841.3
32	254.002	223.021	936.389	1159.410	.078839	12.68	791.8
34	257.523	226.594	933.891	1160.485	.083461	11.98	748.0
36	260.883	230.001	931.508	1161.509	.088067	11.36	708.8
38	264.093	233.261	929.227	1162.488	.092657	10.79	673.7
40	267.168	236.386	927.040	1163.426	.097231	10.28	642.0
42	270.122	239.389	924.940	1164.329	.101794	9.826	613.3
44	272.965	242.275	922.919	1165.194	.106345	9.403	587.0
46	275.704	245.061	920.968	1166.029	.110884	9.018	563.0
48	278.348	247.752	919.084	1166.836	.115411	8.665	540.9
50	280.904	250.355	917.260	1167.615	.119927	8.338	520.5
52	283.381	252.875	915.494	1168.369	.124433	8.037	501.7
54	285.781	255.321	913.781	1169.102	.128928	7.756	484.2
56	288.111	257.695	912.118	1169.813	.133414	7.496	467.9
58	290.374	260.002	910.501	1170.503	.137892	7.252	452.7
60	292.575	262.248	908.928	1171.176	.142362	7.024	438.5
62	294.717	264.433	907.396	1171.829	.146824	6.811	425.2
64	296.805	266.566	905.900	1172.466	.151277	6.610	412.6
66	298.842	268.644	904.443	1173.087	.155721	6.422	400.8
68	300.831	270.674	903.020	1173.694	.160157	6.244	389.8
70	302.774	272.657	901.629	1174.286	.164584	6.076	379.3
72	304.669	274.597	900.269	1174.866	.169003	5.917	369.4
74	306.526	276.493	898.938	1175.431	.173417	5.767	360.0
76	308.344	278.350	897.635	1175.985	.177825	5.624	351.1
78	310.123	280.170	896.359	1176.529	.182229	5.488	342.6
80	311.866	281.952	895.108	1177.060	.186627	5.358	334.5
82	313.576	283.701	893.879	1177.580	.191017	5.235	326.8
84	315.250	285.414	892.677	1178.091	.195401	5.118	319.5
86	316.893	287.096	891.496	1178.592	.199781	5.006	312.5
88	318.510	288.750	890.335	1179.085	.204155	4.898	305.8

1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
90	320.094	290.373	889.196	1179.569	.208525	4.796	299.4
92	321.653	291.970	888.075	1180.045	.212892	4.697	293.2
94	323.183	293.539	886.972	1180.511	.217253	4.603	287.3
96	324.688	295.083	885.887	1180.970	.221604	4.513	281.7
98	326.169	296.601	884.821	1181.422	.225950	4.426	276.3
100	327.625	298.093	883.773	1181.866	.230293	4.342	271.1
105	331.169	301.731	881.214	1182.945	.241139	4.147	258.9
110	334.582	305.242	878.744	1183.986	.251947	3.969	247.8
115	337.874	308.621	876.371	1184.992	.262732	3.806	237.6
120	341.058	311.885	874.076	1185.961	.273500	3.656	228.3
125	344.136	315.051	871.848	1186.899	.284243	3.518	219.6
130	347.121	318.121	869.688	1187.809	.294961	3.390	211.6
135	350.015	321.105	867.590	1188.695	.305659	3.272	204.2
140	352.827	324.003	865.552	1189.555	.316338	3.161	197.3
145	355.562	326.823	863.567	1190.390	.326998	3.058	190.9
150	358.223	329.566	861.634	1191.200	.337643	2.962	184.9
160	363.346	334.850	857.912	1192.762	.358886	2.786	173.9
170	368.226	339.892	854.359	1194.251	.380071	2.631	164.3
180	372.886	344.708	850.963	1195.671	.401201	2.493	155.6
190	377.352	349.329	847.703	1197.032	.422280	2.368	147.8
200	381.636	353.766	844.573	1198.339	.443310	2.256	140.8
210	385.759	358.041	841.556	1199.597	.464295	2.154	134.5
220	389.736	362.168	838.642	1200.810	.485237	2.061	128.7
230	393.575	366.152	835.828	1201.980	.506139	1.976	123.3
240	397.285	370.008	833.103	1203.111	.527003	1.898	118.5
250	400.883	373.750	830.459	1204.209	.547831	1.825	114.0
260	404.370	377.377	827.896	1205.273	.568626	1.759	109.8
270	407.755	380.905	825.401	1206.306	.589390	1.697	105.9
280	411.048	384.337	822.973	1207.310	.610124	1.639	102.3
290	414.250	387.677	820.609	1208.286	.630829	1.585	99.0
300	417.371	390.933	818.305	1209.238	.651506	1.535	95.8

MOMENTS OF INERTIA.

Form of Section.	Dotted Line Shows Position of Neutral Axis.	A	I	c
1. Rectangle...		bd	$\frac{1}{12}bd^3$	$\frac{1}{2}d$
2. Square..		d^2	$\frac{1}{12}d^4$	$\frac{1}{2}d$
3. Axis through Diagonal		d^2	$\frac{1}{12}d^4$	$.707d$
4. Hollow Square		$d^2 - d_1^2$	$\frac{1}{12}(d^4 - d_1^4)$	$\frac{1}{2}d$
5. Hollow Rectangle, I or Channel Iron		$bd - b_1d_1$	$\frac{1}{12}(bd^3 - b_1d_1^3)$	$\frac{1}{2}d$
6. Triangle		$\frac{1}{2}bd$	$\frac{1}{36}bd^3$	$\frac{2}{3}d$
7. Cross....		$td + t_1b$	$\frac{1}{12}(td^3 + bt_1^3)$	$\frac{1}{2}d$
8. Angle Iron.....		$bd - b_1d_1$	$\frac{(bd^2 - b_1d_1^2)^2 - 4bdb_1d_1(d - d_1)^2}{12(bd - b_1d_1)}$	$\frac{d}{2} + \frac{b_1d_1}{2}\left(\frac{d - d_1}{bd - b_1d_1}\right)$
9. Circle....		$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	$\frac{1}{2}d$
10. Hollow Circle...		$\frac{\pi}{4}(d^2 - d_1^2)$	$\frac{\pi(d^4 - d_1^4)}{64}$	$\frac{1}{2}d$
11. Ellipse..		$\frac{\pi}{4}bd$	$\frac{\pi bd^3}{64}$	$\frac{1}{2}d$
12. Hollow Ellipse..		$\frac{\pi}{4}(bd - b_1d_1)$	$\frac{\pi(bd^3 - b_1d_1^3)}{64}$	$\frac{1}{2}d$

BENDING MOMENTS AND DEFLECTIONS.

Manner of Supporting Beams.	Maximum Bending Moment, M.	Maximum Deflection, s.	Remarks.
1. 	Wl	$\frac{1}{3}\frac{Wl^3}{EI}$	Cantilever, load at free end.
2. 	$W_1l_1+W_2l_2$		Cantilever, more than one load.
3. 	$\frac{wl^2}{2}$	$\frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, uniform load w lb. per unit of length. $W'=wl$.
4. 	$\frac{wl^2}{2}+Wl$	$\frac{1}{3}\frac{Wl^3}{EI}+\frac{1}{8}\frac{W'l^3}{EI}$	Cantilever, load partly uniform, partly concentrated.
5. 	$\frac{Wl}{4}$	$\frac{1}{48}\frac{Wl^3}{EI}$	Simple beam, load at middle.
6. 	$W\frac{l_1l_2}{l}$		Simple beam, load at some other point than the middle.
7. 	$\frac{wl^2}{8}$	$\frac{5}{384}\frac{W'l^3}{EI}$	Simple beam, uniformly loaded.
8. 	$\frac{3}{16}Wl$	$.0182\frac{Wl^3}{EI}$	One end fixed, other end supported, load in the middle.
9. 	$\frac{wl^2}{8}$	$.0054\frac{W'l^3}{EI}$	One end fixed, other end supported, uniformly loaded.
10. 	$\frac{Wl}{8}$	$\frac{1}{192}\frac{Wl^3}{EI}$	Both ends fixed, load in the middle.
11. 	$\frac{wl^2}{12}$		-ni.

SPECIFIC GRAVITIES AND WEIGHTS PER CUBIC FOOT.

METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Osmium	23.00	1,437.5
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428.8
Antimony	6.71	419.4
Aluminum	2.50	156.3

WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Acetic Acid	1.062	66.4
Nitric Acid	1.217	76.1
Sulphuric Acid	1.841	115.1
Muriatic Acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea Water (ordinary)	1.026	64.1
Milk	1.032	64.5

GASES.

At 32° F., and under a Pressure of One Atmosphere.

Substance.	Specifio Gravity.	Weight per Cubic Foot in Pounds.
Atmospheric Air	1.0000	.08073
Carbonic Acid	1.5290	.12344
Carbonic Oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03790
Hydrogen	.0692	.00559

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* The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is 0.622.

MISCELLANEOUS.

Substance.	Specific Gravity.	Weight per Cubic Foot in Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster Paris (average)	2.00	125
Sand	1.80	113

MISCELLANEOUS TABLES.

SPECIFIC HEATS UNDER CONSTANT PRESSURE.

SOLIDS.

Copper	0.0951	Cast Iron	0.1298
Gold	0.0324	Lead	0.0314
Wrought Iron	0.1138	Platinum	0.0324
Steel (soft)	0.1165	Silver	0.0570
Steel (hard)	0.1175	Tin	0.0562
Zinc	0.0956	Ice	0.5040
Brass	0.0939	Sulphur	0.2026
Glass	0.1937	Charcoal	0.2410

LIQUIDS.

Water	1.0000	Lead (melted)	0.0402
Alcohol	0.7000	Sulphur (melted)	0.2340
Mercury	0.0333	Tin (melted)	0.0637
Benzine	0.4500	Sulphuric Acid	0.3350
Glycerine	0.5550	Oil of Turpentine	0.4260

GASES.

	Constant Pressure.	Constant Volume.
Air	0.23751	0.16847
Oxygen	0.21751	0.15507
Nitrogen	0.24380	0.17273
Hydrogen	3.40900	2.41226
Superheated Steam	0.48050	0.34600
Carbonic Oxide	0.24790	0.17580
Carbonic Acid	0.40400	0.15350

VOLUMES AND WEIGHTS OF GASES.

Gas.	Volume of one Pound at 32°F. and a Tension of 1 Atmosphere (14.7 Lb. per Sq. In.). Cu. Ft.	Weight of one Cu. Ft. at 32° F. and a Tension of 1 Atmosphere (14.7 Lb. per Sq. In.). Lb.	R.
Air	12.388	.08073	.37052
Oxygen	11.2056	.08925	.33552
Nitrogen	12.7226	.0786	.38143
Hydrogen	178.891	.00559	5.34946

TEMPERATURES OF FUSION AND VAPORIZATION.

Substance.	Temperature of Fusion.	Temperature of Vaporization.	Latent Heat of Fusion.	Latent Heat of Vaporization.
Water	32°	212°	142.65	966.6
Mercury	− 37.8°	662°	5.09	157
Sulphur	228.3°	824°	13.26	
Tin	446°		25.65	
Lead	626°		9.67	
Zinc	680°	1900°	50.63	493
Alcohol	Unknown	173°		372
Oil of Turpentine	14°	313°		124
Linseed Oil		600°		
Aluminum	1400°			
Copper	2100°			
Cast Iron	2192°	3300°		
Wrought Iron	2912°	5000°		
Steel	2520°			
Platinum	3632°			
Iridium	4892°			

COEFFICIENTS OF EXPANSION.

Name of Substance.	Linear Expansion.	Surface Expansion.	Cubic Expansion.
Cast Iron	.00000617	.00001234	.00001850
Copper	.00000955	.00001910	.00002864
Brass	.00001037	.00002074	.00003112
Silver	.00000690	.00001390	.00002070
Bar Iron	.00000686	.00001372	.00002058
Steel (untempered)	.00000599	.00001198	.00001798
Steel (tempered)	.00000702	.00001404	.00002106
Zinc	.00001634	.00003268	.00004903
Tin	.00001410	.00002820	.00004229
Mercury	.00003334	.00006668	.00010010
Alcohol	.00019259	.00038518	.00057778
Gases			.00203252

UNITED STATES STANDARD SCREW THREADS.

Diameter of Screw in Inches. d	Number of Threads per Inch. n	Diameter at Bottom of Threads in Inches. d_1	Area at Bottom of Threads in Square Inches. a
¼	20	.185	.0269
5/16	18	.240	.0452
⅜	16	.294	.0679
7/16	14	.345	.0935
½	13	.400	.1257
9/16	12	.454	.1619
⅝	11	.507	.2019
¾	10	.620	.3019
⅞	9	.731	.4197
1	8	.838	.5515
1⅛	7	.939	.6925
1¼	7	1.064	.8892
1⅜	6	1.158	1.0532
1½	6	1.283	1.2928
1⅝	5½	1.389	1.5153
1¾	5	1.490	1.7437
1⅞	5	1.615	2.0485
2	4½	1.711	2.2993
2¼	4½	1.961	3.0203
2½	4	2.175	3.7154
2¾	4	2.425	4.6186
3	3½	2.629	5.4284
3¼	3½	2.879	6.5099
3½	3¼	3.100	7.5477
3¾	3	3.317	8.6414
4	3	3.567	9.9930
4¼	2⅞	3.798	11.3292
4½	2¾	4.027	12.7366
4¾	2⅝	4.255	14.2197
5	2½	4.480	15.7633
5¼	2½	4.730	17.5717
5½	2⅜	4.953	19.2676
5¾	2⅜	5.203	21.2617
6	2¼	5.423	23.0978

STANDARD DIMENSIONS OF WROUGHT-IRON STEAM, GAS, AND WATER PIPES.

Nominal Diameter in Inches.	Thickness in Inches.	Actual Internal Diameter in Inches.	Actual External Diameter in Inches.	Threads per Inch. *n*	Pitch of Threads.
1/8	.068	.270	.405	27	.037
1/4	.088	.364	.540	18	.056
3/8	.091	.494	.675	18	.056
1/2	.109	.623	.840	14	.071
3/4	.113	.824	1.050	14	.071
1	.134	1.048	1.315	11½	.087
1¼	.140	1.380	1.660	11½	.087
1½	.145	1.611	1.900	11½	.087
2	.154	2.067	2.375	11½	.087
2½	.204	2.468	2.875	8	.125
3	.217	3.061	3.500	8	.125
3½	.226	3.548	4.000	8	.125
4	.237	4.026	4.500	8	.125
4½	.247	4.508	5.000	8	.125
5	.259	5.045	5.563	8	.125
6	.280	6.065	6.625	8	.125
7	.301	7.023	7.625	8	.125
8	.322	7.982	8.625	8	.125
9	.344	9.001	9.688	8	.125
10	.366	10.019	10.750	8	.125

STANDARD PIPE FLANGES.

Inside Diam. of Pipe.	Thickness of Pipe.	Diam. of Bolts.	Length of Bolts.	No. of Bolts.	Thickness of Flange.	Diam. of Bolt Circle.	Diam. of Flange.
2.0	.409	$\frac{5}{8}$	2.0	4	$\frac{5}{8}$	4.75	6.0
2.5	.429	$\frac{5}{8}$	2.25	4	$\frac{11}{16}$	5.25	7.0
3.0	.448	$\frac{5}{8}$	2.5	4	$\frac{3}{4}$	6.0	7.5
3.5	.466	$\frac{5}{8}$	2.5	4	$\frac{13}{16}$	6.5	8.5
4.0	.486	$\frac{3}{4}$	2.75	4	$\frac{15}{16}$	7.25	9.0
4.5	.498	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	7.75	9.25
5	.525	$\frac{3}{4}$	3.0	8	$\frac{15}{16}$	8.5	10.0
6	.563	$\frac{3}{4}$	3.0	8	1	9.625	11.0
7	.600	$\frac{3}{4}$	3.25	8	$1\frac{1}{16}$	10.75	12.5
8	.639	$\frac{3}{4}$	3.5	8	$1\frac{1}{8}$	11.75	13.5
9	.678	$\frac{3}{4}$	3.5	12	$1\frac{1}{8}$	13.0	15.0
10	.713	$\frac{7}{8}$	3.625	12	$1\frac{3}{16}$	14.25	16.0
12	.790	$\frac{7}{8}$	3.75	12	$1\frac{1}{4}$	16.5	19.0
14	.864	1	4.25	12	$1\frac{3}{8}$	18.75	21.0
15	.904	1	4.25	16	$1\frac{3}{8}$	20.0	22.25
16	.946	1	4.25	16	$1\frac{7}{16}$	21.25	23.5
18	1.020	$1\frac{1}{8}$	4.75	16	$1\frac{9}{16}$	22.75	25.0
20	1.090	$1\frac{1}{8}$	5.0	20	$1\frac{11}{16}$	25.0	27.5
22	1.180	$1\frac{1}{4}$	5.5	20	$1\frac{13}{16}$	27.25	29.5
24	1.250	$1\frac{1}{4}$	5.5	20	$1\frac{7}{8}$	29.5	32.0
26	1.300	$1\frac{1}{4}$	5.75	24	2	31.75	34.25
28	1.380	$1\frac{1}{4}$	6.0	28	$2\frac{1}{16}$	34.0	36.5
30	1.480	$1\frac{3}{8}$	6.25	28	$2\frac{1}{8}$	36.0	38.75
36	1.710	$1\frac{3}{8}$	6.5	32	$2\frac{3}{8}$	42.75	45.75
42	1.870	$1\frac{1}{2}$	7.25	36	$2\frac{5}{8}$	49.5	52.75
48	2.170	$1\frac{1}{2}$	7.75	44	$2\frac{3}{4}$	56.0	59.5

LAP-WELDED AMERICAN CHARCOAL IRON BOILER TUBES.

STANDARD DIMENSIONS.

External Diameter.	Standard Thickness.	Wire Gauge.	Internal Diameter.	Internal Circumference.	External Circumference.	Length of Tube per sq. ft. Inside Surface.	Length of Tube per sq. ft. Out. Surface.	Internal Area.	External Area.	Weight per foot.
In.	In.		In.	In.	In.	Ft.	Ft.	Sq. In.	Sq. In.	Lb.
1	.072	15	.856	2.689	3.142	4.463	3.819	.575	.785	.708
$1\frac{1}{2}$	.083	14	1.334	4.191	4.712	2.863	2.547	1.396	1.767	1.250
2	.098	13	1.804	5.667	6.283	2.118	1.909	2.556	3.142	1.981
$2\frac{1}{2}$	.109	12	2.283	7.172	7.854	1.673	1.528	4.094	4.909	2.755
3	.109	12	2.783	8.743	9.425	1.373	1.273	6.083	7.069	3.333
$3\frac{1}{2}$	.119	11	3.262	10.248	10.996	1.171	1.091	8.357	9.621	4.272
4	.130	10	3.741	11.753	12.566	1.021	.955	10.992	12.566	5.320
$4\frac{1}{2}$	.130	10	4.241	13.323	14.137	.901	.849	14.126	15.904	6.010
5	.140	9.5	4.720	14.828	15.708	.809	.764	17.497	19.635	7.276
6	.151	9	5.699	17.904	18.850	.670	.637	25.509	28.274	9.346

PROPORTIONS OF RIVETED JOINTS.

Thickness of Plate. t	Diameter of Rivet.	Diameter of Hole. d	Pitch.		Efficiency of Joint.	
			Single.	Double.	Single.	Double.
$\frac{1}{4}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$2''$	$3''$	.66	.77
$\frac{5}{16}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$2\frac{1}{16}''$	$3\frac{1}{8}''$	.64	.76
$\frac{3}{8}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$2\frac{1}{8}''$	$3\frac{1}{4}''$	.62	.75
$\frac{7}{16}''$	$\frac{13}{16}''$	$\frac{7}{8}''$	$2\frac{3}{16}''$	$3\frac{3}{8}''$	.60	.74
$\frac{1}{2}''$	$\frac{7}{8}''$	$\frac{15}{16}''$	$2\frac{1}{4}''$	$3\frac{1}{2}''$	.58	.73

PROPORTIONS OF GEAR TEETH.

C = Circular Pitch.

	Rough Cast Gears.	Cut Gears.
Addendum	$0.30\,C$	$0.30\,C$
Root	$0.40\,C$	$0.35\,C$
Whole Depth	$0.70\,C$	$0.65\,C$
Thickness of Tooth	$0.48\,C$	$0.495\,C$
Width of Space	$0.52\,C$	$0.505\,C$

RELATION BETWEEN DIAMETRAL AND CIRCULAR PITCH.

Diametral Pitch.	Circular Pitch.	Circular Pitch.	Diametral Pitch.
2	1.571 inches.	2 inches	1.571
$2\frac{1}{4}$	1.396 "	$1\frac{7}{8}$ "	1.676
$2\frac{1}{2}$	1.257 "	$1\frac{3}{4}$ "	1.795
$2\frac{3}{4}$	1.142 "	$1\frac{5}{8}$ "	1.933
3	1.047 "	$1\frac{1}{2}$ "	2.094
$3\frac{1}{2}$	.898 "	$1\frac{7}{16}$ "	2.185
4	.785 "	$1\frac{3}{8}$ "	2.285
5	.628 "	$1\frac{5}{16}$ "	2.394
6	.524 "	$1\frac{1}{4}$ "	2.513
7	.449 "	$1\frac{3}{16}$ "	2.646
8	.393 "	$1\frac{1}{8}$ "	2.793
9	.349 "	$1\frac{1}{16}$ "	2.957
10	.314 "	1	3.142
11	.286 "	$\frac{15}{16}$ "	3.351
12	.262 "	$\frac{7}{8}$ "	3.590
14	.224 "	$\frac{13}{16}$ "	3.867
16	.196 "	$\frac{3}{4}$ "	4.189
18	.175 "	$\frac{11}{16}$ "	4.570
20	.157 "	$\frac{5}{8}$ "	5.027

BREAKING STRENGTH OF AND POWER TRANSMITTED BY WIRE ROPES (42 WIRES).

Diameter of Ropes, Inches.	Diameter of Pulleys, Feet.	Revolutions per Minute.	Breaking Stress of Rope per Pound.	Horse-power Transmitted.	Velocity of Rope in Feet per Second.
$\frac{7}{16}$	5	100	4,260	8.6	26
$\frac{15}{32}$	6	100	5,660	13.4	31
$\frac{1}{2}$	7	100	8,200	21.1	36
$\frac{5}{8}$	8	100	11,600	27.5	42
$\frac{5}{8}$	8	120	11,600	33.0	50
$\frac{5}{8}$	9	100	11,600	51.9	47
$\frac{5}{8}$	9	120	11,600	62.2	56
$\frac{11}{16}$	10	100	15,200	73.0	52
$\frac{11}{16}$	10	120	15,200	87.6	62
$\frac{11}{16}$	10	140	15,200	102.2	73
$\frac{11}{16}$	12	100	15,200	116.7	63
$\frac{3}{4}$	12	120	17,600	148.9	75
$\frac{3}{4}$	12	140	17,600	173.7	87
$\frac{3}{4}$	14	100	17,600	185.0	73
$\frac{3}{4}$	14	120	17,600	222.0	87
$\frac{3}{4}$	15	120	17,600	300.0	94

SAFE PRESSURE ON BEARINGS AND SLIDES

Kind of Journal Bearing.	Pressure per Sq. In. of Projected Area, p.
Bearings on which the load is intermittent and the speed slow, such as crank-pins of shearing machines	3,000 lb.
Cross-head neck journals	1,200 lb.
Crank-pins of large, slow-speed engines	800 to 900 lb.
Crank-pins of marine engines, usually	400 to 500 lb.
Main crank-shaft bearings—marine engines (slow)	600 lb.

Main crank-shaft bearings—marine engines (fast)		400 lb.
Locomotive driving-axle journal	180 to	350 lb
Railway journals		200 lb.
Fly-wheel shaft journals	150 to	250 lb.
Small engine crank-pins	150 to	200 lb.
Slipper slide blocks, marine engines		100 lb.
Stationary engine slide blocks	25 to	125 lb.
Stationary engine slide blocks, usually	30 to	60 lb.
Propeller thrust bearings	50 to	70 lb.
Shaft in cast-iron steps or seats		15 lb.

RULES AND FORMULAS.

FORMULAS USED IN ALGEBRA.

Let a and b be any two quantities, then,

$(a+b)^2 = a^2 + 2ab + b^2.$ **(1.)** Art. **432.**

$(a-b)^2 = a^2 - 2ab + b^2.$ **(2.)** Art. **432.**

$(a+b)(a-b) = a^2 - b^2.$ **(3.)** Art. **432.**

$a^2+2ab+b^2=(a+b)(a+b)=(a+b)^2.$ **(4.)** Art. **455.**

$a^2-2ab+b^2=(a-b)(a-b)=(a-b)^2.$ **(5.)** Art. **455.**

$a^2 - b^2 = (a+b)(a-b).$ **(6.)** Art. **462.**

Let $ax^2 + bx = c$ be any quadratic equation (it must be borne in mind that b and c may be positive or negative); then,

$$x = -\frac{b}{2a} \pm \sqrt{\left(\frac{b}{2a}\right)^2 + \frac{c}{a}} = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}.$$ Art. **597.**

THE TRIGONOMETRIC FUNCTIONS.

Art. **754.**

Rule 1.—*Sine* $= \dfrac{\textit{side opposite}}{\textit{hypotenuse}}.$

Rule 2.—*Side opposite = hypotenuse × sine.*

Rule 3.—*Cosine* $= \dfrac{\textit{side adjacent}}{\textit{hypotenuse}}.$

Rule 4.—*Side adjacent = hypotenuse × cosine.*

Rule 5.—*Tangent* $= \dfrac{\textit{side opposite}}{\textit{side adjacent}}$.

Rule 6.—*Side opposite = side adjacent* × *tangent.*

Rule 7.—*Cotangent* $= \dfrac{\textit{side adjacent}}{\textit{side opposite}}$.

Rule 8.—*Side adjacent = side opposite* × *cotangent.*

Rule 9.—*Hypotenuse* $= \dfrac{\textit{side opposite}}{\textit{sine}}$.

Rule 10.—*Hypotenuse* $= \dfrac{\textit{side adjacent}}{\textit{cosine}}$.

RULES FOR USING TABLE OF LOGARITHMS OF NUMBERS.

Arts. **625–636.**

I. **To find the Characteristic.**—*For a number greater than 1 the characteristic is one less than the number of integral places in the number. For a number wholly decimal the characteristic is negative, and is numerically one greater than the number of ciphers between the decimal point and the first digit of the decimal.*

II. **To find the Logarithm of a Number not having more than four figures.**—*Find the first three significant figures of the number whose logarithm is desired in the left-hand column; find the fourth figure in the column at the top (or bottom) of the page, and in the column under (or above) this figure, and opposite the first three figures previously found, will be the mantissa, or decimal part, of the logarithm. The characteristic being found as described above, write it at the left of the mantissa, and the resulting expression will be the logarithm of the required number.*

III. **To find the Logarithm of a Number consisting of five or more figures.**

(**a**) *If the number consists of more than five figures, and the sixth figure is 5 or greater, increase the fifth figure by 1, and write ciphers in place of the sixth and remaining figures.*

(**b**) *Find the mantissa corresponding to the logarithm of the first four figures, and subtract this mantissa from the next greater mantissa in the table; the remainder is the difference.*

(**c**) *Find in the secondary table headed P. P. a column headed by the same number as that just found for the difference, and in this column opposite the number corresponding to the fifth figure (or fifth figure increased by 1) of the given number (this figure is always situated at the left of the dividing line of the column) will be found the P. P. (proportional part) for that number. The P. P. thus found is to be added to the mantissa found in* (**b**), *and the result is the mantissa of the logarithm of the given number, as nearly as may be found with five-place tables.*

IV. **To find a Number whose Logarithm is given.**—

(**a**) *Consider the mantissa first. Glance along the different columns of the table which are headed O until the first two figures of the mantissa are found. Then glance down the same column until the third figure is found (or 1 less than the third figure). Having found the first three figures, glance to the right along the row in which they are situated until the last three figures of the mantissa are found. Then the number which heads the column in which the last three figures of the mantissa are found is the fourth figure of the required number, and the first three figures lie in the column headed N, and in the same row in which lie the last three figures of the mantissa.*

(**b**) *If the mantissa cannot be found in the table, find the mantissa which is nearest to, but less than, the given mantissa, and which call the* **next less mantissa.** *Subtract the next less mantissa from the next greater mantissa in the table to obtain the difference. Also subtract the next less mantissa from the mantissa of the given logarithm, and call the remainder the P. P. Looking in the secondary table headed P. P. for the column headed by the difference just found, find the number opposite the P. P. just found (or the P. P. corresponding most* *nearly to that just found*) ; *this number is the fifth figure of the required number ; the fourth figure will be found at the top of the column containing the next less mantissa, and the first three figures in the column headed N, and in the same row which contains the next less mantissa.*

(*c*) *Having found the figures of the number as above directed, locate the decimal point by the rules for the characteristic, annexing ciphers to bring the number up to the required number of figures if the characteristic is greater than 4.*

RULES FOR USING TRIGONOMETRIC TABLES.

Given, an angle, to find its sine, cosine, tangent. and cotangent.

Rule 11.—*Find in the table the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes of the angle.*

For the seconds, find the difference of the values of the sine, cosine, tangent, or cotangent taken from the table, between which the seconds of the angle fall ; multiply this difference by a fraction whose numerator is the number of seconds in the given angle, and whose denominator is 60.

If sine or tangent, add this correction to the value first found; if cosine or cotangent, subtract the correction. Art. **756.**

Given, the sine, cosine, tangent, or cotangent to find the angle corresponding.

To find the angle corresponding to a given sine, cosine, tangent, or cotangent whose exact value is not contained in the table:

Rule 12.—*Find the difference of the two numbers in the table between which the given sine, cosine, tangent, or cotangent falls, and use the number of parts in this difference as the denominator of a fraction.*

Find the difference between the number belonging to the **smaller angle,** *and the given sine, cosine, tangent, or cotangent, and use the number of parts in the difference just found as the numerator of the fraction mentioned above. Multiply this fraction by 60, and the result will be the number of seconds to be added to the* **smaller angle.** Art. **758.**

RULES FOR MENSURATION.

THE TRIANGLE.

Rule.—*The area of any triangle equals one-half the product of the base and the altitude.* Art. **766.**

THE QUADRILATERAL.

To find the area of a parallelogram:

Rule.—*The area of any parallelogram equals the product of the base and the altitude.* Art. **777.**

To find the area of a trapezoid:

Rule.—*The area of a trapezoid equals one-half the sum of the parallel sides multiplied by the altitude.* Art. **778.**

To find the area of an irregular figure bounded by straight lines:

Rule.—*Divide the figure into triangles, and find the area of each triangle separately. The sum of the areas of all the triangles will be the area of the figure.* Art. **779.**

THE CIRCLE.

To find the circumference or diameter of a circle:

Rule.—*The circumference of a circle equals the diameter multiplied by 3.1416.* Art. **780.**

Rule.—*The diameter of a circle equals the circumference divided by 3.1416.* Art. **780.**

To find the length of an arc of a circle:

Rule.—*The length of an arc of a circle equals the circumference of the circle of which the arc is a part multiplied by the number of degrees in the arc, and divided by 360.* Art. **781.**

To find the area of a circle:

Rule.—*Square the diameter, and multiply by .7854.* Art. **782.**

Given, the area of a circle to find its diameter:

Rule.—*Divide the area by .7854, and extract the square root of the quotient.* Art. **783.**

To find the area of a sector:

Rule.—*Divide the number of degrees in the arc of a sector by 360. Multiply the result by the area of the circle of which the sector is a part.* Art. **784.**

To find the area of a segment of a circle:

Rule.—*Draw radii from the center of the circle to the extremities of the arc of the segment; find the area of the sector thus formed, subtract from this the area of the triangle formed by the radii and the chord of the arc of the segment, and the result is the area of the segment.* Art. **785.**

THE ELLIPSE.

To find the perimeter of an ellipse:

Rule.—*Multiply the major axis by 1.82, and the minor axis by 1.315. The sum of the results will be the perimeter.* Art. **788.**

To find the exact area of an ellipse:

Rule.—*The area of an ellipse is equal to the product of its two diameters multiplied by .7854.* Art. **789.**

To find the area of any plane figure:

Rule.—*The area of any plane figure may be found by dividing it into triangles, quadrilaterals, circles or parts of circles, and ellipses, finding the area of each part separately, and adding them together.* Art. **790.**

THE PRISM AND CYLINDER.

To find the area of the convex surface of any right prism or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.* Art. **803.**

To find the volume of a right prism or cylinder:

Rule.—*The volume of any right prism or cylinder equals the area of the base multiplied by the altitude.* Art. **804.**

THE PYRAMID AND CONE.

To find the area of a right pyramid or right cone:

Rule.—*The convex area of a right pyramid or cone equals the perimeter of the base multiplied by one-half the slant height.* Art. **809.**

To find the volume of a right pyramid or cone:

Rule.—*The volume of a right pyramid or cone equals the area of the base multiplied by one-third of the altitude.* Art. **810.**

THE FRUSTUM OF A PYRAMID OR CONE.

To find the convex area of a frustum of a right pyramid or right cone:

Rule.—*The convex area of a frustum of a right pyramid or right cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.* Art. **814.**

To find the volume of the frustum of a pyramid or cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.* Art. **815.**

THE SPHERE.

To find the area of the surface of a sphere:

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by 3.1416.* Art. **817.**

To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by .5236.* Art. **818.**

FORMULAS USED IN ELEMENTARY MECHANICS.

UNIFORM MOTION.

Let $S =$ the length of space passed over uniformly;

$t =$ the time occupied in passing over the space S;

$V =$ the velocity.

$V = \frac{S}{t}$. (7.) Art. **859.**

$S = Vt$. (8.) Art. **859.**

$t = \frac{S}{V}$. (9.) Art. **859.**

MASS, WEIGHT, AND GRAVITY.

If the mass of the body be represented by m, its weight by W, and the force of gravity at the place where the body was weighed by g, we have

$\text{mass} = \frac{\text{weight of body}}{\text{force of gravity}}$, or $m = \frac{W}{g}$. (10.) Art. **888.**

FORMULAS FOR GRAVITY PROBLEMS.

Let W = weight of body at the surface;
w = weight of a body at a given distance above or below the surface;
d = distance between the center of the earth and the center of the body;
R = radius of the earth = 4,000 miles.

Formula for weight when the body is below the surface:

$w R = d W$. (11.) Art. **891.**

Formula for weight when the body is above the surface:

$w d^2 = W R^2$. (12.) Art. **891.**

FALLING BODIES.

Let g = force of gravity = constant accelerating force due to the attraction of the earth;
t = number of seconds the body falls;
v = velocity at the end of the time t;
h = distance that a body falls during the time t.

$v = g t$. (13.) Art. **896.**

That is, the velocity acquired by a freely falling body at the end of t seconds equals 32.16 multiplied by the time in seconds.

$t = \frac{v}{g}$. (14.) Art. **896.**

That is, the number of seconds during which a body must have fallen to acquire a given velocity equals the given velocity in feet per second divided by 32.16.

$$h = \frac{v^2}{2g}. \qquad (15.) \text{ Art. } \mathbf{896.}$$

That is, the height from which a body must fall to acquire a given velocity equals the square of the given velocity divided by 2 × 32.16.

$$v = \sqrt{2gh}. \qquad (16.) \text{ Art. } \mathbf{896.}$$

That is, the velocity that a body will acquire in falling through a given height equals the square root of the product of twice 32.16 and the given height.

$$h = \tfrac{1}{2} g t^2. \qquad (17.) \text{ Art. } \mathbf{896.}$$

That is, the distance a body will fall in a given time equals 32.16 ÷ 2 multiplied by the square of the number of seconds.

$$t = \sqrt{\frac{2h}{g}}. \qquad (18.) \text{ Art. } \mathbf{896.}$$

That is, the time it will take a body to fall through a given height equals the square root of twice the height divided by 32.16.

CENTRIFUGAL FORCE.

The value of the centrifugal force of any revolving body, expressed in pounds, is

$$F = .00034\, W R N^2; \qquad (19.) \text{ Art. } \mathbf{903.}$$

in which F = centrifugal force;
W = total weight of body in pounds;
R = radius, usually taken as the distance between the center of motion and the center of gravity of the revolving body, in feet;
N = number of revolutions per *minute*.

THE CENTER OF GRAVITY OF TWO BODIES.

Let l = the distance between the centers of the bodies;
l_1 = the short arm;
w = weight of small body;
W = weight of large body.

$$l_1 = \frac{w\,l}{W + w}. \qquad \textbf{(20.)} \quad \text{Art. } \mathbf{911.}$$

THE EFFICIENCY OF A MACHINE.

Let F = the force applied to the machine;
V = the velocity ratio of the machine;
W = the weight actually lifted or equivalent resistance overcome;
E = the efficiency of the machine;

$$\text{Then, } \mathsf{E} = \frac{W}{F\,V}. \qquad \textbf{(22.)} \quad \text{Art. } \mathbf{950.}$$

WORK.

If the force necessary to overcome the resistance be represented by F, the space through which the resistance acts by S, and the work done by U, then $U = F\,S$.

If W = the weight of a body, and h = the height through which it is raised, $U = W\,h$. Hence the work done

$$U = F\,S = W\,h. \qquad \textbf{(23.)} \quad \text{Art. } \mathbf{953.}$$

POWER.

The power of a machine may always be determined by *dividing the work done in foot-pounds by the time in minutes required to do the work; i. e.*,

$$\text{Power} = \frac{F\,S}{T}. \qquad \textbf{(24.)} \quad \text{Art. } \mathbf{954.}$$

KINETIC ENERGY.

Let W = the weight of the body in pounds;
v = its velocity in feet per second;
h = the height in feet through which the body must fall to produce the velocity v;
m = the mass of the body = $\frac{W}{g}$. (See formula **10.**)

The work necessary to raise a body through a height h is Wh. The velocity produced in falling a height h is $v = \sqrt{2gh}$, and $h = \frac{v^2}{2g}$. (See formulas **15** and **16.**)

Therefore, work $= Wh = W\frac{v^2}{2g} = \frac{1}{2} \times \frac{W}{g} \times v^2 = \frac{1}{2} m v^2$, or

$$Wh = \frac{1}{2} m v^2. \qquad \text{(25.) Art. 957.}$$

DENSITY.

The **density of a body** is its mass divided by its volume in cubic feet.

Let D be the density; then the density of a body is

$$D = \frac{m}{V}. \quad \text{Since } m = \frac{W}{g}, \; D = \frac{W}{gV}. \qquad \text{(26.) Art. 962.}$$

RULES AND FORMULAS USED IN HYDROMECHANICS.

PASCAL'S LAW.

Rule I.—*The pressure per unit of area exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all surfaces in a direction at* **right angles** *to those surfaces.*

THE GENERAL LAW FOR THE DOWNWARD PRESSURE UPON THE BOTTOM OF ANY VESSEL.

Rule II.—*The pressure upon the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a prism of the fluid whose base has the same area as the bottom of the vessel, and whose altitude is the distance between the bottom and the upper surface of the fluid plus the pressure per unit of area upon the upper surface of the fluid multiplied by the area of the bottom of the vessel.*

GENERAL LAW FOR UPWARD PRESSURE.

Rule III.—*The upward pressure on any submerged horizontal surface equals the weight of a prism of the liquid* *whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid plus the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface.*

GENERAL LAW FOR LATERAL PRESSURE.

Rule IV.—*The pressure upon any vertical surface due to the weight of a liquid is equal to the weight of a prism of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the level of the liquid.*

Any additional pressure is to be added as in the previous cases.

GENERAL LAW FOR PRESSURE.

Rule V.—*The pressure exerted by a fluid in any direction upon* **any** *surface is equal to the weight of a prism of the fluid whose base is the projection of the surface at right angles to the direction considered, and whose height is the depth of the center of gravity of the surface below the level of the liquid.*

SPECIFIC GRAVITY.

Let W be the weight of the solid in air and W' the weight in water; then, $W - W'$ = the weight of a volume of water equal to the volume of the solid, and

$$\text{Sp. Gr.} = \frac{W}{W - W'}. \qquad \textbf{(27.) Art. 991.}$$

If the body be lighter than water, a piece of iron or other heavy substance must be attached to it sufficiently heavy to sink both. *Then weigh both bodies in air and both in water.*

Let W = weight of both bodies in air;
W' = weight of both bodies in water;
w = weight of light body in air;
W_1 = weight of heavy body in air;
W_2 = weight of heavy body in water

Then, the specific gravity of the light body is given by

$$\text{Sp. Gr.} = \frac{w}{(W - W') - (W_1 - W_2)}. \quad \text{(28.) Art. } \mathbf{992.}$$

To find the specific gravity of a liquid:

Weigh an empty flask; fill it with water, then weigh it, and find the difference between the two results; this will equal the weight of the water. Then weigh the flask filled with the liquid and subtract the weight of the flask; the result is the weight of a volume of the liquid equal to the volume of the water. The weight of the liquid divided by the weight of the water is the specific gravity of the liquid.

Let W = the weight of the flask and liquid;
W' = the weight of the flask and water;
w = the weight of the flask.

Then,
$$\text{Sp. Gr.} = \frac{W - w}{W' - w}. \quad \text{(29.) Art. } \mathbf{993.}$$

FORMULAS FOR FLOW OF WATER.

MEAN VELOCITY.

Let Q = the quantity in cubic feet which passes any section in 1 second;
A = the area of the section in square feet;
v_m = the mean velocity in feet per second.

Then,
$$Q = A\,v_m, \quad \text{(31.) Art. } \mathbf{1003.}$$

and
$$v_m = \frac{Q}{A}. \quad \text{(32.) Art. } \mathbf{1003.}$$

VELOCITY OF EFFLUX FROM AN ORIFICE.

Let v = the velocity of efflux in feet per second;
h = the head in feet on the orifice considered;
h_1 = the head equivalent to a pressure p;
W = the weight of the water in pounds flowing through the aperture per second.

The kinetic energy of the issuing water $= \frac{W v^2}{2g}$.

The work the issuing water can do $= Wh$.

$$Wh = \frac{Wv^2}{2g}, \text{ or } v = \sqrt{2gh}.$$

$h = \frac{p}{.434}$, where h_1 is in feet, and p in pounds per square inch.

$h_1 = \frac{p}{62.5}$, where h_1 is in feet, and p in pounds per square foot.

$h + h_1 =$ the *total head.*

$$v = \sqrt{2g(h_1 + h)}. \qquad \textbf{(33.)} \text{ Art. } \textbf{1007.}$$

If a is the area of a large orifice in the bottom of a small vessel whose area is A, the velocity is

$$v\sqrt{\frac{2gh}{1 - \frac{a^2}{A^2}}}. \qquad \textbf{(35.)} \text{ Art. } \textbf{1010.}$$

Actual velocity of efflux from a small square-edged orifice:

$$v = .98\sqrt{2gh}. \qquad \textbf{(36.)} \text{ Art. } \textbf{1013.}$$

Actual quantity discharged from a small square-edged orifice:

$$Q = .615Av. \qquad \textbf{(37.)} \text{ Art. } \textbf{1015.}$$

THEORETICAL AND ACTUAL DISCHARGE.

Let Q = theoretical number of cubic feet discharged per second;

v_m = theoretical mean velocity through orifice in feet per second $= Q \div A$;

A = area of orifice in square feet;

h = theoretical head necessary to give a mean velocity v_m.

Q_a = actual quantity discharged in cubic feet per second.

Then, for an orifice in a thin plate, or a square-edged orifice (the hole itself may be of any shape, triangular, square, circular, etc., but the edges must not be rounded), the actual quantity discharged is:

$$Q_a = .615\,Q = .615\,A\,v_m = .615\,A\sqrt{2gh}.$$ **(38.)** Art. **1017.**

For a discharge through a short tube,

$$Q_a = .815\,Q = .815\,A\,v_m = .815\,A\sqrt{2gh}.$$ **(39.)** Art. **1017.**

For a discharge through a mouthpiece,

$$Q_a = .97\,Q = .97\,A\,v_m = .97\,A\sqrt{2gh}.$$ **(40.)** Art. **1017.**

For a discharge through the compound mouthpiece, the area of the orifice being taken as the area of the smallest section,

$$Q_a = 1.5526\,Q = 1.5526\,A\,v_m = 1.5526\,A\sqrt{2gh}.$$ **(41.)** Art. **1017.**

In these four formulas it is taken for granted that there is a constant head.

FLOW OF WATER THROUGH WEIRS.

If $d =$ the depth of the opening in feet, and b its breadth in feet, the area of the opening is $A = d \times b$, and the theoretical discharge is $Q = d \times b \times v_m = db \times \frac{2}{3}\sqrt{2gh} = \frac{2}{3}\,bd\sqrt{2gd}$, the head for this case being taken as d.

The actual discharge is

$$Q_a = .615\,Q = .615 \times \frac{2}{3}\,bd\sqrt{2gd} = .41\,b\sqrt{2gd^3}.$$ **(42.)** Art. **1020.**

That is, *the actual discharge through a weir in cubic feet per second, whose top is on a level with the upper surface of the water, is equal to .41 multiplied by the breadth of the weir, multiplied by the square root of 2 g times the cube of the depth of the weir. All dimensions are to be taken in feet.*

To obtain the mean velocity v_m, divide the actual discharge by the area of the weir, or

$$v_m = \frac{Q_a}{A} = \frac{Q_a}{bd}.$$ **(43.)** Art. **1020.**

For a weir whose upper edge is below the level of the upper surface of the water, let h_1 be the depth in feet of the top of the weir below the surface of the water, and h the depth in feet of the bottom of the weir below the surface of the water. The actual discharge Q_a in cubic feet per second is

$$Q_a = .41\, b \sqrt{2g}\,(\sqrt{h^3} - \sqrt{h_1^3}). \qquad \textbf{(44.)}\ \text{Art. } \mathbf{1022.}$$

FLOW OF WATER IN PIPES.

THE MEAN VELOCITY OF DISCHARGE.

For straight cylindrical pipes of uniform diameter:

Let v_m = mean velocity of discharge in feet per second;
h = total head in feet = the vertical distance between the level of the water in the reservoir and the point of discharge;
l = length of pipe in feet;
d = diameter of pipe in inches;
f = coefficient of friction.

Then,
$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l + \frac{1}{8}d}}. \qquad \textbf{(45.)}\ \text{Art. } \mathbf{1024.}$$

When the pipe is very long, compared with the diameter, the following formula may be used:

$$v_m \doteq 2.315\sqrt{\frac{h\,d}{f\,l}}. \qquad \textbf{(46.)}\ \text{Art. } \mathbf{1027.}$$

THE ACTUAL HEAD.

The actual head necessary to produce a certain velocity v_m may be calculated by the formula

$$h = \frac{f\,l\,v_m^2}{5.36\,d} + .0233\,v_m^2. \qquad \textbf{(47.)}\ \text{Art. } \mathbf{1029.}$$

THE QUANTITY DISCHARGED FROM PIPES.

Let d = the diameter of the pipe in inches, then the discharge Q in gallons per second is

$$Q \doteq .0408\,d^2\,v_m. \qquad \textbf{(48.)}\ \text{Art. } \mathbf{1030.}$$

If the diameter of the pipe and the discharge are known, the mean velocity v_m is

$$v_m = \frac{24.51\,Q}{d^2}. \qquad \textbf{(49.)} \text{ Art. } \textbf{1031.}$$

If the head, the length of the pipe, and the diameter of the pipe are given, to find the discharge use the formula

$$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + \frac{1}{8}d}}. \qquad \textbf{(50.)} \text{ Art. } \textbf{1032.}$$

To find the value of f, calculate v_m by formula **46,** assuming that $f = .025$, and get the final value of f from the following table. Art. **1033.**

$v_m =$	0.1	0.2	0.3	0.4	0.5	0.6
$f =$	.0686	.0527	.0457	.0415	.0387	.0365
$v_m =$	0.7	0.8	0.9	1	1¼	1½
$f =$	.0349	.0336	.0325	.0315	.0297	.0284
$v_m =$	2	3	4	6	8	12
$f =$	.0265	.0243	.0230	.0214	.0205	.0193

Note.—The values given in the above table are calculated by the formula $f = .014393 + \frac{.017156}{\sqrt{v_m}}$.

TO CALCULATE THE DIAMETER OF A PIPE.

Neglecting the fraction $\frac{1}{8}\,d$ in formula **50,** and solving for d,

$$d = 2.57\sqrt[5]{\frac{f\,l\,Q^2}{h}}.$$

Assuming that $f = .025$, for a trial value, the above equation then becomes

$$d = 1.229\sqrt[5]{\frac{l\,Q^2}{h}}. \qquad \textbf{(51.)} \text{ Art. } \textbf{1035.}$$

Formula **50** may also be written

$$d = 2.57 \sqrt[5]{\frac{(f\,l + \frac{1}{8}d)\,Q^2}{h}}. \quad \textbf{(52.)} \text{ Art. } \mathbf{1035.}$$

By aid of formulas **51** and **52**, the diameter of a pipe may be approximated as follows:

Rule.—*Find the value of d by formula **51**; substitute this value in formula **49**, and find the value of* v_m. *Then find from the table the value of f corresponding to this value of* v_m. *Substitute the values of d and f just found in the right-hand member of formula **52**, and solve for d; the result will be the diameter of the pipe, nearly enough for all practical purposes.*

FORMULAS USED IN PNEUMATICS.

PRESSURE, VOLUME, DENSITY, AND WEIGHT OF AIR WHEN THE TEMPERATURE IS CONSTANT:

Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

Let p = pressure for one position of the piston;
p_1 = pressure for any other position of the piston;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

$$\text{Then, } p\,v = p_1\,v_1. \quad \textbf{(53.)} \text{ Art. } \mathbf{1049.}$$

Let D be the density corresponding to the pressure p and volume v, and D_1 be the density corresponding to the pressure p_1 and volume v_1; then,

$$p : D = p_1 : D_1, \text{ or } p\,D_1 = p_1\,D, \quad \textbf{(54.)} \text{ Art. } \mathbf{1052.}$$

$$\text{and } v : D_1 = v_1 : D, \text{ or } v\,D = v_1\,D_1. \quad \textbf{(55.)} \text{ Art. } \mathbf{1052.}$$

Thus, let W be the weight of a cubic foot of air or other gas, whose volume is v, and pressure is p; let W_1 be the weight of a cubic foot when the volume is v_1, and pressure is p_1; then,

$$p\,W_1 = p_1\,W. \quad \textbf{(56.)} \text{ Art. } \mathbf{1052.}$$

$$v\,W = v_1\,W_1. \quad \textbf{(57.)} \text{ Art. } \mathbf{1052.}$$

PRESSURE AND VOLUME OF A GAS WITH VARIABLE TEMPERATURE:

Gay-Lussac's Law.—*If the pressure remains constant, every increase of temperature of 1° F. produces in a given quantity of gas an expansion of $\frac{1}{492}$ of its volume at 32° F.*

If the pressure remains constant it will also be found that every decrease of temperature of 1° F. will cause a decrease of $\frac{1}{492}$ of the volume at 32° F.

Let v = original volume of gas;
v_1 = final volume of gas;
t = temperature corresponding to volume v;
t_1 = temperature corresponding to volume v_1.

Then, $$v_1 = v\left(\frac{460 + t_1}{460 + t}\right). \quad \textbf{(58.)} \text{ Art. } \textbf{1054.}$$

That is, *the volume of gas after heating (or cooling) equals the original volume multiplied by 460 plus the final temperature divided by 460 plus the original temperature.*

Let p = the original tension;
t = the corresponding temperature;
p_1 = final tension;
t_1 = final temperature.

Then, $$p_1 = p\left(\frac{460 + t_1}{460 + t}\right). \quad \textbf{(59.)} \text{ Art. } \textbf{1055.}$$

Let p = pressure in pounds per square inch;
V = volume of air in cubic feet;
T = absolute temperature;
W = weight in pounds.

Then, $$p\,V = .37052\,T. \quad \textbf{(60.)} \text{ Art. } \textbf{1056.}$$

If the weight of the air be greater or less than 1 pound, the following formula must be used:

$$p\,V = .37052\,W\,T. \quad \textbf{(61.)} \text{ Art. } \textbf{1057.}$$

Let p_1, V_1, and T_1 represent the pressure, volume, and temperature of the same weight of air in another state; then,

$$\frac{p\,V}{T} = \frac{p_1\,V_1}{T_1}. \quad \textbf{(62.)} \text{ Art. } \textbf{1058.}$$

MIXTURE OF TWO GASES HAVING UNEQUAL VOLUMES AND PRESSURES.

Let v and p be the volume and pressure, respectively, of one of the gases.

Let v_1 and p_1 be the volume and pressure, respectively, of the other gas.

Let V and P be the volume and pressure, respectively, of the mixture. Then, if the temperature remains the same,

$$VP = vp + v_1 p_1. \quad \textbf{(63.)} \quad \text{Art. } \mathbf{1062}.$$

MIXTURE OF TWO VOLUMES OF AIR HAVING UNEQUAL PRESSURES, VOLUMES, AND TEMPERATURES.

If a body of air having a temperature t_1, a pressure p_1, and a volume v_1, be mixed with another volume of air having a temperature t_2, a pressure p_2, and a volume v_2, to form a volume V having a pressure P, and a temperature t, then, either the new temperature t, the new volume V, or the new pressure P may be found, if the other two quantities are known, by the following formula, in which T_1, T_2, and T are the absolute temperatures corresponding to t_1, t_2, and t:

$$PV = \left[\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right] T. \quad \textbf{(64.)} \quad \text{Art. } \mathbf{1063}.$$

FORMULAS USED IN HEAT.

TO CHANGE FAHRENHEIT TEMPERATURES TO CENTIGRADE, AND CENTIGRADE TO FAHRENHEIT.

Let t_c = temperature Centigrade, and t_f = temperature Fahrenheit. Then,

$$t_f = \tfrac{9}{5}\, t_c + 32°, \quad \textbf{(65.)} \quad \text{Art. } \mathbf{1104}.$$

$$\text{and } t_c = (t_f - 32°)\, \tfrac{5}{9}. \quad \textbf{(66.)} \quad \text{Art. } \mathbf{1104}.$$

EXPANSION OR CONTRACTION OF BODIES FROM CHANGES OF TEMPERATURE.

Let L = length of any body;

l = amount of expansion or contraction due to heating or cooling the body;

A = area of any section of the body;

a = increase or decrease of area of the same section after heating or cooling the body;
V = volume of the body;
v = increase or decrease in volume due to heating or cooling the body;
C_1 = coefficient of expansion taken from column 1, Table 1;
C_2 = coefficient taken from column 2, Table 1;
C_3 = coefficient taken from column 3, Table 1;
t = difference in degrees of temperature between the original temperature and the temperature of the body after it has been heated or cooled.

Then, $l = L\,C_1\,t$; **(67.)** Art. **1113.**

$a = A\,C_2\,t$; **(68.)** Art. **1113.**

$v = V\,C_3\,t$. **(69.)** Art. **1113.**

RESULTING TEMPERATURES AND SPECIFIC HEATS OF THE MIXTURE OF SEVERAL BODIES HAVING UNEQUAL TEMPERATURES.

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3 + \text{etc.}}{W_1 s_1 + W_2 s_2 + W_3 s_3 + \text{etc.}}, \quad \left\{ \begin{array}{c} \textbf{(73.)} \\ \text{Art. } \textbf{1136.} \end{array} \right.$$

$$\text{or } s_3 = \frac{W_1 s_1 (t_1 - t) + W_2 s_2 (t_2 - t) + \text{etc.}}{W_3 (t - t_3)}, \quad \left\{ \begin{array}{c} \textbf{(74.)} \\ \text{Art. } \textbf{1137.} \end{array} \right.$$

in which t is the final temperature of the mixture; W_1, s_1, and t_1, the weight, specific heat, and temperature, respectively, of one body; W_2, s_2, and t_2 the same for second body; and W_3, s_3, and t_3 the same for a third body, etc.

WORK DONE BY THE ISOTHERMAL EXPANSION OF A GAS.

L = the work in foot-pounds;
P = the total initial pressure in pounds per square foot;
P_1 = the total final pressure in pounds per square foot;
V = the initial volume in cubic feet;
V_1 = the final volume in cubic feet.

$$L = 2.3026\,P\,V \log \frac{V_1}{V}. \qquad \textbf{(75.)} \text{ Art. } \textbf{1160.}$$

Since $PV = P_1 V_1$, $\frac{P}{P_1} = \frac{V_1}{V}$, and formula **75** might be written

$$L = 2.3026\,PV \log \frac{P}{P_1}. \qquad \textbf{(76.)} \text{ Art. } \textbf{1160.}$$

Whichever formula is used it must be kept in mind that the fraction $\frac{V_1}{V}$ or $\frac{P}{P_1}$ must *always be greater than 1; that is, the numerator must always be greater than the denominator.*

WORK REQUIRED TO COMPRESS A GAS ISOTHERMALLY.

$$L = 2.3026\,PV \log \frac{V}{V_1}, \qquad \textbf{(77.)} \text{ and}$$

$$L = 2.3026\,PV \log \frac{P_1}{P}, \qquad \textbf{(78.)} \text{ Art. } \textbf{1162.}$$

in which the letters have the same meaning as before, and *the larger volume or pressure is always in the numerator.*

Formulas **75, 76, 77,** and **78** will be easier to use if the pressure be taken in pounds per square inch, and $144 \times 2.3026 = 331.5744$ be substituted for 2.3026. As before, the volume must always be taken in cubic feet. Formulas **75** and **76** then become

$$L = 331.5744\,p\,V \log \frac{V_1}{V}, \qquad \textbf{(79.)}$$

$$L = 331.5744\,p\,V \log \frac{p}{p_1}, \qquad \textbf{(80.)} \text{ Art. } \textbf{1163.}$$

in which p = pressure in pounds per square inch.

RELATION BETWEEN PRESSURES AND VOLUMES OF AIR DURING ADIABATIC EXPANSION OR COMPRESSION.

Let p and v = the volume and pressure during one stage;

p_1 and v_1 = the volume and pressure during the next stage;

p_2 and v_2 = the volume and pressure during the third stage, etc.;

then $p\,v^{1.41} = p_1\,v_1^{1.41} = p_2\,v_2^{1.41} = \text{etc.} \qquad \textbf{(81.)}$ Art. **1164.**

WORK DONE BY THE ADIABATIC EXPANSION OF AIR.

Let L = work in foot-pounds;
P = initial pressure in pounds per square foot;
P_1 = final pressure in pounds per square foot;
V = initial volume in cubic feet;
V_1 = final volume in cubic feet.

$$L = 2.44\,PV\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right], \qquad \textbf{(82.)}\ \text{Art. } \textbf{1165.}$$

$$L = 2.44\,PV\left[1 - \left(\frac{P_1}{P}\right)^{.29078}\right]. \qquad \textbf{(83.)}\ \text{Art. } \textbf{1166.}$$

Let p and p_1 be the initial and final pressures, respectively, in pounds per square inch, then,

$$L = 351.36\,p\,V\left[1 - \left(\frac{V}{V_1}\right)^{.41}\right], \qquad \textbf{(84.)}$$

$$L = 351.36\,p\,V\left[1 - \left(\frac{p_1}{p}\right)^{.29078}\right]. \qquad \textbf{(85.)}\ \text{Art. } \textbf{1167.}$$

The relation between the pressures and temperatures, and the volumes and temperatures, during the adiabatic expansion of air, are given by the following formulas:

$$\left(\frac{P}{P_1}\right)^{.29078} = \frac{T}{T_1}. \qquad \textbf{(86.)}\ \text{Art. } \textbf{1170.}$$

$$\left(\frac{V_1}{V}\right)^{.41} = \frac{T}{T_1}. \qquad \textbf{(87.)}\ \text{Art. } \textbf{1170.}$$

To obtain the area of the diagram representing the adiabatic compression of air, the following formula may be used which gives it directly when p and p_1 are the greater and lesser pressures, respectively, and V and V_1 their corresponding volumes:

$$\frac{pV - p_1V_1}{.41} = \text{area.} \qquad \textbf{(88.)}\ \text{Art. } \textbf{1172.}$$

EFFICIENCY OF A PERFECT HEAT ENGINE.

Let $c = \frac{P_1V_1}{T_1}$, and $r = \frac{V_2}{V_1}$, then, the efficiency of a perfect

heat engine $= \frac{2.3026\,c \log r_1\,(T_1 - T_2)}{2.3026\,c\,T_1 \log r_1} = E = \frac{T_1 - T_2}{T_1}$. **(89.)**
Art. **1182.**

That is, *for a perfect heat engine, operating through a reversible cycle process, the efficiency of the machine is the ratio of the difference of the absolute temperatures of the sources of heat and of cold to the absolute temperature of the source of heat.*

FORMULAS USED IN STEAM AND STEAM ENGINES.

RELATION BETWEEN THE PRESSURE AND TEMPERATURE OF SATURATED STEAM.

Let t = temperature, Fahrenheit;

T = absolute temperature = $t + 460°$;

p = absolute pressure (pressure above vacuum) in pounds per sq. in.

$$\operatorname{Log} p = 6.1007 - \frac{2,719.78}{T} - \frac{400,215}{T^2}. \quad \left\{ \begin{array}{c} \textbf{(90.)} \\ \text{Art. } \textbf{1197.} \end{array} \right.$$

TOTAL HEAT OF VAPORIZATION.

Let H = total heat of vaporization;

t = temperature;

$H = 1,081.94 + .305\,t$. **(91.)** Art. **1200.**

SPECIFIC VOLUME OF SATURATED STEAM.

$p\,V^{\frac{17}{16}} = 475$, **(92.)** Art. **1203.**

in which p is the pressure in pounds per square inch, and V the volume in cubic feet of a pound of steam at the given pressure.

WORK DONE IN THE CYLINDER OF A STEAM ENGINE.

Let P = the pressure on the piston in pounds per sq. ft.;

p = pressure on piston in pounds per sq. in.;

V = volume swept through by the piston;

W = work in foot-pounds.

Then, $W = P\,V = 144\,p\,V$. **(93.)** Art. **1212.**

RELATION BETWEEN CLEARANCE, CUT-OFF, AND NUMBER OF EXPANSIONS, OR RATIO OF EXPANSION.

Let e = the number of expansions;

i = the clearance, expressed as a per cent. of the stroke;

k = the real cut-off;
k_1 = the apparent cut-off;
r = the apparent number of expansions $= \frac{1}{k_1}$.

Then, $e = \frac{1}{k}$ and $k = \frac{1}{e}$. **(96.)** Art. **1254.**

$$k = \frac{k_1 + i}{1 + i}. \quad \textbf{(97.)} \text{ Art. } \textbf{1254.}$$

TO FIND THE AREA OF A THEORETICAL DIAGRAM AND THE WORK REPRESENTED BY THE DIAGRAM.

Let A = total area of diagram in square inches;
p_1 = initial pressure measured in inches;
V_1 = volume at cut-off measured in inches;
L = work in foot-pounds;
a = net area of diagram;
h = scale used to lay off pressures;
h_1 = scale used to lay off volumes.

$$A = 2.3026\, p_1 V_1 \log \frac{V_2}{V_1}. \quad \textbf{(94.)} \text{ Art. } \textbf{1215.}$$

$$L = 144\, a h h_1. \quad \textbf{(95.)} \text{ Art. } \textbf{1215.}$$

INDICATED HORSEPOWER AND MEAN EFFECTIVE PRESSURE.

Let P = the M. E. P. in lb. per sq. in.;
A = the area of piston in sq. in.;
L = the length of stroke in ft.
N = the No. of strokes per min.

Then, the work done per minute is $P L A N$ foot-pounds.

One horsepower = 33,000 foot-pounds per min.

Therefore, the indicated horsepower of the engine is found from the formula

$$\text{I. H. P.} = \frac{PLAN}{33{,}000}. \quad \textbf{(98.)} \text{ Art. } \textbf{1268.}$$

When the point of real cut-off, and the steam pressure at the beginning of the stroke, are known, the M. E. P. may be found approximately by the following formula:

$$\text{M. E. P.} = \frac{.9\,P(1 + 2.3 \log e)}{e} - .9\,p, \qquad (99.)$$

Art. **1269.**

in which P = absolute steam pressure = gauge pressure + 14.7 pounds;
e = ratio of expansion;
p = absolute back pressure.

p is usually taken as about 3 pounds for condensing engines, and 17 pounds for non-condensing engines.

RELATION BETWEEN LENGTH OF STROKE, NUMBER OF REVOLUTIONS, AND PISTON SPEED.

Let L = length of stroke in inches;
R = number of revolutions per minute;
S = piston speed in feet per minute.

$$\left.\begin{aligned} S &= \frac{LR}{6}. \\ L &= \frac{6S}{R}. \\ R &= \frac{6S}{L}. \end{aligned}\right\} \qquad (100.)$$

Art. **1270.**

RATIO OF EXPANSION IN A COMPOUND OR TRIPLE-EXPANSION ENGINE.

Let e = ratio of expansion in high-pressure cylinder;
E = total ratio of expansion;
v = volume of cylinder receiving steam from the boiler;
V = volume of cylinder exhausting into atmosphere or condenser.

Then, $$E = \frac{eV}{v}. \qquad (102.)$$

Art. **1298.**

RATIO BETWEEN THE CYLINDERS OF A COMPOUND ENGINE.

Letting the letters have the same meaning as in the last formula, then either of the following formulas may be used:

$$E = 2.72\frac{V}{v}, \text{ or } \frac{V}{v} = \frac{E}{2.72}. \qquad \textbf{(103.)} \text{ Art. } \textbf{1306.}$$

$$\frac{V}{v} = \sqrt{E}. \qquad \textbf{(104.)} \text{ Art. } \textbf{1306.}$$

AMOUNT OF WATER AND THE COOLING SURFACE FOR CONDENSERS.

Water Required by a Condenser.

Let t_1 = the temperature of departing condensing water;

t_2 = the temperature of entering condensing water;

t_3 = the temperature of the condensed steam upon leaving the condenser;

H = total heat of one pound of steam at the pressure of the exhaust;

W = the weight of water required per pound of steam condensed.

$$W = \frac{H - t_3 + 32}{t_1 - t_2}. \qquad \textbf{(105.)} \text{ Art. } \textbf{1323.}$$

Cooling Surface.

Let S = the required surface in square feet;

W = total weight of steam used per hour.

Then, $S = .0944\ W.$ **(106.)** Art. **1325.**

TO FIND THE WEIGHT OF A FLY-WHEEL.

Let V_1 = the greatest velocity of the crank-pin in ft. per sec.;

V_2 = the least velocity of the crank-pin in ft. per sec.;

V_0 = average velocity of crank-pin in ft. per sec.;

W = required weight of fly-wheel in pounds;

H = the number of foot-pounds per sq. in. of piston represented by the excess of crank effect over the resistance;

A = area of piston in square inches;

n = ratio between radius of fly-wheel and length of crank;

$E = \frac{V_1 - V_2}{V_0}$ = coefficient of unsteadiness.

$$W = \frac{A\,H\,g}{n^2\,E\,V_0^2}. \qquad \textbf{(107.)} \text{ Art. } \textbf{1328.}$$

FORMULAS USED IN STRENGTH OF MATERIALS.

UNIT STRESS, UNIT STRAIN, AND COEFFICIENT OF ELASTICITY.

Let P = the total stress in pounds;
A = area of cross-section in square inches;
S = unit stress in pounds per square inch;
l = length of body in inches;
e = elongation in inches;
s = unit strain;
E = coefficient of elasticity.

$$S = \frac{P}{A}, \text{ or } P = A\,S. \qquad \textbf{(108.)} \text{ Art. } \mathbf{1345.}$$

$$s = \frac{e}{l}, \text{ or } e = l s. \qquad \textbf{(109.)} \text{ Art. } \mathbf{1346.}$$

$$E = \frac{S}{s} = \frac{P}{A} \div \frac{e}{l} = \frac{P l}{A e}. \qquad \textbf{(110.)} \text{ Art. } \mathbf{1352.}$$

STRENGTH OF PIPES AND CYLINDERS.

Let d = inside diameter of pipe in inches;
l = length of pipe in inches;
p = pressure in pounds per square inch;
P = total pressure; then $P = p\,l\,d$;
t = thickness of pipe;
S = working strength of the material.

For longitudinal rupture

$$p\,l\,d = 2\,t\,l\,S, \text{ or}$$

$$p\,d = 2\,t\,S. \qquad \textbf{(111.)} \text{ Art. } \mathbf{1365.}$$

For transverse rupture

$$p\,d = 4\,t\,S. \qquad \textbf{(112.)} \text{ Art. } \mathbf{1366.}$$

Since, for longitudinal rupture, $p\,d = 2\,t\,S$, it is seen that a cylinder is twice as strong against transverse rupture as against longitudinal rupture.

For pipes and cylinders whose thickness is greater than $\frac{1}{10}$ of the radius, use the following formula, in which r = the inner radius, and the other letters have the same meaning as before.

$$p = \frac{S t}{r + t}. \qquad \textbf{(113.)} \text{ Art. } \textbf{1367.}$$

The following formula gives the collapsing pressure in lb. per sq. in. for wrought-iron pipe:

$$p = 9,600,000 \frac{t^{2.18}}{l d}. \qquad \textbf{(114.)} \text{ Art. } \textbf{1368.}$$

MOMENT OF INERTIA, RESISTING MOMENT, AND BENDING MOMENT OF BEAMS.

Let I = moment of inertia;
A = area of cross-section;
r = radius of gyration;
c = distance from neutral axis to outermost fiber;
S_4 = ultimate strength of flexure;
f = factor of safety;
M = bending moment.

$$I = A r^2. \qquad \textbf{(115.)} \text{ Art. } \textbf{1396.}$$

The resisting moment is given by the expression

$$\frac{S}{c} A r^2 = \frac{S}{c} I, \text{ or } S \frac{I}{c}.$$

For the bending moment

$$M = S_4 \frac{I}{c}, \qquad \textbf{(116.)} \text{ Art. } \textbf{1398.}$$

and, when a factor of safety is used,

$$M = \frac{S_4 I}{f c}. \qquad \textbf{(117.)} \text{ Art. } \textbf{1401.}$$

DEFLECTION OF A BEAM.

Let a = a constant depending on the manner of loading the beam and the condition of the ends;
s = the deflection;
E = the coefficient of elasticity;
l = the length of the beam in inches;
W = the total weight supported in pounds;
I = moment of inertia about the neutral axis.

$$s = a \frac{W l^3}{E I}. \qquad \textbf{(118.)} \text{ Art. } \textbf{1404.}$$

STRENGTH OF COLUMNS.

Let W = load on a column;
S_c = ultimate strength for compression;
A = area of section of column;
f = factor of safety;
l = length of column in inches;
g = a constant to be taken from table;
I = least moment of inertia of cross-section;
b = length of longer side of a rectangular column;
d = length of shorter side of a rectangular column, or the diameter of a circular column;
c = length of one side of a square column.

$$W = \frac{S_c A}{f\left(1 + \frac{A l^2}{g I}\right)}. \qquad \textbf{(119.)}\ \text{Art. } \mathbf{1411.}$$

$$c = \sqrt{\frac{Wf}{2S_c} + \sqrt{\frac{Wf}{S_c}\left(\frac{Wf}{4S_c} + \frac{12 l^2}{g}\right)}}. \qquad \left\{ \begin{array}{l} \textbf{(120.)} \\ \text{Art. } \mathbf{1413.} \end{array} \right.$$

For a circular column,

$$d = 1.4142\sqrt{\frac{.3183\,Wf}{S_c} + \sqrt{\frac{.3183\,Wf}{S_c}\left(\frac{.3183\,Wf}{S_c} + \frac{16 l^2}{g}\right)}}.$$

(121.) Art. **1413.**

For a rectangular column, assume d, then,

$$b = \frac{Wf\left(1 + \frac{12 l^2}{d^2 g}\right)}{d S_c}. \qquad \textbf{(122.)}\ \text{Art. } \mathbf{1413.}$$

STRENGTH OF SHAFTS.

Let d = diameter of a round shaft, or side of a square shaft, in inches;
c = a constant taken from Table 31;
c_1 = a constant taken from Table 31;
P = a force applied at the end of a lever arm in pounds;
r = length of lever arm in inches;

H = horsepower transmitted by shaft;
N = number of revolutions per minute;
k = a constant taken from Table 32;
k_1 = a constant taken from Table 32;
q = a constant taken from Table 32;
q_1 = a constant taken from Table 32.

For all solid shafts below 11 inches in diameter use the formula

$$d = c\sqrt[4]{Pr} = c_1\sqrt[4]{\frac{H}{N}}. \qquad \textbf{(123.)} \text{ Art. } \mathbf{1415.}$$

If the diameter of a wrought-iron shaft is greater than 12.4″, of a cast-iron shaft greater than 10.3″, or of a steel shaft greater than 13.6″, the following formula should be used:

$$d = k\sqrt[3]{Pr} = k_1\sqrt[3]{\frac{H}{N}}. \qquad \textbf{(124.)} \text{ Art. } \mathbf{1416.}$$

If the shaft is hollow (round) either of the two following formulas may be used:

$$P = q\left(\frac{d_1^4 - d_2^4}{d_1 r}\right), \qquad \textbf{(125.)} \text{ Art. } \mathbf{1416.}$$

$$\text{or } H = q_1 N\left(\frac{d_1^4 - d_2^4}{d_1}\right). \qquad \textbf{(126.)} \text{ Art. } \mathbf{1416.}$$

SHAFTS SUBJECTED TO COMBINED BENDING AND TWISTING.

Let B = bending moment;
T = twisting moment;
T_1 = the twisting moment which would have the same effect as B and T acting together.

$$\text{Then, } T_1 = B + \sqrt{B^2 + T^2}. \qquad \textbf{(255.)} \text{ Art. } \mathbf{1991.}$$

The twisting moment T_1 is called the **ideal twisting moment,** and should be used in place of Pr in formulas **123** and **124** to determine the diameter of a shaft subjected to combined bending and twisting.

VALUES OF c AND c_1 TO BE USED IN FORMULA 123.

Material.	c		c_1	
	Round.	Square.	Round.	Square.
Wrought Iron	.31	.272	4.92	4.31
Cast Iron	.353	.309	5.59	4.89
Steel	.297	.26	4.7	4.11

VALUES OF k, k_1, q, AND q_1 TO BE USED IN FORMULAS 124, 125, AND 126.

Material.	k	k_1	q	q_1
Wrought Iron	.0909	3.62	1,335	.0212
Cast Iron	.1145	4.56	669	.0106
Steel	.0828	3.3	1,767	.028

STRENGTH OF ROPES AND CHAINS.

Let $P =$ working or safe load in pounds;
$C =$ circumference of rope in inches;
$d =$ diameter of the link of a chain in inches.

For manila ropes, hemp ropes, or tarred hemp ropes,

$$P = 100\,C^2. \qquad \textbf{(127.)}\ \text{Art. } \mathbf{1417.}$$

For iron wire rope of 7 strands, 19 wires to the strand,

$$P = 600\,C^2. \qquad \textbf{(128.)}\ \text{Art. } \mathbf{1418.}$$

For the best steel wire rope, 7 strands, 19 wires to the strand,

$$P = 1{,}000\,C^2. \qquad \textbf{(129.)}\ \text{Art. } \mathbf{1418.}$$

For open-link chains made from a good quality of wrought iron,

$$P = 12{,}000\,d^2, \qquad \textbf{(130.)}\ \text{Art. } \mathbf{1421.}$$

and for stud-link chains,

$$P = 18{,}000\,d^2. \qquad \textbf{(131.)}\ \text{Art. } \mathbf{1421.}$$

FORMULAS USED IN APPLIED MECHANICS.

RELATION BETWEEN THE DIAMETERS AND NUMBER OF REVOLUTIONS OF PULLEYS.

Let D_1 = diameter of driving pulley;
N_1 = number of revolutions of driving pulley;
F_1 = diameter of driven pulley;
n_1 = number of revolutions of driven pulley.

$$\frac{N_1}{n_1} = \frac{F_1}{D_1}. \qquad \textbf{(136.)} \text{ Art. } \textbf{1477.}$$

$$D_1 N_1 = F_1 n_1.$$

HORSEPOWER OF BELTS.

Let H = horsepower to be transmitted;
W = width of belt in inches;
S = belt speed in feet per minute.

FOR SINGLE BELTS.

$$H = \frac{WS}{900}. \qquad \textbf{(138.)} \text{ Art. } \textbf{1488.}$$

$$W = \frac{900\,H}{S}. \qquad \textbf{(139.)} \text{ Art. } \textbf{1488.}$$

$$S = \frac{900\,H}{W}. \qquad \textbf{(140.)} \text{ Art. } \textbf{1488.}$$

FOR DOUBLE BELTS.

$$H = \frac{WS}{630}. \qquad \textbf{(141.)} \text{ Art. } \textbf{1489.}$$

$$W = \frac{630\,H}{S}. \qquad \textbf{(142.)} \text{ Art. } \textbf{1489.}$$

$$S = \frac{630\,H}{W}. \qquad \textbf{(143.)} \text{ Art. } \textbf{1489.}$$

TEETH OF GEAR-WHEELS.

Let P = diametral pitch;
D = diameter of pitch circle of large gear;
d = diameter of pitch circle of small gear;
$O\,D$ = outside diameter;

C = circular pitch;
N = number of teeth;
A = distance between centers of wheels;
V = velocity; i. e., revolutions per minute of large gear.
v = velocity; i. e., revolutions per minute of small gear.

$$P=\frac{3.1416}{C}, \text{ and} \qquad \textbf{(149.)}$$

$$C=\frac{3.1416}{P}. \qquad \textbf{(150.)} \text{ Art. } \textbf{1556.}$$

$$N=DP. \qquad \textbf{(151.)} \text{ Art. } \textbf{1557.}$$

$$OD=\frac{N}{P}+2\times\frac{1}{P}=\frac{N+2}{P}. \qquad \textbf{(152.)} \text{ Art. } \textbf{1557.}$$

$$D=\frac{C\times N}{3.1416}. \qquad \textbf{(153.)} \text{ Art. } \textbf{1557.}$$

Given the distance between the centers of two gears and their velocities, the formulas for their diameters are

$$d=\frac{2AV}{V+v}; \qquad \textbf{(154.)}$$

$$\text{and } D=\frac{2Av}{v+V}. \qquad \textbf{(155.)} \text{ Art. } \textbf{1557.}$$

THE PRONY BRAKE.

Let $H.\ P.$ = number of horsepower absorbed;
R = length of lever arm in feet;
W = weight in scale pan;
N = number of revolutions per minute;
r = length of lever arm in inches.

$$H.\ P.=\frac{2\times 3.1416\times R\times W\times N}{33{,}000}, \text{ or}$$

$$H.\ P.=.0001904\ R\ W N. \qquad \textbf{(156.)} \text{ Art. } \textbf{1593.}$$

$$H.\ P.=.00001586\ r\ W N. \qquad \textbf{(157.)} \text{ Art. } \textbf{1595.}$$

FORMULAS USED IN STEAM BOILERS.

STRENGTH OF BOILER SHELLS AND TUBES.

Let p = the pressure of steam in pounds per square inch;
t = thickness of plate in shell or tube;
d = diameter of shell or tube;
S_1 = ultimate tensile strength of material;
f = factor of safety;
y = efficiency of joint;
l = length of a tube subjected to external pressure

For tubes without riveted joints,

$$t = \frac{fpd}{2S_1}. \qquad \textbf{(164.)} \text{ Art. } \textbf{1730.}$$

$$p = \frac{2tS_1}{fd}. \qquad \textbf{(166.)} \text{ Art. } \textbf{1731.}$$

For tubes or shells with riveted joints,

$$t = \frac{fpd}{2S_1y}, \qquad \textbf{(165.)}$$

$$p = \frac{2tS_1y}{fd}. \qquad \textbf{(167.)} \text{ Art. } \textbf{1731.}$$

For tubes subjected to external pressure,

$$p = 1{,}600{,}000\frac{t^2}{ld} \qquad \textbf{(168.)} \text{ Art. } \textbf{1732.}$$

RIVETED JOINTS.

Let d = diameter of rivet hole in inches;
h = pitch of rivets in inches;
t = thickness of plate in inches.

$$h = \left(1 + \frac{7d}{8t}\right)d. \qquad \textbf{(169.)} \text{ Art. } \textbf{1742.}$$

For double-riveted lap joints or single butt joints, formula **169** becomes

$$h = \left(1 + \frac{7d}{4t}\right)d. \qquad \textbf{(170.)} \text{ Art. } \textbf{1742.}$$

For double-riveted double butt joints, formula **170** becomes

$$h = \left(1 + \frac{7d}{2t}\right)d. \qquad \textbf{(171.)} \text{ Art. } \textbf{1742.}$$

The *d* in the above formulas refers to the diameter of the rivet hole, or of the rivet after being headed.

The diameter of the cold rivet is $\frac{1}{16}''$ less.

For single and double-riveted lap joints and butt joints, with one cover plate, the diameter of the rivet is

$$d = t + \tfrac{3}{8}'' \qquad \textbf{(172.)} \text{ Art. } \textbf{1742.}$$

for values of *t* between $\frac{1}{4}''$ and $\frac{3}{4}''$. For butt joints with double cover plates, the diameter of rivet may be taken about $1\frac{1}{4}$ times the thickness of the plate.

FLAT SURFACES.

The strength of a circular plate supported at the edges, as, for example, the head of a boiler shell, is given by the following formula:

$$S = \frac{2r^2p}{3t^2}, \qquad \textbf{(174.)} \text{ Art. } \textbf{1751.}$$

$$t = r\sqrt{\frac{2p}{3S}}, \qquad \textbf{(175.)} \text{ Art. } \textbf{1751.}$$

where S represents the safe tensile strength of the material, r the radius of the plate in inches, t the thickness of the plate in inches, and p the pressure in lb. per sq. in.

The following formula gives the strength of a square unstayed flat plate fixed along the edge (the length of side is a):

$$\left.\begin{aligned} S &= \frac{a^2p}{4t^2}, \\ \text{or } t &= \frac{a}{2}\sqrt{\frac{p}{S}}. \end{aligned}\right\} \qquad \textbf{(176.)} \text{ Art. } \textbf{1752.}$$

BOILER STAYS.

Let A = area in sq. in. of plate supported by one stay;

d = smallest diameter of stay in inches;

p = pressure of steam in pounds per sq. in.;

T = safe stress per sq. in. allowed on stay;
a = pitch for stays arranged at equal distances apart, both horizontally and vertically.

$$\left.\begin{aligned} \tfrac{1}{4}\pi d^2\, T &= Ap, \\ \text{or } d &= 1.13\sqrt{\frac{Ap}{T}}. \end{aligned}\right\} \qquad \textbf{(177.)}\ \text{Art. } \mathbf{1756.}$$

$$d = 1.13\, a\sqrt{\frac{p}{T}}, \qquad \textbf{(178.)}\ \text{Art. } \mathbf{1756.}$$

$$\text{and } a = .885\, d\sqrt{\frac{T}{p}}. \qquad \textbf{(179.)}\ \text{Art. } \mathbf{1756.}$$

The values of T used in formulas **177, 178,** and **179** may be taken as follows:

For copper screw staybolts.......................$T = 4{,}000$

Iron Stays.—For screw stays, and for other stays not exceeding 1¼ inches in diameter, and for all stays which are welded...........$T = 6{,}000$

For unwelded stays above 1½ inches effective diameter...................................$T = 7{,}500$

Steel Stays.—For screw stays, and for other stays not exceeding 1½ inches effective diameter......................... ....$T = 8{,}000$

For stays above 1½ inches effective diameter.....$T = 9{,}000$

No steel stays are to be welded.

The size of a diagonal stay may be calculated from formula **178** by substituting $\frac{p}{\cos B}$ for p, B being the angle between the stay and the shell. Formula **178** thus becomes

$$d = 1.13\sqrt{\frac{Ap}{T\cos B}}. \qquad \textbf{(180.)}\ \text{Art. } \mathbf{1758.}$$

For the strength of a girder stay:

Let l = length of girder in inches;
h = distance between adjacent girders in inches;
p = steam pressure in lb. per sq. in.;
b = breadth of girder in inches = ½ d;
d = depth of girder in inches.

$$d=\sqrt[3]{\frac{p\,h\,l^2}{6,000}}.\qquad \textbf{(181.)}\ \text{Art. }\mathbf{1761.}$$

$$p=\frac{6,000\,d^3}{h\,l^2}.\qquad \textbf{(182.)}\ \text{Art. }\mathbf{1761.}$$

For the safe pressure allowable on a flat surface strengthened by screw staybolts:

Let p = pressure in lb. per sq. in.;
h = pitch of staybolts;
t = thickness of plate;
k = a constant.

$$\text{Then, } p=k\frac{t^2}{h^2},\qquad \textbf{(183.)}$$

$$\text{and } t=h\sqrt{\frac{p}{k}}.\qquad \textbf{(184.)}\ \text{Art. }\mathbf{1763.}$$

The constant k has the following values:
For iron plates and iron bolts .... k = 24,000
For low steel plates and iron bolts .... k = 25,000
For low steel plates and low steel bolts .... k = 28,000
For iron plates and iron bolts with nuts .... k = 40,000
For copper plates and iron bolts .... k = 14,500

SAFETY VALVES.

For a dead-weight safety valve:

Let A = area of opening in valve seat in sq. in.;
p = pressure at which the valve is to blow off (in lb per sq. in.);
W = dead weight in pounds (including weight of valve).

$$W=A\,p.\qquad \text{Art. }\mathbf{1774.}$$

For a weighted-lever safety valve:

Let d = FB = distance from fulcrum to weight in inches;
c = FG = distance from fulcrum to center of gravity of lever in inches;

a $= F\,V =$ distance from fulcrum to center of valve in inches;
A $=$ area of orifice beneath bottom of valve in inches;
W $=$ weight of weight P in pounds;
W_1 $=$ weight of valve and stem in pounds;
W_2 $=$ weight of lever in pounds;
P $=$ blow-off pressure in pounds per sq. in.;
$W_3 = W_1 + W_2 =$ combined weight of valve and lever;
b $=$ distance from fulcrum of center of gravity of W_3.

$$W = \frac{a\,(p\,A - W_1) - W_2\,c}{d}, \qquad \textbf{(185.)}\ \text{Art. }\mathbf{1776.}$$

and $$d = \frac{a\,(p\,A - W_1) - W_2\,c}{W}. \qquad \textbf{(186.)}\ \text{Art. }\mathbf{1776.}$$

Also $$d = \frac{p\,A\,a - W_3\,b}{W}. \qquad \textbf{(187.)}\ \text{Art. }\mathbf{1778.}$$

To find the area of a safety valve:

Let $a =$ area of the valve opening in square inches;
$w =$ maximum weight of steam generated in pounds per hour;
$p =$ gauge pressure in pounds per square inch.

$$a = \frac{.5\,w}{p + 10}. \qquad \textbf{(188.)}\ \text{Art. }\mathbf{1781.}$$

DIMENSIONS OF CHIMNEYS.

Let H $=$ height of chimney in feet;
$H.\,P.$ $=$ horsepower of boiler or boilers;
A $=$ actual area of chimney in square feet;
E $=$ effective area of chimney in square feet;
S $=$ side of square chimney in inches;
d $=$ diameter of round chimney in inches;
p $=$ pressure of draft in inches of water;
T_a $=$ absolute temperature of outside air;
T_c $=$ absolute temperature of gas in chimney.

$$p = H\left(\frac{7.6}{T_a} - \frac{7.9}{T_c}\right). \qquad \textbf{(192.)}\ \text{Art. }\mathbf{1825.}$$

$$H = \frac{p}{\left(\dfrac{7.6}{T_a} - \dfrac{7.9}{T_c}\right)}. \qquad \textbf{(193.)}\ \text{Art. }\mathbf{1826.}$$

$$E = \frac{.3\,H.\,P.}{\sqrt{H}} = A - .6\sqrt{A}. \qquad \textbf{(194.)}\ \text{Art. } \textbf{1827.}$$

$$H.\,P. = 3.33\,E\sqrt{H}. \qquad \textbf{(195.)}\ \text{Art. } \textbf{1827.}$$

$$S = 12\sqrt{E} + 4. \qquad \textbf{(196.)}\ \text{Art. } \textbf{1827.}$$

$$d = 13.54\sqrt{E} + 4. \qquad \textbf{(197.)}\ \text{Art. } \textbf{1827.}$$

The *maximum* rates of combustion attainable under natural draft are given by the following formulas:

Let F = weight in pounds of coal per hour per square foot of grate area;

H = height in feet of chimney or stack.

Then, for anthracite coal burned under the most favorable conditions,

$$F = 2\sqrt{H} - 1, \qquad \textbf{(198.)}\ \text{Art. } \textbf{1829.}$$

and under ordinary conditions,

$$F = 1.5\sqrt{H} - 1. \qquad \textbf{(199.)}\ \text{Art. } \textbf{1829.}$$

For best semianthracite and bituminous coals,

$$F = 2.25\sqrt{H}, \qquad \textbf{(200.)}\ \text{Art. } \textbf{1829.}$$

and for less valuable soft coals,

$$F = 3\sqrt{H}. \qquad \textbf{(201.)}\ \text{Art. } \textbf{1829.}$$

GRATE AREA.

Let G = area of grate in sq. ft.;

F = rate of combustion in lb. per sq. ft. of grate surface per hour;

W = weight of steam generated per hour by boiler or boilers;

e = evaporation in pounds of water per pound of coal per hour.

Then.

$$G = \frac{W}{Fe}. \qquad \textbf{(202.)}\ \text{Art. } \textbf{1830.}$$

In using this formula care must be taken that W and e are taken at the same pressure and temperature.

TO REDUCE THE ACTUAL EVAPORATION TO AN EQUIVALENT EVAPORATION.

Let W = the actual evaporation;

H = total heat of steam above 32° at pressure of actual evaporation;

t = observed temperature of feed-water;

W' = equivalent evaporation from and at 212°.

Then, $W(H-t+32)=966.1\,W'$,

$$\text{or } W'=\frac{W(H-t+32)}{966.1}. \qquad \textbf{(203.)} \text{ Art. } \textbf{1838.}$$

The quantity $\left(\frac{H-t+32}{966.1}\right)$ which changes actual evaporation to equivalent evaporation from and at 212° is called the **factor of evaporation.**

TO FIND THE QUALITY OF STEAM BY MEANS OF THE BARREL CALORIMETER.

Let W = original weight of water in barrel;

w = weight of steam and water condensed in the cold water;

l = latent heat of steam at observed pressure;

t = temperature of steam at observed pressure;

t_1 = initial temperature of water;

t_2 = final temperature of water;

x = the portion of w which is dry steam;

Q = quality of steam; that is, the percentage of the mixture evaporated, which is pure dry steam.

$$Q=\frac{x}{w}=\frac{1}{l}\left[\frac{W}{w}(t_2-t_1)-(t-t_2)\right]. \qquad \textbf{(204.)} \text{ Art. } \textbf{1853.}$$

TO FIND THE QUALITY OF STEAM BY MEANS OF THE SEPARATOR CALORIMETER.

Let W = weight of dry steam discharged from the instrument;

w = weight of water drawn from separator;

R = water condensed by radiation:

Q = quality of steam.

$$Q=\frac{W+R}{W+R+w}. \qquad \textbf{(205.)} \text{ Art. } \textbf{1855.}$$

FORMULAS USED IN MACHINE DESIGN.

PROPORTIONS OF BOLT HEADS, NUTS, AND WASHERS.

$d =$ diameter of bolt.

Diameter of nut or head across flats:

$$\left.\begin{array}{l} D = 1\tfrac{1}{2}d + \tfrac{1}{8}'' \text{ for rough work.} \\ D = 1\tfrac{1}{2}d + \tfrac{1}{16}'' \text{ for finished work.} \end{array}\right\} \quad \textbf{(220.)} \text{ Art. } \textbf{1937.}$$

Height of nut:

$$\left.\begin{array}{l} h = d \text{ for rough work.} \\ h = d - \tfrac{1}{16}'' \text{ for finished work.} \end{array}\right\} \quad \textbf{(221.)} \text{ Art. } \textbf{1937.}$$

Thickness of washer, $t = .15\,d$. **(222.)** Art. **1937.**

Diameter of washer, $D_2 = 1\tfrac{1}{8}\,D_1$. **(223.)** Art. **1937.**

The above proportions for diameters D hold for both hexagonal and square nuts. The diameter across corners D_1 is as follows:

For hexagonal nuts:

$$D_1 = \left\{\begin{array}{l} 1.73\,d + .14'' \text{ for rough nuts.} \\ 1.73\,d + .07'' \text{ for finished nuts.} \end{array}\right. \quad \textbf{(224.)} \text{ Art. } \textbf{1937.}$$

For square nuts:

$$D_1 = \left\{\begin{array}{l} 2.12\,d + .18'' \text{ for rough nuts.} \\ 2.12\,d + .09'' \text{ for finished nuts.} \end{array}\right. \quad \textbf{(225.)} \text{ Art. } \textbf{1937.}$$

Height of head:

$$\left.\begin{array}{l} h' = \tfrac{3}{4}d + \tfrac{1}{16}'' \text{ for rough bolts.} \\ h' = d - \tfrac{1}{16}'' \text{ for finished bolts.} \end{array}\right\} \quad \textbf{(226.)} \text{ Art. } \textbf{1937.}$$

PROPORTIONS OF KEYS.

Let b = width of key in inches;
t = thickness of key in inches;
l = length of key in inches;
N = number of revolutions per minute;
H = horsepower transmitted by a pulley.

$$\left.\begin{array}{l} b = \dfrac{d}{7} + \dfrac{1}{4}''. \\ t = \dfrac{d}{12} + \dfrac{3}{16}''. \end{array}\right\} \text{For driven pulleys.} \quad \textbf{(234.)} \text{ Art. } \textbf{1965.}$$

$$\left.\begin{array}{l} b = .2d + .16''. \\ t = .1d + .16''. \end{array}\right\} \text{For driving pulleys.} \qquad \textbf{(235.)}\ \text{Art. } \textbf{1965.}$$

$$\text{When } d \text{ is less than } 1\tfrac{3}{16} \text{ in.,} \quad \left.\begin{array}{l} b = \dfrac{d}{3}. \\ t = \dfrac{d}{5}. \end{array}\right\} \qquad \textbf{(236.)}\ \text{Art. } \textbf{1965.}$$

When the pulley transmits but a small part of the power of a shaft, use the formula $d = 5\sqrt[3]{\frac{H}{N}}$ from which to obtain the value of d to use in formula **236.**

For sliding feather keys,

$$\left.\begin{array}{l} b = \dfrac{d}{8} + \dfrac{1'}{16}. \\ t = \dfrac{3d}{16} + \dfrac{1'}{16}. \end{array}\right\} \qquad \textbf{(237.)}\ \text{Art. } \textbf{1965.}$$

PROPORTIONS AND STRENGTH OF GEAR-WHEELS.

Let p = pressure acting at pitch line;
b = breadth of tooth;
n = number of revolutions per minute;
H = horsepower transmitted;
D = diameter of gear in inches;
C = the circular pitch;
P = diametral pitch;
S = allowable working stress in tooth;
v = velocity of a point in the pitch circle in feet per minute;
c = thickness of rim.

$$bC = 16.8\frac{p}{S}. \qquad \textbf{(266.)}\ \text{Art. } \textbf{2024.}$$

If v is greater than 100 feet per minute make

$$S = \frac{9,600,000}{v + 2,160}. \qquad \textbf{(267.)}\ \text{Art. } \textbf{2024.}$$

To insure durability make

$$\frac{pn}{b} = 28,000, \text{ or } b = 4\tfrac{1}{2}\frac{H}{D}. \qquad \textbf{(268.)}\ \text{Art. } \textbf{2025.}$$

To calculate the dimensions of the arms·

Let z = number of arms in wheel;
a = width of arm, measured at center of wheel;
x = thickness of arm;
S_1 = allowable safe stress in arm;
R = radius of wheel in inches;
p = pressure between gears at pitch line.

For cross-shaped, **T**, or **I** sections:

$$\left.\begin{aligned} a &= \sqrt{\frac{Rb}{z}}. \\ x &= \tfrac{1}{2}C. \end{aligned}\right\} \qquad \textbf{(270.)}\ \text{Art. } \textbf{2030.}$$

For oval or elliptical sections:

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{bCR}{z}}. \\ a &= 1.75\sqrt[3]{\frac{bR}{Pz.}} \end{aligned}\right\} \qquad \textbf{(271.)}\ \text{Art. } \textbf{2030.}$$

$$x = \frac{a}{2}.$$

For thickness of rim:

$c = \frac{1}{2}\ C$ for large gears and coarse pitch.
$c = .4\ C \times \frac{1}{8}'$ for gears with small pitch.

PROPORTIONS OF BEVEL GEARS.

Denote the mean circular pitch by C_m.

Then, $$b\,C_m = 16.8\frac{p}{S}. \qquad \textbf{(266}a.\textbf{)}\ \text{Art. } \textbf{2033.}$$

$$c = .4\,C_m + \tfrac{1}{8}' = \frac{5}{4P} + \tfrac{1}{8}'. \qquad \textbf{(269}a.\textbf{)}\ \text{Art. } \textbf{2033.}$$

$$\left.\begin{aligned} a &= \sqrt{\frac{Rb}{z}}. \\ x &= \tfrac{1}{2}C_m. \end{aligned}\right\} \qquad \textbf{(270}a.\textbf{)}\ \text{Art. } \textbf{2033.}$$

$$\left.\begin{aligned} a &= 1.2\sqrt[3]{\frac{bC_mR}{z}}. \\ a &= 1.75\sqrt[3]{\frac{bR}{P_m z}}. \end{aligned}\right\} \qquad \textbf{(271}a.\textbf{)}\ \text{Art. } \textbf{2033.}$$

RULES AND FORMULAS USED IN DYNAMOS AND MOTORS.

DIRECTION OF LINES OF FORCE AROUND A CONDUCTOR.

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch. Art.* **2157.**

TO DETERMINE THE POLARITY OF A SOLENOID.

Rule.—*In looking at the end of the helix, if it is so wound that the current circulates around the helix in the direction of the hands of a watch, that end will be a south pole; if in the other direction it will be a north pole. Art.* **2160.**

RESISTANCE OF CONDUCTORS.

Let r_1 = the original resistance of a conductor;
r_2 = the changed resistance:
l_1 = the original length;
l_2 = the changed length;
a_1 = the original sectional area;
a_2 = the changed sectional area;
D = the original diameter;
d = the changed diameter;
k = temperature coefficient;
t = rise or fall in temperature, degrees Fahrenheit.

For a change in the length of a conductor:

$$r_2 = \frac{r_1 l_2}{l_1}. \qquad \textbf{(306.)} \quad \text{Art. } \textbf{2171.}$$

For a change in the sectional area of a conductor:

$$r_2 = \frac{r_1 a_1}{a_2}. \qquad \textbf{(307.)} \quad \text{Art. } \textbf{2172.}$$

For a change in the diameter of a conductor:

$$r_2 = \frac{r_1 D^2}{d^2}. \qquad \textbf{(308.)} \quad \text{Art. } \textbf{2173.}$$

For a rise in the temperature of a conductor:

$$r_2 = r_1 (1 + t k). \qquad \textbf{(309.)} \text{ Art. } \mathbf{2177.}$$

For a fall in the temperature of a conductor:

$$r_2 = \frac{r_1}{1 + t k}. \qquad \textbf{(310.)} \text{ Art. } \mathbf{2178.}$$

RESISTANCES AND TEMPERATURE COEFFICIENTS OF DIFFERENT METALS.

Name of Metal.	Resistance, Microhms per Cu. In.	Relative Resistance.	Temperature Coefficient.
Silver, annealed	.5921	1.	.002094
Copper, annealed	.6292	1.063	.002155
Silver, hard-drawn	.6433	1.086	.002094
Copper, hard-drawn	.6433	1.086	.002155
Gold, annealed	.8102	1.369	.002028
Gold, hard-drawn	.8247	1.393	.002028
Aluminum, annealed	1.1470	1.935	....
Zinc, pressed	2.215	3.741	.002028
Platinum, annealed.	3.565	6.022	....
Iron, annealed	3.825	6.460	....
Nickel, annealed	4.907	8.285	....
Tin, pressed	5.202	8.784	.002028
Lead, pressed	7.728	13.05	.002150
German silver	8.240	13.92	.000244
Antimony, pressed	13.98	23.60	.002161
Mercury	37.15	62.73	.000400
Bismuth, pressed	51.65	87.23	.001967

CURRENT STRENGTH, ELECTROMOTIVE FORCE, AND RESISTANCE.

Let C = strength of current flowing in a closed circuit;
E = electromotive force;
R = resistance.

$$C = \frac{E}{R}. \qquad \textbf{(311.)} \text{ Art. } \mathbf{2192.}$$

$R = \frac{E}{C}$. **(312.)** Art. **2193.**

$E = C R$. **(313.)** Art. **2194.**

TO FIND THE AVAILABLE ELECTROMOTIVE FORCE IN A CELL.

Let E = the total generated E. M. F.;
E' = *available* E. M. F. when the circuit is closed;
C = the current flowing when the circuit is closed;
r_i = the internal resistance of the cell.

$E' = E - C r_i$. **(314.)** Art. **2198.**

THE CURRENT AND RESISTANCE IN BRANCHES OF DIVIDED CONDUCTORS.

Let r_1 = resistance of first branch;
r_2 = resistance of second branch;
r_3 = resistance of third branch;
c_1 = current in first branch;
c_2 = current in second branch;
C = sum of the currents in the two branches;
R' = joint resistance of two branches in parallel;
R''' = joint resistance of three branches in parallel.

$c_1 = \frac{C r_2}{r_1 + r_2}$. **(315.)** Art. **2200.**

$c_2 = \frac{C r_1}{r_1 + r_2}$. **(316.)** Art. **2200.**

$R'' = \frac{r_1 r_2}{r_1 + r_2}$. **(317.)** Art. **2202.**

$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$. **(318.)** Art. **2203.**

ELECTRICAL QUANTITY.

Let Q = quantity of electricity in coulombs;
C = current strength in amperes;
t = time in seconds;

$Q = C t$. **(319.)** Art. **2207.**

ELECTRICAL WORK AND POWER.

J = electrical work in joules;
$F\text{-}P$ = work in foot-pounds;
Q = quantity of electricity in coulombs;
C = current in amperes;
t = time in seconds during which the current flows;
E = potential, or E. M. F., of circuit;
R = resistance of circuit;
W = power in watts;
$H. P.$ = horsepower.

$$J = CEt. \qquad (320.)$$ Art. **2209.**

$$J = C^2Rt. \qquad (321.)$$ Art. **2209.**

$$J = \frac{E^2t}{R}. \qquad (322.)$$ Art. **2209.**

$$F\text{-}P = .7373J. \qquad (323.)$$ Art. **2210.**

$$W = CE. \qquad (324.)$$ Art. **2211.**

$$W = C^2R. \qquad (325.)$$ Art. **2211.**

$$W = \frac{E^2}{R}. \qquad (326.)$$ Art. **2211.**

$$H. P. = \frac{W}{746}. \qquad (327.)$$ Art. **2212.**

$$W = H. P. \times 746. \qquad (328.)$$ Art. **2213.**

TO DETERMINE THE DIRECTION OF THE CURRENT GENERATED IN A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the right hand so that each will be perpendicular to the other two; if the forefinger points in the direction of the lines of force, and the thumb points in the direction towards which the conductor is moving, then the middle finger will point in the direction towards which the current generated in the conductor tends to flow.* Art. **2221.**

DETERMINATION OF ELECTROMOTIVE FORCE.

Let E = maximum electromotive force obtained at the brushes;

N = total number of lines of force passing from north pole through the core to the south pole;

S = number of outside wires on the periphery through which the current flows *in series;*

n = number of complete revolutions per second of core.

$$E = \frac{2NSn}{10^8}. \qquad \textbf{(329.)} \quad \text{Art. } \textbf{2236.}$$

TO DETERMINE THE DIRECTION OF MOTION IMPARTED TO A CONDUCTOR.

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two; if the forefinger points in the direction of the lines of force, and the middle finger points in the direction towards which the current flows, then the thumb will point in the direction of movement imparted to the conductor.* Art. **2239.**

EFFICIENCY OF A DYNAMO.

Let I = input of a dynamo;

O = output;

E = efficiency, per cent.

$$\mathsf{E} = \frac{100 \times O}{I}. \qquad \textbf{(330.)} \quad \text{Art. } \textbf{2273.}$$

PER CENT. LOSS IN A DYNAMO.

Let L = per cent. loss;

I = input;

O = output.

$$L = \frac{100\,(I - O)}{I}. \qquad \textbf{(331.)} \quad \text{Art. } \textbf{2274.}$$

When the output and efficiency are given:

$$I = \frac{100 \times O}{\mathsf{E}}. \qquad \textbf{(332.)} \quad \text{Art. } \textbf{2275.}$$

When the input and efficiency are given:

$$O = \frac{I E}{100}. \qquad \textbf{(333.)} \quad \text{Art. } \textbf{2276.}$$

HORSEPOWER, TORQUE, AND NUMBER OF REVOLUTIONS OF MOTORS.

Let $H.\ P.$ = horsepower.
T = torque;
S = number of revolutions per minute.

$$H.\ P. = .0001904\, T S. \qquad \textbf{(334.)} \quad \text{Art. } \textbf{2346.}$$

$$S = \frac{H.\ P.}{.0001904\, T}. \qquad \textbf{(335.)} \quad \text{Art. } \textbf{2346.}$$

$$T = \frac{H.\ P.}{.0001904\, S}. \qquad \textbf{(336.)} \quad \text{Art. } \textbf{2346.}$$

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